Computing with Words in Linguistic Decision Making: Analysis of Linguistic Computing Models

Luis Martínez
School of Computing
University of Jaén
Jaén, Spain 23071
Email: luis.martinez@ujaen.es

***Abstract*—Decision Making is a core area in different fields in the real world. This plenary lecture focuses mainly on those problems dealing with vague and uncertain information, that often is based on perceptions. In such problems the linguistic information is a very helpful and flexible tool to model such a type of information but it implies the accomplishment of processes of computing with words. In the literature there exist different linguistic computing models to deal with linguistic information. This contribution reviews, analyzes and discusses different features of computing models in linguistic decision making, to verify if they can be branded as computing with words models.**

I. Introduction

Decision making is inherent to mankind, as human beings daily face situations in which they should choose among different alternatives. It can be seen as a process composed of different phases such as information gathering, analysis and selection based on different mental and reasoning processes that led to choose a *suitable* alternative among a set of possible alternatives [3], [7], [17].

Remarkably, decision making is a core area in a wide range of disciplines such as engineering, psychology, operations research, artificial intelligence, etc. A basic scheme of a decision making process [29], consists of two main phases (see graphically Fig 1): (i) An aggregation process of the information gathered and (ii) an exploitation process that choose the best alternatives from the aggregated information.

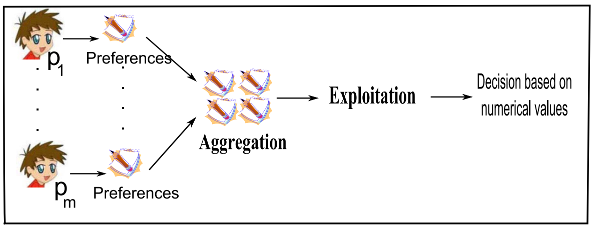

Fig. 1. Basic Decision Making Scheme

Decision problems have been classified in decision theory attending to their framework and elements [6]. Sometimes the solving process of a decision making problem is straightforward by using an algorithmic approach, these situations are so-called well-structured problems. However many decision problems cannot be solved in this way because decisions might be related to changing environments, the existence of vagueness, knowledge based on human perceptions, uncertainty in the decision framework, and so on. The latter problems, so-called ill-structured problems [47], are quite common in real world problems.

This lecture focuses on ill-structured decision problems dealing with vague and imprecise information, i.e., decision making under uncertainty. Classical decision theory provides probabilistic models to manage uncertainty in decision problems but in many of them it is easy to observe that a lot of aspects of these uncertainties have a non-probabilistic character since they are related to imprecision and vagueness of meanings [21]. Linguistic descriptors are often used by experts in such a type of problems, due to this fact some authors claim that is not adequate to make decisions based on numerical values when the decision process has been based on linguistic information [19], [23], [30], [31], [41], [40].

Therefore, taking into account that linguistic terms are fuzzy judgments rather than probabilistic values among the appropriate tools to overcome these difficulties of managing and modelling this type of uncertainties, fuzzy logic and fuzzy set theory [15], [44] arise to facilitate the managing of uncertainty in decision processes [2], [17] and the fuzzy linguistic approach [45] provides a direct way to represent the linguistic information by means of linguistic variables. The use of linguistic information thus enhances the reliability and flexibility of classical decision models [22].

It is clear that the linguistic information plays a key role in linguistic decision making [9], [11], [20] that demands processes of Computing with Words (CW) to solve the related decision problems. A initial proposal introduced by Tong and Bonisonne [31] proposed a computing scheme with fuzzy linguistic terms in decision making to manage the uncertainty that assumed that results should be quantifiable in natural language (see Fig. 2).

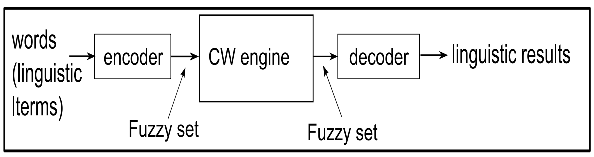

Fig. 2. Fuzzy Linguistic Computing Scheme

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By using the previous scheme the basic decision making scheme presented in Fig. 1 turns into a linguistic decision process similar to the one shows in Fig. 3, where the inputs and outputs are linguistically expressed.

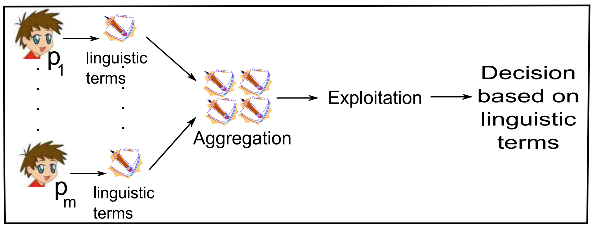

Fig. 3. Basic Linguistic Decision Making Scheme

Much discussion about linguistic decision making has arisen in the specialized literature, for instance in similar ways Schmucker [30] and mainly Yager [40], [41] have proposed linguistic schemes to deal with uncertainty (see Fig. 4). Yager [27] points out the importance of the translation and retranslation processes in CW, because the former involves taking information linguistically and translation into machine manipulative format. Meanwhile the latter involves taking the results from the manipulation machine format and transforms them into linguistic information understandable by human beings that is one of the main objectives of CW (*make easier to understand the results of information processing*). The retranslation process includes techniques of linguistic summarization too.

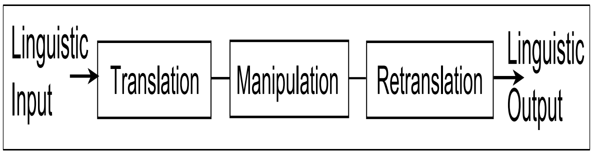

Fig. 4. Computing with Words Scheme

Consequently, different schemes for CW has been developed and applied as computational basis to linguistic decision making [11] to improve the resolution of decision making under uncertainty as linguistic decision making.

Recently and due to the fact that in many decision problems under uncertainty, such a uncertainty was caused by information based on human perceptions. It has been considering the use of CW to deal with such perceptions in decision making or other problems.

In perceptual computing different ideas and proposals regarding CW have been issued [18], [24], [26], [34], [46].

In [23], [25] Mendel adopts a similar scheme to the proposed by Tong and Bonisonne [31] showed in Fig. 2 and in [27] provides some *guidelines* that in his view must be passed any work to be branded as CW or else should not be called CW, such guidelines are the following ones:

G1. *A word must lead to a membership function rather than a membership function leading to a word.*

G2. *Numbers alone may not activate the CW engine.*

G3. *The output from a CW must be at least a word and not just a number.*

Taking into account the previous considerations and applications of CW to linguistic decision making and perceptual computing. In this lecture it is reviewed, analyzed and discussed the most spread linguistic computing models used in linguistic decision making, such as:

1) Linguistic computational models based on membership functions [1], [4], [8], [18], [42].
2) Linguistic symbolic computational models based on ordinal scales:
   a) Linguistic symbolic computational model based on ordinal scales and max-min operators [38], [39], [43].
   b) Linguistic symbolic computational model based on convex combinations [5], [10], [28], [14].
   c) Linguistic symbolic computational model based on virtual linguistic terms [36], [35], [37].
3) Linguistic symbolic models extending the use of indexes:
   a) Linguistic 2-tuple model [12], [13].
   b) Proportional linguistic 2-tuple [32], [33].
   c) Extended 2-tuple [16].

Despite there are others linguistic computational models [9], this work focuses on the previous ones in order to check if such models can be branded as CW models according to Mendel's guidelines [27] together other features to point out their weakness, strengths and usefulness in CW.

II. Analyzing Linguistic Computing Models

In order to analyze the different linguistic computing models previously enumerated and check if they can be branded as CW models. This section defines a simple linguistic decision making problem that will be solved following the decision scheme presented in Fig. 3 and by using different linguistic computing models.

An analysis of the models is then carried out attending to the obtained results checking if they fulfil Mendel's guidelines and as well as are analyzed the features of *accuracy* and *interpretability*.

A. Solving a Linguistic Decision Problem

Let be a linguistic decision problem with four experts $P = \{p_1, p_2, p_3, p_4\}$ that provides their opinions about four alternatives $X = \{x_1, x_2, x_3, x_4\}$ by using a linguistic term set $S = \{s_0 : Nothing, s_1 : VeryLow, s_2 : Low, s_3 : Medium, s_4 : High, s_5 : VeryHigh, s_6 : Perfect\}$ (see Fig. 5).

The decision matrix provided by the experts is the following one:

	alternatives			
	x_1	x_2	x_3	x_4
p_1	Low	$Medium$	$Medium$	Low
p_2	$Medium$	Low	$VeryLow$	$High$
p_3	$High$	$VeryLow$	$Medium$	$Medium$
p_4	$High$	$High$	Low	Low

TABLE I
SOLUTIONS OF THE LDM PROBLEM WITH DIFFERENT CW MODELS

	$\overline{x}_1^e$	$\overline{x}_2^e$	$\overline{x}_3^e$	$\overline{x}_4^e$
Memb.Func.[1]	$(\mathbf{0.375}, \mathbf{0.543}, \mathbf{0.702})$	(0.25,0.417,0.582)	(0.207,0.375,0.542)	(0.292,0.457,0.625)
Convex Comb.[2]	**Medium**	**Medium**	Very Low	Low
Ling. Virtual[3]	$(\mathbf{s_{3.25}})$	$(s_{2.5})$	$(s_{2.25})$	$(s_{2.75})$
2-tuple[4]	**(Medium,.25)**	$(Medium, -.5)$	$(Low, .25)$	$(Medium, -.25)$
2-T Prop.[5]	**(0.75Medium,0.25High)**	$(0.5Low, 0.5Medium)$	$(0.75Low, 0.25Medium)$	$(0.25Low, 0.75Medium)$

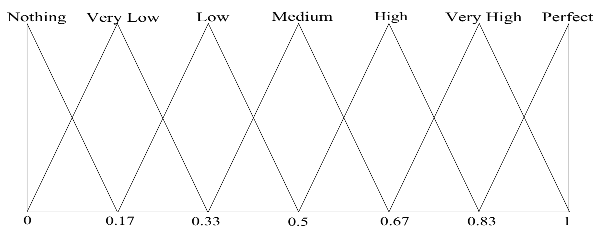

Fig. 5. Computing with Words Scheme

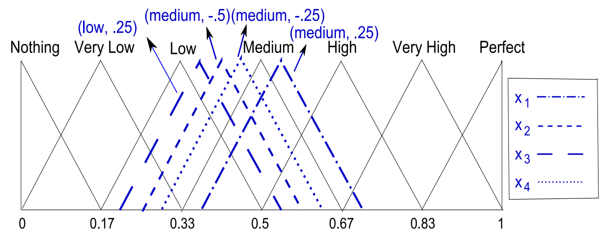

Fig. 7. Results expressed by 2-tuples

The results obtained of applying the different linguistic computing models (see footnotes 1-5) and the decision scheme presented in Fig. 3 are showed in Table I.

B. Analysis

From the previous results it will be analyzed the Mendel's guidelines **(G1,G2,G3)** and the *accuracy* and *interpretability* features of the linguistic computational models applied to solve the previous decision problem.

In order to clarify the discussion about the different models is noteworthy to remark that the representation of the results obtained by the different CW models used to solve the decision problem posses the following characteristics:

- Linguistic computational model based on membership functions: the results are fuzzy numbers (see Fig. 6).

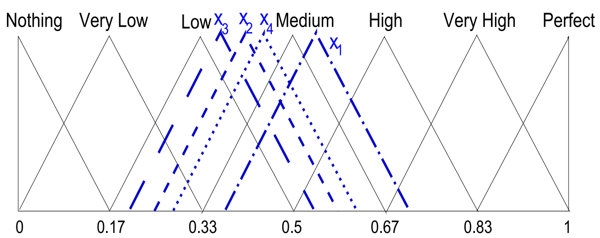

Fig. 6. Results expressed by fuzzy numbers

- Linguistic symbolic computational model based on convex combination: the results are fuzzy number similar to the ones presented in Fig. 5.
- Linguistic symbolic computational model based on virtual linguistic terms: these results cannot be represented in a fuzzy way because Xu [36] did not provide a syntax either any fuzzy semantic representation for virtual terms.
- Linguistic 2-tuple model: the results can be represented by means of fuzzy linguistic labels (see Fig. 7).
- Proportional linguistic 2-tuple: in this model Wang and Hao [32] did not provide either any semantics based representation of the linguistic information.

Taking into account the previous representations can be analyzed the features and guidelines aforementioned to check if the models can be branded as CW. The summary of this analysis is showed in the Table II.

TABLE II
COMPARATIVE ANALYSIS

	M.F.[1]	**C.C.**[2]	**2-tuple**[3]	Virtual[4]	Prop. 2-t [5]
G1	No	Yes	Yes	No	No
G2	No	Yes	Yes	Yes	Yes
G3	No	Yes	Yes	No	Yes
Accuracy	Yes/No	No	Yes	Yes	Yes
Interpretability	No/Yes	Yes	Yes	No	Yes

The analysis of Table II shows that just the convex combination and the linguistic 2-tuple models fulfil somehow Mendel's guidelines. Furthermore the tuple model possesses better characteristics regarding *accuracy*. Hence, between both models the 2-tuple model is a better CW approach quite suitable to deal with uncertainty in decision making.

III. CONCLUSIONS

The modelling and treatment of linguistic information for necessary computing with words processes are crucial in decision and perception based problems. Many proposals have been provided to accomplish such CW processes. In this contribution has been evaluated different CW models according to several features and guidelines to verify if such models can be branded as CW. The results are quite interesting because it can be argued that different CW proposals in the literature do not fulfil such guidelines.

[1]Model based on membership function
[2]Model based on the convex combination
[3]Model based on linguistic 2-tuple model
[4]Model based on linguistic virtual model
[5]Model based on proportional linguistic 2-tuple

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