A model of consensus reaching process for Group Decision Problems with multi-granular linguistic information

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Abstract In the literature, group decision making problems have been solved by carrying out two processes: a consensus process and a selection process. In the consensus one, decision makers or experts try to achieve an agreement among their opinions in order to solve the problem. In the selection process, the experts choose the solution set with the best alternative/s to solve the problem.

The consensus is a process composed of several discussion rounds where the experts provide and change their opinions. This process is guided by the figure of a human moderator that helps to the experts to make their opinion closer.

In this paper we propose a model for the consensus reaching process in group decision making problems defined in a multi-granular linguistic context. The proposed model measures the agreement and defines a set of rules which will be used by the moderator to suggest changes to the experts in order to improve the agreement in the next consensus round.

Keywords: Consensus, multi-granular linguistic information, group decision-making, linguistic modelling, fuzzy preference relation.

1 Introduction

In today's business environments, the decision making process has become a essential activity to ensure the success of the a company. We can see as the decision making process has changed from a sole decision maker toward groups of decision makers or experts. A group decision making process in which all experts agree about the solution, produces higher quality decisions than those from a single expert [5].

In group decision making (GDM) problems, usually the experts express their opinions by means of quantitative assessments. However, in some decision situations, the experts deal with vague or imprecise information for instance when they have to assess qualitative aspects that cannot be assessed by means of quantitative values. In these cases, the use of linguistic terms instead of

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precise numerical values seems to be more adequate. The use of the Fuzzy Linguistic Approach [7] to assess qualitative aspects using linguistic variables, i.e., variables whose values are not numbers but words or sentences in a natural or an appropriate artificial language, has provided successful results in handling fuzziness and uncertainty in decision making problems [3].

In the problems with multiple experts it is possible that they do not belong to the same research areas. This involves that they have different background and degrees of knowledge about the problem. In these cases, it seems logic that each expert may prefer to use different linguistic term sets to express their individual opinions. In these situations, we say that the decision making problem is defined in a multi-granular linguistic context [3, 6]. In this contribution we deal with GDM problems which the experts give their assessment using multi-granular linguistic term sets.

In the literature, we can differentiate two processes in order to solve a GDM problem (see Figure 1):

- *Consensus process*: It is a discussion process that consists of several rounds where the experts exchange their preferences on the alternatives to solve the problem. The purpose of this process is to reach the maximum agreement before making a decision. Normally, this process is guided by a moderator.

- *Selection process*: It obtains a solution set of alternatives from the opinions given by the experts. Clearly, it is preferable that the experts agree about the alternatives before applying the selection process.

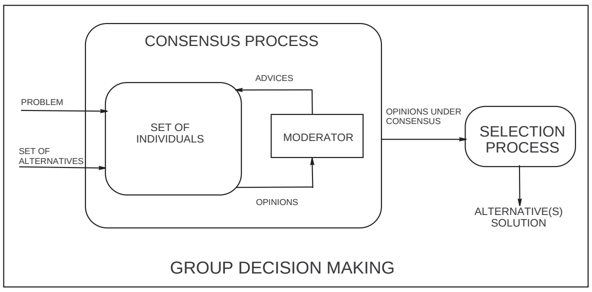

Figure 1: Resolution process of a group decision-making problem

The selection process with multi-granular linguistic information was studied in [3]. In this paper, we focus on the consensus process. The consensus is defined as a state of mutual agreement among members of a group where all opinions have been heard and addressed to the satisfaction of the group [5]. The consensus reaching process should be the first activity to carry out in a group decision making process. We deal with a decision making process where several experts participate and given their opinions, that usually will be different. Each expert may have a different point of view on each alternative. Through the consensus reaching process all experts' opinions will be considered to obtain the solution. So, before making a decision, it is necessary to carry out a discussion process in which the experts change their opinions in order to reach an agreement on the alternatives that allow to solve the problem.

Traditionally, the consensus reaching process has been coordinated by the figure of human moderator [5, 8]. The role of the moderator is very important. On the one hand he/she has to evaluate the agreement among experts and on the other hand he/she is in charge of addressing the consensus reaching process towards the success, i.e, to achieve the highest agreement possible. In the Figure 2 is shown an overall schema of the different phases of a consensus reaching process.

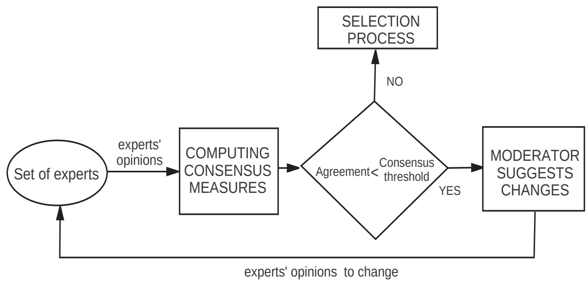

Figure 2: Phases of the consensus reaching process

First, the experts provide their opinions. Afterwards, several consensus measures are computed to know the level of agreement among experts. If a consensus threshold fixed in advance is achieved, then the consensus reaching process will finish and the selection process will obtain the best solution of the problem. Otherwise, the moderator will propose to several experts to change some of their opinions.

To compute the level of agreement it is necessary to use suitable consensus measures. In [4], we presented two type of consensus measures able to measure the similarity among experts's opinions in multi-granular linguistic contexts.

a) Consensus degree. This measure evaluates the agreement among the experts' opinions in each consensus round. Also, it is used to identify the experts' preferences where exist disagreement.

b) Proximity measure. This measure evaluates the distance between the experts' individual opinions and the group opinion. It identifies the furthest experts' preferences from the group opinion.

In this contribution we propose a consensus reaching process for GDM problems where the experts use different linguistic term sets to express their opinions. This consensus process is composed of several discussion rounds that are carried out until reaching the fixed agreement or until reaching a maximum number of rounds.

This contribution is set out as follows. A description of the GDM problems in multi-granular linguistic context is shown in Section 2. The proposed model of consensus reaching process is presented in Section 3, and finally, in Section 4 we draw some conclusions.

2 Multi-granular Linguistic GDM Problems

A GDM problem may be defined as a decision making process which two or more experts, $E = \{e_1, e_2, \ldots, e_m\}$ $(m \geq 2)$, try to choose the best alternative(s) from a set of alternatives $X = \{x_1, x_2, \ldots, x_n\}$ $(n \geq 2)$. An usual preference structure used by the experts to express their opinions in this type of problems is the preference relation, $P_{e_i} \subset X \times X$, where each value p_i^{lk} of the matrix represents the preference of the alternative x_l over the alternative x_k provided by expert e_i [2].

We assume that the experts use linguistic terms to assess their preferences, $\mu_{P_{e_i}} : X \times X \to S$, where $S = \{s_0, s_1, \ldots, s_g\}$ is an appropriate linguistic term set characterized by its cardinality or *granularity*, $\#(S) = g + 1$. The granularity represents the level of discrimination among different degrees of uncertainty. Additionally, S has the following properties [3]:

1. The set S is ordered: $s_i \geq s_j$, if $i \geq$ j.
2. There is a negation operator: $Neg(s_i) = s_j$ such that $j = g - i$.

The semantics of the terms is represented by fuzzy numbers defined on the [0,1] interval. One way to characterize a fuzzy number is by using a representation based on parameters of its membership function [1]. For example, the following semantics, represented in the Figure 3, can be assigned to a set of seven terms via triangular fuzzy numbers:

$P = Perfect = (.83, 1, 1)$ $VH = Very\_High = (.67, .83, 1)$
$H = High = (.5, .67, .83)$ $M = Medium = (.33, .5, .67)$
$L = Low = (.17, .33, .5)$ $VL = Very\_Low = (0, .17, .33)$
$N = None = (0, 0, .17)$.

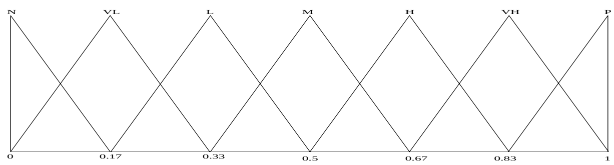

Figure 3: A set of seven terms with their semantics

The ideal situation in a GDM problem defined in a linguistic context would be that all the experts use the same linguistic term set S to provide their opinions. However, in some cases, experts may belong to distinct research areas and have different levels of knowledge about the alternatives. As consequence of this fact, the preferences may be assessed by means of linguistic term sets with different granularity, which involves that appropriate tools to manage and model multi-granular linguistic information become essential [3, 6].

In this paper, we deal with multi-granular linguistic GDM problems, where each expert e_i may express his/her opinions on the set of alternatives by using preference relations $\mathbf{P_{e_i}} = (p_i^{jk})$, $p_i^{jk} \in S_i$, where each $S_i = \{s_0^i, \ldots, s_g^i\}$ has different cardinality $\#(S_i) = g + 1$.

3 A model of consensus reaching process with multi-granular information

Here, we present our proposal for a consensus reaching process for GDM problems defined in a multi-granular linguistic context (see Figure 4). This model is composed of four phases:

1. *Making the linguistic information uniform.* In this phase, the multi-granular linguistic preferences are unified into a single linguistic domain in order to make calculi with them. We use transformation functions to unify the multi-granular linguistic information.
2. *Computation of the consensus degree.* In this phase, we evaluate the agreement among experts. To do so, we compute the consensus degree among the experts at different levels as pairs of alternatives, alternatives and preference relations.
3. *Consensus control.* This phase controls the consensus process and decides to continue or finish the consensus reaching process.
4. *Moderator's actions.* In this phase, the proximity measures are computed and the moderator identifies the furthest experts' preferences. The moderator uses a set of direction rules to recommend the changes in the experts' opinions.

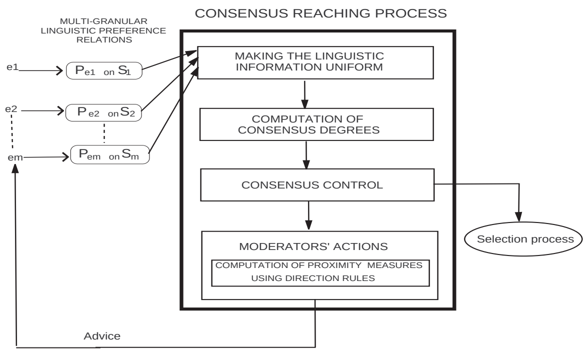

Figure 4: A model of consensus reaching process in a multi-granular linguistic context

These phases are described in further detail in the next subsections.

3.1 Making the linguistic information uniform

To manage the multi-granular linguistic information, first we need to unify it into a single linguistic domain that we will call basic linguistic term set, S_T. S_T should have a granularity high enough to maintain the uncertainty degrees associated to each one of the possible domains S_i. To make the multi-granular linguistic information uniform, we use the multi-granular transformation function, $\tau_{S_i S_T}$. This transformation function transforms each linguistic terms $s_g^i \in S_i$ into fuzzy sets defined on S_T:

$$\tau_{S_i S_T} : S_i \longrightarrow F(S_T), \quad \forall\ S_i$$

$$\tau_{S_i S_T}(p_i^{lk}) = \{(c_h, \alpha_h^{lk})\ /\ h = 0, \ldots, g\}.$$

where at least $\exists\ \alpha_h^{lk} > 0$ and $\forall\ \alpha_h^{lk} \in [0, 1]$.

The conditions to choose an appropriate S_T and the characteristic of the multi-granular transformation functions were defined in [3]. We will continue to denote $\tau_{S_i S_T}(p_i^{lk})$ by $\tilde{p}_i^{lk}$, and the representation of the fuzzy sets by means of its memberships degrees $(\alpha_0^{lk}, \ldots, \alpha_g^{lk})$. Once all linguistic terms have been unified on S_T, the unification process is carried out for all the experts' opinions and so we obtain a preference relation P_{e_i} for each expert e_i, where the expert's preferences will be represented by means of fuzzy sets:

$$\tilde{P}_{e_i} = \begin{pmatrix} \tilde{p}_i^{11} = (\alpha_0^{11}, \ldots, \alpha_g^{11}) & \cdots & \tilde{p}_i^{1n} = (\alpha_0^{1n}, \ldots, \alpha_g^{1n}) \\ \vdots & \cdots & \vdots \\ \tilde{p}_i^{n1} = (\alpha_0^{n1}, \ldots, \alpha_g^{n1}) & \cdots & \tilde{p}_i^{nn} = (\alpha_0^{nn}, \ldots, \alpha_g^{nn}) \end{pmatrix}$$

3.2 Computation of Consensus Degrees

The consensus degree measures the agreement among all the experts. To evaluate the agreement, it is calculated the distance or similarity among experts' preferences, i.e, in this case among fuzzy sets. To do so, we use the value cv_i^{lk} that represents the centre of gravity of the information contained in the fuzzy set of the preference $\tilde{p}_i^{lk} = (\alpha_0^{lk}, \ldots, \alpha_g^{lk})$ that is obtained as

$$cv_i^{lk} = \frac{\sum_{h=0}^{g} index(s_h^i) \cdot \alpha_h^{lk}}{\sum_{h=0}^{g} \alpha_h^{lk}},\ being\ index\ (s_h^i) = h. \tag{1}$$

The distance between two preferences given by two experts $\tilde{p}_i^{lk}, \tilde{p}_j^{lk}$, is computed by means of the similarity function s presented in [4],

$$s(\tilde{p}_i^{lk}, \tilde{p}_j^{lk}) = 1 - \left| \frac{cv_i^{lk} - cv_j^{lk}}{g} \right| \tag{2}$$

The agreement is computed in the three different levels of representation of information of the preference relations, i.e., on pairs of alternatives, alternatives and relations. In this way we can know with precision the preferences in which there exist agreement or disagreement.

The consensus measures are computed according to the following steps:

1. Computation of the distance matrixes, DM_{ij}: We calculate a distance matrix DM_{ij} for each pair of experts e_i, e_j. Each element of this matrix dm_{ij}^{lk} represents the distance among the experts in the level of pairs of alternatives:
$$dm_{ij}^{lk} = s(\tilde{p}_i^{lk}, \tilde{p}_j^{lk}) \tag{3}$$

2. Computation of the consensus matrix, CM: This matrix is obtained by aggregating all DM_{ij} at level of pairs of alternatives. We use the arithmetic mean as as the aggregation function ϕ.

$$cm^{lk} \quad = \quad \phi(dm_{ij}^{lk}, i, j = 1, \ldots, m \ , \ i < j), \ \forall \ l, k = 1, \ldots, n.$$

3. Computation of the consensus degrees at the three levels:

Level 1. *Consensus on pairs of alternatives, cp^{lk}*: It measures the consensus degree on each pair of alternatives among all experts. In our case, this is expressed by the element (l, k) of the consensus matrix CM, i.e.,

$$cp^{lk} = cm^{lk}, \quad \forall l, k = 1, \ldots, n \quad \wedge \quad l \neq k$$

This measure will allow the identification of those pairs of alternatives with a poor level of consensus.

Level 2. *Consensus on alternatives, ca^l*: It measures the consensus degree on each alternative among all experts. For this, we compute the average of each row of the consensus matrix CM.

$$ca^l = \frac{\sum_{k=1}^{n} cp^{lk}}{n} \tag{4}$$

The value of ca^l is used to identify the alternatives with poor level of consensus.

Level 3. *Consensus among the experts, ce*: It measures the global consensus degree among the experts' opinions. It is computed as the average of all the values of consensus on alternatives, i.e,

$$ce = \frac{\sum_{l=1}^{n} ca^l}{n} \tag{5}$$

This value is used to control the level of consensus in each consensus round.

If some of the above consensus degrees is near to 1, then it means that there exist a great agreement on that elements.

3.3 Consensus Control

In this phase we control the consensus process and decide to continue it or otherwise carry out the selection process. To do so, we fix in advance a parameter called consensus threshold ($\gamma / \gamma \in [0, 1]$) and establish the following conditions:

- If $ce >= \gamma$, then the experts have achieved the wanted agreement and the selection process of the best alternatives solutions can start.
- If $ce < \gamma$, then the agreement is still low and the consensus reaching process should continue.

It is necessary to highlight that if γ is to much high, it is possible that is never comply with the first condition and the consensus reaching process does not finish. In order to above this situation, we have defined other parameter, Max_cicles, that represents the maximum number of consensus rounds to carry out before starting the selection process.

3.4 Moderator's actions

In this section, we propose a solution for the following questions:

a) How does the moderator identify the experts' preferences that should change?. To answer this question, we use the proximity measures.

b) How does the moderator know the direction of the changes in order to make the experts' opinions closer?. To answer this question, the moderator will use a set of direction rules that we provide here.

Each one of these questions are further developed in the next subsections.

3.4.1 Proximity measures

If it does not achieve the wanted agreement γ in the current consensus round, the moderator needs to identify the furthest individual experts' preferences and suggests that the experts change them. To do so, the moderator computes the proximity measures to measure the similarity or distance among experts' opinions and the group's opinion. The proximity measures are also calculated at level of pairs of alternatives, alternatives and relations. Before calculating them, it is necessary to obtain a collective preference relation that represents the opinions of all the experts. This is calculated by aggregating all opinions following the method presented in [3]. So we obtain the matrix P_{e_c},

$$\tilde{P}_{e_c} = \begin{pmatrix} \tilde{p}_c^{11} & \cdots & \tilde{p}_c^{1n} \\ \vdots & \cdots & \vdots \\ \tilde{p}_c^{n1} & \cdots & \tilde{p}_c^{nn} \end{pmatrix}$$

where each $\tilde{p}_c^{lk}$ is a fuzzy set.

Following the next steps, proximity measures can be calculated:

1. Computing of the proximity matrixes, PM_i: To compute the proximity measures it is necessary to calculate the proximity matrix of each expert. This represents the distance between the group opinion and individual opinion. The values of PM_i are calculated by using the distance function defined in expression (2), such that:

$$PM_i = \begin{pmatrix} s(\tilde{p}_i^{11}, \tilde{p}_c^{11}) & \cdots & s(\tilde{p}_i^{1n}, \tilde{p}_c^{1n}) \\ \vdots & \cdots & \vdots \\ s(\tilde{p}_i^{n1}, \tilde{p}_c^{n1}) & \cdots & s(\tilde{p}_i^{nn}, \tilde{p}_c^{nn}) \end{pmatrix}$$

2. Computing of the proximity measures: We calculates the proximity measures at level of pairs of alternatives, alternatives and relations:

Level 1. *Proximity on pairs of alternatives, pp_i^{jk}*: It measures the proximity between each pair of alternatives. In our case, this is expressed by the element (l, k) of the proximity matrix PM_i, i.e.,

$$pp_i^{lk} \quad = \quad s(\tilde{p}_i^{lk}, \tilde{p}_c^{lk}), \ where \ l, k = 1, \ldots, n \ and \ l \neq k$$

Level 2. *Proximity on alternatives, pa^l*: It measures the proximity between alternatives. To do this, we take the average of each row of the proximity matrix PM_i.

$$pa_i^l = \frac{\sum_{k=1}^n pp_i^{lk}}{n} \tag{6}$$

Level 3. *Proximity on the relation, pr_i*: It measures the global proximity between the preferences of each individual expert, e_i, and the group's ones. It is computed as the average of all proximity on alternative values, i.e,

$$pr_i = \frac{\sum_{l=1}^{n} pa_i^l}{n} \tag{7}$$

If any of the proximity measures is close to 1, then that element is close to the group opinion and therefore has a positive contribution for the consensus, while if is close to 0 then has a negative contribution to consensus. Once we have defined the proximity measures, we can establish a set of steps and conditions in order to the moderator can identify the experts that will should change their opinions:

1. Identification of the furthest experts. Obviously, the first experts to change their opinions are those with the lowest proximity values pr_i. Therefore the moderator sort the experts according to pr_i and decides the number or % of experts that should modify their opinions, for instance the 50% of the experts.

2. Identification of the disagreement alternatives. We propose changing only the preferences of the alternatives where exist disagreement. So the moderator identifies the alternatives which the agreement is smaller than the consensus threshold, $ca^l < \gamma$. In this way we avoid to change alternatives where exist a high consensus degree.

3. Identification of pair of alternatives to change. Once the moderator knows the expert e_i and the alternative x_l that should change, the moderator suggests to change all pairs of alternatives whose proximity $pp_i^{lk} < \beta$, being β a maximum proximity threshold.

3.4.2 Direction rules

To find out the direction of the changes that the moderator will propose to the experts, we have defined two pairs of direction parameters, one for the individual preference $\tilde{p}_i^{lk}$, and the other for the group preference $\tilde{p}_c^{lk}$. These pairs of direction parameters will contain both the position and membership degree associated to a main-label (ml) and a secondary-label (sl) of each fuzzy set respectively. The main-label will correspond to that with maximum membership degree while the secondary-label will correspond to that with second greatest membership degree. Therefore, for each preference assessment $\tilde{p}_i^{lk}$ to be changed, $\big(\tilde{p}_i^{lk}(ml_{pos}), \tilde{p}_i^{lk}(ml_{val}), \tilde{p}_i^{lk}(sl_{pos}), \tilde{p}_i^{lk}(sl_{val})\big)$ and $\big(\tilde{p}_c^{lk}(ml_{pos}), \tilde{p}_c^{lk}(ml_{val}), \tilde{p}_c^{lk}(sl_{pos}), \tilde{p}_c^{lk}(sl_{val})\big)$ are compared to define the following four direction rules:

DR.1. If $\tilde{p}_i^{lk}(ml_{pos}) > \tilde{p}_c^{lk}(ml_{pos})$ then the expert e_i should decrease the assessment associated to the pair of alternatives (x_l, x_k), i.e. $\tilde{p}_i^{lk}$.

DR.2. If $\tilde{p}_i^{lk}(ml_{pos}) < \tilde{p}_c^{lk}(ml_{pos})$ then the expert e_i should increase the assessment associated to the pair of alternatives (x_l, x_k), i.e. $\tilde{p}_i^{lk}$.

DR.3. If $\tilde{p}_i^{lk}(ml_{pos}) = \tilde{p}_c^{lk}(ml_{pos})$ then rules DR.1, DR.2 and DR.3 are applied using the membership values of the main-labels, $\tilde{p}_i^{lk}(ml_{val})$ and $\tilde{p}_c^{lk}(ml_{val})$.

DR.4. If $\left(\tilde{p}_i^{lk}(ml_{pos}) = \tilde{p}_c^{lk}(ml_{pos}), \tilde{p}_i^{lk}(ml_{val}) = \tilde{p}_c^{lk}(ml_{val})\right)$, then rules DR.1, DR.2, and DR.3 are applied using the position and membership values of the secondary-labels *sl*.

The above direction rules will not be used when a decrease or increase are suggested to an assessment represented by the first or last label of a linguistic term set, respectively.

4 Conclusions

In this contribution we have presented a model of consensus reaching process for GDM problems in which the experts use different linguistic term sets to express their opinions. We use two types of measures (consensus degrees and proximity measures) to evaluate the agreement among experts and the proximity between the experts' opinion and group's opinion. Finally, a set of direction rules are defined in order to be used by the moderator to suggest the changes in the experts' opinion in order to improve the agreement.

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