9

A Consensus Support System for Group Decision Making Problems with Heterogeneous Information

F. Mata[1], L. Martínez[2], and E. Herrera-Viedma[3]

[1] Department of Computer Science, University of Jaén, Spain fmata@ujaen.es
[2] Department of Computer Science, University of Jaén, Spain martin@ujaen.es
[3] Department of Computer Science and A.I., University of Granada, Spain viedma@decsai.ugr.es

Summary. A group decision making (GDM) problem is a decision process where several decision makers (experts, judges, etc.) participate and try to reach a common solution. In the literature these problems have been solved carrying out a selection process that returns the solution set of alternatives from the preferences given by the experts. In order to achieve an agreement on the solution set of alternatives among the experts, it would be adequate to carry out a consensus process before the selection process. In the consensus process the experts discuss and change their preferences in order to achieve a big agreement. Due to the fact that the experts may belong to different research areas, they may express their preferences in different information domains. In this contribution we focus on the consensus process in GDM problems defined in heterogeneous contexts where the experts express their preferences by means of numerical, linguistic and interval-valued assessments. We propose a consensus support system model to automate the consensus reaching process, which provides two main advantages: (1) firstly, its ability to cope with GDM problems with heterogeneous information by means of the Fuzzy Sets Theory, and, (2) secondly, it assumes the moderator's tasks, figure traditionally presents in the consensus reaching process.

9.1 Introduction

Group decision-making (GDM) problems may be defined as decision situations where two or more experts try to achieve a common solution about a problem taking into account their opinions or preferences.

In the literature we can find many proposals to solve decision problems where experts use the same information domain to express their preferences (Bui 1987; Herrera and Herrera-Viedma 2000; Kacprzyk 1986; Kim et al. 1999). However, in several occasions it may be more suitable that experts express their opinions in different expression domains according to their own

F. Mata et al.: *A Consensus Support System for Group Decision Making Problems with Heterogeneous Information*, Studies in Computational Intelligence (SCI) **97**, 229–257 (2008)
www.springerlink.com

knowledge or nature of the alternatives. For example, if experts belong to different departments (marketing, accounting, psychology, etc.), they may prefer to provide their opinions by using an information domain closer their research topics. Moreover, in a decision problem we can deal with alternatives whose nature is quantitative and others whose nature is qualitative. The first ones can be assessed by means of precise values like crisp values (Kacprzyk 1986; Yager 1988). However, when alternatives are related to qualitative aspects, it may be difficult to qualify them using precise values. In such cases, where the uncertainty is present, the experts can use interval-valued (Kundu 1997; Le Téno and Mareschal 1998) or linguistic values (Herrera and Herrera-Viedma 2000; Yager 1995) to express their preferences. In such situations, the decision problem is defined into a heterogenous context.

Usually GDM problems have been solved carrying out *Selection Processes* where experts obtain the best solution set of alternatives from the preferences given by themselves (Fodor and Roubens 1994; Roubens 1997). However it may happen that some experts consider that their preferences have not been taken into account to obtain the solution, and therefore they do not agree with it. To avoid this situation, it is suitable to carry out a consensus process (see Fig. 9.1) where the experts discuss and change their preferences in order to reach a sufficient agreement before making a decision (Carlsson et al. 1992; Herrera et al. 1996; Herrera-Viedma et al. 2002; Kacprzyk et al. 1997).

Different methods have been proposed to deal with *Selection Processes* in heterogeneous GDM problems in the literature (Delgado et al. 1998; Herrera et al. 2005; Zhang et al. 2004), but there are not defined specific consensus processes for this kind of problems. Consequently, in this chapter we focus on the *Consensus Process* on GDM problems dealing with heterogeneous information.

The consensus is an important area of research in GDM (Bordogna et al. 1997; Bryson 1996; Carlsson et al. 1992; Fan and Chen 2005; Herrera-Viedma et al. 2002; Kacprzyk et al. 1997; Szmidt and Kacprzyk 2003; Yager 1997).

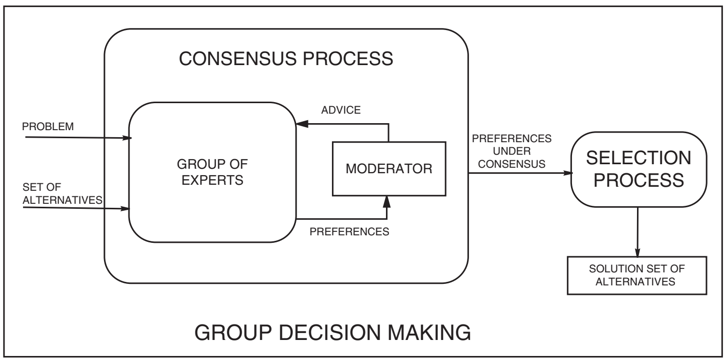

Fig. 9.1. Resolution process of a GDM problem

The consensus is defined as a state of mutual agreement among members of a group where all opinions have been heard and addressed to the satisfaction of the group (Saint and Lawson 1994). The consensus reaching process is a dynamic and iterative process composed of several rounds, where the experts express and discuss their opinions. Traditionally this process is coordinated by a human moderator, who computes the agreement among experts in each round using different consensus measures (Herrera-Viedma et al. 2004; Kuncheva 1991). If the agreement is not enough then the moderator recommends the experts to change their furthest preferences from the group opinion in an effort to make them closer in the next consensus round (Bryson 1996; Zadrozny 1997).

The moderator has been a controversial figure because experts may have complaints about his lack of objectivity. Moreover, in heterogeneous contexts, he may have problems to understand all the different domains and scales in a proper way. Therefore, the aim of this chapter is to present a consensus support system (CSS) model for GDM problems such that:

- The experts can express their preferences by means of linguistic, numerical or interval-valued preference relations, i.e., into a heterogeneous context.
- The moderator's tasks are assumed by an automatic guided advice generator.

The chapter is set out as follows. First, we introduce the GDM problems defined in heterogeneous contexts in the Sect. 2. In the Sect. 3 the different phases of the consensus model are explained in detail. Finally, in the Sect. 4, a practical example is proposed in order to show the performance of the CSS.

9.2 Preliminaries

Let's begin this section introducing the GDM problems based on fuzzy preference relations. Afterwards, it is briefly reviewed different approaches proposed in the literature to express the experts' preferences and finally it is presented the heterogeneous GDM problems.

9.2.1 Group Decision Making Problems

GDM problems are decision situations in which two or more individuals or experts, $E = \{e_1, e_2, \ldots, e_m\}$ $(m \geq 2)$, provide their preferences on a set of alternatives, $X = \{x_1, x_2, \ldots, x_n\}$ $(n \geq 2)$, to derive a solution (an alternative or set of alternatives). Depending on the nature or the knowledge on the alternatives, experts may express their preferences using different approaches.

In fuzzy contexts, experts' preferences are usually expressed by means of fuzzy preference relations (Kacprzyk 1986). A preference relation may be defined as a matrix $P_{e_i} \subset X \times X$

$$\mathbf{P_{e_i}} = \begin{pmatrix} p_i^{11} & \cdots & p_i^{1n} \\ \vdots & \ddots & \vdots \\ p_i^{n1} & \cdots & p_i^{nn} \end{pmatrix},$$

where the value $\mu_{P_{e_i}}(x_l, x_k) = p_i^{lk}$ is interpreted as the preference degree of the alternative x_l over x_k expressed by the expert e_i.

Let's suppose $p^{lj} \in [0, 1]$, then:

1. $p^{lj} = 1$ indicates the maximum degree of preference of x_l over x_j.
2. $0.5 \leq p^{lj} \leq 1$ indicates a definitive preference of x_l over x_j.
3. $p^{lj} = 0.5$ indicates the indifference between x_l and x_j.

The fuzzy preference relations may satisfy some of the following properties:

- Reciprocity: $p^{lj} + p^{jl} = 1, \forall l, j$
- Completeness: $p^{lj} + p^{jl} \geq 1, \forall l, j$
- Max–Min Transitivity: $p^{lk} \geq Min(p^{lj}, p^{jk}), \forall l, j, k$
- Max–Max Transitivity: $p^{lk} \geq Max(p^{lj}, p^{jk}), \forall l, j, k$
- Restricted Max–Min Transitivity: $p^{lj} \geq 0.5, p^{lk} \geq 0.5 \Rightarrow p^{lk} \geq Min(p^{lj}, p^{jk})$
- Restricted Max–Max Transitivity: $p^{lj} \geq 0.5, p^{lk} \geq 0.5 \Rightarrow p^{lk} \geq Max(p^{lj}, p^{jk})$
- Additive Transitivity: $p^{lj} + p^{jk} - 0.5 = p^{lk}, \forall l, j, k$

9.2.2 Preferences Modeling

Fuzzy Preference Relations

A valued fuzzy preference relation R on X is defined as a fuzzy subset of the direct product $X \times X$, i.e, $R : X \times X \rightarrow [0, 1]$. The value, $R(x_l, x_k) = p^{lk}$ denotes the degree in which an alternative x_l is preferred to alternative x_k.

$$\mathbf{P_{e_i}} = \begin{pmatrix} 0.5 & \cdots & 0.7 \\ \vdots & \ddots & \vdots \\ 0.3 & \cdots & 0.5 \end{pmatrix}$$

These were the first type of fuzzy preference relations used in decision making (Kacprzyk 1986) to deal with uncertainty, but soon appeared other approaches to express the uncertainty that will be reviewed in the following subsections.

Interval-Valued Preference Relations

A first approach to add some flexibility to the uncertainty representation problem was by means of interval-valued preferences relations:

$$R : X \times X \rightarrow I([0, 1]),$$

where $R(x_l, x_k) = p^{lk}$ denotes the interval-valued preference degree of the alternative x_l over x_k. In these approaches (Kundu 1997; Le Téno and Mareschal 1998), the preferences provided by the experts are interval values assessed in $I([0,1])$, where the preference is expressed as $[\underline{a}, \overline{a}]^{lk}$, with $\underline{a} \leq \overline{a}$

$$\mathbf{P}_{\mathbf{e_i}} = \begin{pmatrix} [0.5, 0.5] & \cdots & [0.7, 0.9] \\ \vdots & \ddots & \vdots \\ [0.1, 0.3] & \cdots & [0.5, 0.5] \end{pmatrix}.$$

Fuzzy Linguistic Preference Relations

A fuzzy linguistic preference relation is defined as

$$R : X \times X \rightarrow S$$

being $S = (s_0, \ldots, s_g)$ a set of labels.

There are situations in which a better approach to qualify aspects of many activities may be to use linguistic assessments instead of numerical values. The fuzzy linguistic approach represents the information as linguistic values by means of linguistic variables (Zadeh 1975). This approach is adequate to qualify phenomena related to human perception that we often assess using words in natural language. This may arise for different reasons. There are some situations where the information may be unquantifiable due to its nature, and thus, it may be stated only in linguistic terms (e.g., when evaluating the “comfort” or “design” of a car, terms like “bad”, “poor”, “tolerable”, “average”, “good” can be used (Levrat et al. 1997)). In other cases, according to (Zadeh 1996) there is a tolerance for imprecision which can be exploited to achieve tractability, robustness, low solution cost, and better rapport with reality (e.g., when evaluating the speed of a car, linguistic terms like “fast”, “very fast”, “slow” are used instead of numerical values).

We have to choose the appropriate linguistic descriptors for the term set and their semantics. One possibility of generating the linguistic term set consists in directly supplying the term set by considering all terms distributed on a scale on which a total order is defined (Yager 1995). For example, a set of seven terms S, could be given as follows:

$$S = \{s_0 = none, s_1 = very\ low, s_2 = low, s_3 = medium,$$
$$s_4 = high, s_5 = very\ high, s_6 = perfect\}.$$

In these cases, it is usually required that there exist:

1. A negation operator $\text{Neg}(s_i) = s_j$ such that $j = g$-i (g+1 is the cardinality of the term set)
2. A maximization operator: $\text{Max}(s_i, s_j) = s_i$ if $s_i \geq s_j$
3. A minimization operator: $\text{Min}(s_i, s_j) = s_i$ if $s_i \leq s_j$

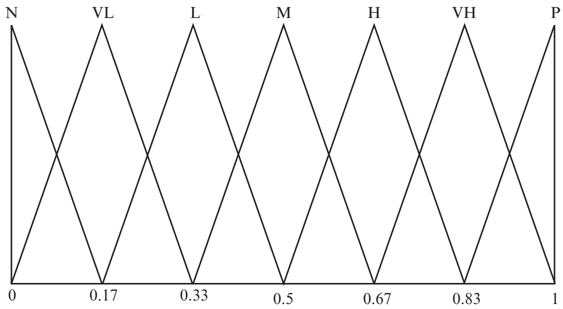

Fig. 9.2. A set of seven linguistic terms with its semantics

The semantics of the terms is given by fuzzy numbers defined in the [0,1] interval. A way to characterize a fuzzy number is to use a representation based on parameters of its membership function (Bonissone and Decker 1986). For example, we may assign the following semantics to the set of seven terms via triangular fuzzy numbers:

$$\begin{array}{ll} Perfect(P) = (0.83, 1, 1) & Very\_High(VH) = (0.67, 0.83, 1) \\ High(H) = (0.5, 0.67, 0.83) & Medium(M) = (0.33, 0.5, 0.67) \\ Low(L) = (0.17, 0.33, 0.5) & Very\_Low(VL) = (0, 0.17, 0.33) \\ None(N) = (0, 0, 0.17), & \end{array}$$

which is graphically shown in Fig. 9.2.

Therefore a linguistic preference relation $R(x_l, x_k)$ denotes the linguistic preference degree of the alternative x_l over x_k. Using the linguistic term set shown in the Fig. 9.2, a linguistic preference relation could be:

$$\mathbf{P_{e_i}} = \begin{pmatrix} M & \cdots & VH \\ \vdots & \ddots & \vdots \\ VL & \cdots & M \end{pmatrix}.$$

9.2.3 Group Decision Making Problems Defined on Heterogeneous Contexts

The ideal situation in a GDM problem is that all experts have a wide knowledge about the alternatives and provide their opinions in a numerical precise scale. However, in some cases, experts may belong to distinct research areas and have different levels of knowledge about the alternatives. Due to this, the experts may prefer to express their preferences by means of different information domains. In such cases, the problem is defined in a heterogeneous context.

In this chapter we deal with heterogenous GDM problems where the experts express their preferences using different expression domains (numerical, interval-valued or linguistic), $D_i \in \{N|I|L\}$. Each expert gives their opinions by means of a fuzzy preference relation defined on an unique expression domain, $\mathbf{P_{e_i}}$:

$$\mathbf{P_{e_i}} = \begin{pmatrix} p_i^{11} & \cdots & p_i^{1n} \\ \vdots & \ddots & \vdots \\ p_i^{n1} & \cdots & p_i^{nn} \end{pmatrix},$$

where $p_i^{lk} \in D_i$ represents the preference of the alternative x_l over the alternative x_k given by the expert e_i.

9.3 A Consensus Support System Model for GDM Problems with Heterogeneous Information

In this section we present the model of a consensus support system for GDM problems with heterogeneous information. The CSS model has two main features:

1. It is able to carry out the consensus process in heterogeneous GDM problems with numerical, interval-valued and linguistic assessments.
2. It includes a guided advice generator that assumes the moderator's tasks and recommends the changes in experts' preferences in order to obtain a high consensus degree.

The CSS model will be built up using:

- A methodology to unify the heterogeneous information into a single domain.
- Different measurements to cope with the agreement: *consensus degree* and *proximity measure*. The first one is used to evaluate the agreement amongst the experts, while the second one is used to measure the distance between the collective and individual expert's preferences.
- A set of advice rules based on the these measurements are used to guide the direction of the changes in the experts' opinions.

The CSS model consists of the following phases depicted in Fig. 9.3:

1. *Making the information uniform.* In this phase, the experts' heterogeneous preferences are unified into an unique domain.
2. *Computing consensus degree.* In this phase consensus degree amongst the experts is computed. To do so, a similarity function is used to calculate the coincidence amongst experts' preferences.
3. *Checking the agreement.* In this phase the CSS controls the level of agreement achieved amongst experts. If the agreement is greater than a specified consensus threshold (γ) then the consensus process will stop and the selection process will be applied to obtain the solution of the problem. Otherwise, in the following phase the experts' preferences must be modified.

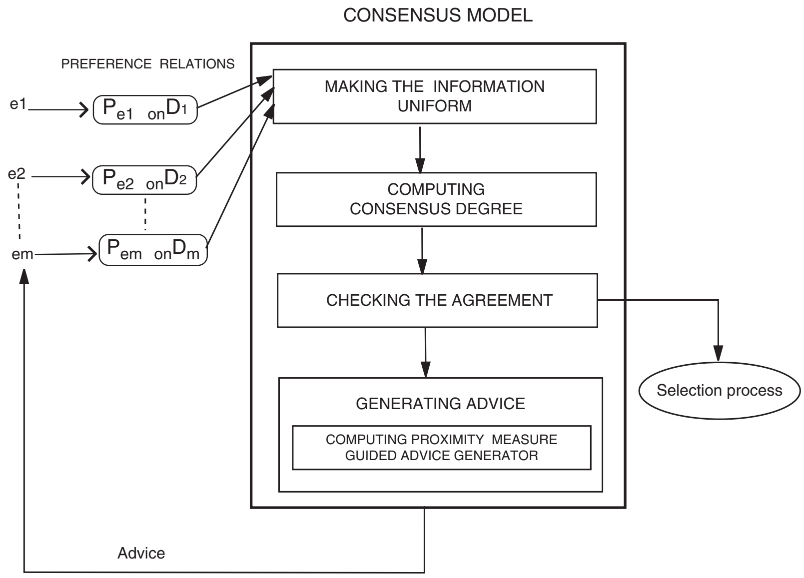

Fig. 9.3. A CSS model with heterogeneous information

4. *Generating advice.* To help experts change their preferences, the CSS generates a set of recommendations or advice. To do this, a proximity measure is used in conjunction with the consensus degree to build a guided advice generator in charge of identifying the preferences to be changed and recommending experts, how should be the changes in order to increase the agreement in the next consensus round.

9.3.1 Making the Information Uniform

Considering that we are dealing with heterogeneous contexts with numerical, interval-valued and linguistic information and because of there are not standard operators to manage directly heterogeneous information, we need to unify this into a common utility space that we will call basic linguistic term set (BLTS), $S_T = \{s_0, \ldots, s_g\}$ (Fig. 9.4). To do so, as it was proposed in (Herrera et al. 2005), we define different transformation functions to transform each numerical, interval-valued and linguistic preference value into a fuzzy set defined in BLTS, $F(S_T)$.

Transforming Numerical Values in $[0, 1]$ into $F(S_T)$

To transform a numerical value into a fuzzy set on S_T, we use the following function. Let ϑ be a numerical value, $\vartheta \in [0, 1]$, and $S_T = \{s_0, \ldots, s_g\}$ the BLTS. The function τ_{NS_T} that transforms a numerical value ϑ into a fuzzy set on S_T is defined as (Herrera and Martínez 2000):

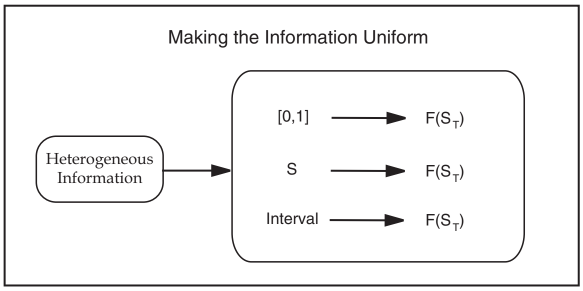

Fig. 9.4. Unification process of heterogeneous information

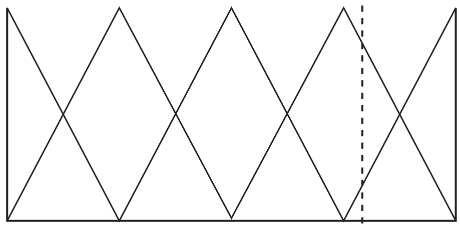

Fig. 9.5. Transforming a numerical value into a fuzzy set in S

$$\tau_{NS_T} : [0,1] \rightarrow F(S_T)$$
$$\tau_{NS_T}(\vartheta) = \{(s_0, \gamma_0), \ldots, (s_g, \gamma_g)\}, s_i \in S_T \, and \, \gamma_i \in [0,1]$$
$$\gamma_i = \mu_{s_i}(\vartheta) = \begin{cases} 0, & if \, \vartheta \notin Support(\mu_{s_i}(x)) \\ \frac{\vartheta - a_i}{b_i - a_i}, & if \, a_i \leq \vartheta \leq b_i \\ 1, & if \, b_i \leq \vartheta \leq d_i \\ \frac{c_i - \vartheta}{c_i - d_i}, & if \, d_i \leq \vartheta \leq c_i \end{cases}$$

Remark 1. We consider membership functions, $\mu_{s_i}(\cdot)$, for linguistic labels, $s_i \in S_T$, represented by a parametric function (a_i, b_i, d_i, c_i). A particular case are the linguistic assessments whose membership functions a triangular, i.e., $b_i = d_i$.

Example 1 *Let $\vartheta = 0.78$ be a numerical value to be transformed into a fuzzy set in $S = \{s_0, \ldots, s_4\}$. The semantic of this term set is:*

$$s_0 = (0, 0, 0.25), s_1 = (0, 0.25, 0.5), s_2 = (0.25, 0.5, 0.75), s_3 = (0.5, 0.75, 1)$$
$$s_4 = (0.75, 1, 1)$$

Therefore, the fuzzy set obtained is (see Fig. 9.5):

$$\tau_{NS_T}(0.78) = \{(s_0, 0), (s_1, 0), (s_2, 0), (s_3, 0.88), (s_4, 0.12)\}$$

Transforming Linguistic Terms in S into $F(S_T)$

To transform a linguistic value into a fuzzy set on S_T, we use the following function. Let $S = \{l_0, \ldots, l_p\}$ and $S_T = \{s_0, \ldots, s_g\}$ be two linguistic term sets, such that, $g \geq p$. Then, the function τ_{SS_T} that transforms $l_i \in S$ into a fuzzy set on S_T is defined as:

$$\tau_{SS_T} : S \rightarrow F(S_T)$$
$$\tau_{SS_T}(l_i) = \{(s_k, \gamma_k^i) \,/\, k \in \{0, \ldots, g\}\}, \, \forall l_i \in S$$
$$\gamma_k^i = \max_y \min\{\mu_{l_i}(y), \mu_{s_k}(y)\},$$

where $F(S_T)$ is the set of fuzzy sets defined in S_T, and $\mu_{l_i}(\cdot)$ and $\mu_{s_k}(\cdot)$ are the membership functions of the fuzzy sets associated with the terms l_i and s_k, respectively.

Therefore, the result of τ_{SS_T} for any linguistic value of S is a fuzzy set defined in S_T.

Remark 2. In the case that the linguistic term set S of the non-homogeneous contexts let be chosen as S_T then the fuzzy set that represents a linguistic term will be all **0** except the value correspondent to the ordinal of the linguistic label that will be **1**.

Example 2 *Let $S = \{l_0, l_1, \ldots, l_4\}$ and $S_T = \{s_0, s_1, \ldots, s_6\}$ be two term set, with 5 and 7 labels, respectively, and with the following semantics associated:*

$l_0 = (0, 0, 0.25)$	$s_0 = (0, 0, 0.16)$
$l_1 = (0, 0.25, 0.5)$	$s_1 = (0, 0.16, 0.34)$
$l_2 = (0.25, 0.5, 0.75)$	$s_2 = (0.16, 0.34, 0.5)$
$l_3 = (0.5, 0.75, 1)$	$s_3 = (0.34, 0.5, 0.66)$
$l_4 = (0.75, 1, 1)$	$s_4 = (0.5, 0.66, 0.84)$
	$s_5 = (0.66, 0.84, 1)$
	$s_6 = (0.84, 1, 1)$

The fuzzy set obtained after applying τ_{SS_T} for l_1 is (see Fig. 9.6):

$$\tau_{SS_T}(l_1) = \{(s_0, 0.39), (s_1, 0.85), (s_2, 0.85), (s_3, 0.39), (s_4, 0), (s_5, 0), (s_6, 0)\}.$$

Transforming Interval-Valued into $F(S_T)$

To transform an interval-valued into a fuzzy set on S_T, we use the following function. Let $I = [\underline{i}, \overline{i}]$ an interval valued in $[0, 1]$ and $S_T = \{s_0, \ldots, s_g\}$ the BLTS . Then, the function τ_{IS_T} that transforms the interval-valued I into a fuzzy set on S_T is defined as:

$$\tau_{IS_T} : I \rightarrow F(S_T)$$
$$\tau_{IS_T}(I) = \{(s_k, \gamma_k^i) \,/\, k \in \{0, \ldots, g\}\},$$
$$\gamma_k^i = \max_y \min\{\mu_I(y), \mu_{s_k}(y)\},$$

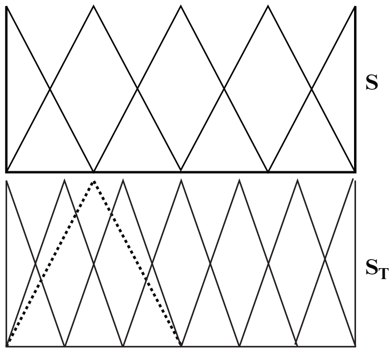

Fig. 9.6. Transforming $l_1 \in S$ into a fuzzy set in S_T

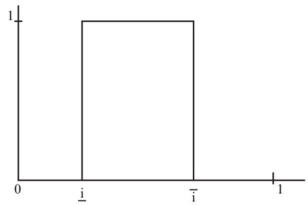

Fig. 9.7. Membership function of $I = [\underline{i}, \overline{i}]$

where $F(S_T)$ is the set of fuzzy sets defined in S_T, and $\mu_I(\cdot)$ and $\mu_{s_k}(\cdot)$ are the membership functions associated with the interval-valued I and terms s_k, respectively.

Remark 3. We assume that the interval-valued has a representation inspired in the membership function of fuzzy sets (Kuchta 2000):

$$\mu_I(\vartheta) = \begin{cases} 0, if\, \vartheta < \underline{i} \\ 1, if\, \underline{i} \leq \vartheta \leq \overline{i}, \\ 0, if\, \overline{i} < \vartheta \end{cases}$$

where ϑ is a value in $[0, 1]$. In Fig. 9.7 can be observed the graphical representation of an interval.

Example 3 *Let* $I = [0.6, 0.78]$ *be an interval-valued to be transformed into a fuzzy set in* S_T *with five terms symmetrically distributed. The fuzzy set obtained after applying* τ_{IS_T} *is (see Fig. 9.8):*

$$\tau_{IS_T}([0.6, 0.78]) = \{(s_0, 0), (s_1, 0), (s_2, 0.6), (s_3, 1), (s_4, 0.2)\}.$$

Results of the Unification Process

Once we have introduced in the previous subsections each one of the different transformation functions, to note that after the unification process and

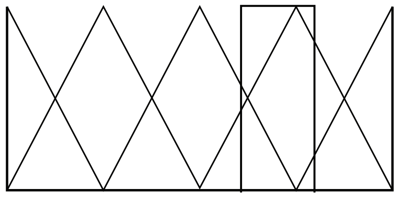

Fig. 9.8. Transforming $[0.6, 0.78]$ into a fuzzy set in S_T

assuming that each fuzzy set will be shown by means of its membership degrees $(\alpha_{i0}^{lk}, \ldots, \alpha_{ig}^{lk})$, the preferences of each expert will be represented as a matrix of fuzzy sets, $\tilde{\mathbf{P}}_{\mathbf{e_i}}$:

$$\tilde{\mathbf{P}}_{\mathbf{e_i}} = \begin{pmatrix} \tilde{p}_i^{11} = (\alpha_{i0}^{11}, \ldots, \alpha_{ig}^{11}) & \cdots & \tilde{p}_i^{1n} = (\alpha_{i0}^{1n}, \ldots, \alpha_{ig}^{1n}) \\ \vdots & \ddots & \vdots \\ \tilde{p}_i^{n1} = (\alpha_{i0}^{n1}, \ldots, \alpha_{ig}^{n1}) & \cdots & \tilde{p}_i^{nn} = (\alpha_{i0}^{nn}, \ldots, \alpha_{ig}^{nn}) \end{pmatrix}.$$

9.3.2 Computing Consensus Degree

The consensus degree evaluates the level of existent agreement among the experts. So, if experts' preferences are similar, the consensus degree will be high, else, if preferences are very different, the consensus degree will be low. To compute the level of agreement, a consensus matrix is obtained aggregating the values which represent the similarity or distance among the experts' preferences, comparing each other.

The distance between two preferences $\tilde{p}_i^{lk}$ and $\tilde{p}_j^{lk}$ is computed by means of the similarity function $s(\tilde{p}_i^{lk}, \tilde{p}_j^{lk})$ measured in the unit interval $[0, 1]$ (Herrera-Viedma et al. 2005):

$$s(\tilde{p}_i^{lk}, \tilde{p}_j^{lk}) = 1 - \left| \frac{cv_i^{lk} - cv_j^{lk}}{g} \right|. \tag{9.1}$$

The cv_i^{lk} is the central value of the fuzzy set:

$$cv_i^{lk} = \frac{\sum_{h=0}^{g} index(s_h^i) \cdot \alpha_{ih}^{lk}}{\sum_{h=0}^{g} \alpha_{ih}^{lk}}, \tag{9.2}$$

and represents the average position or center of gravity of the information contained in the fuzzy set $p_i^{lk} = (\alpha_{i0}^{lk}, \ldots, \alpha_{ig}^{lk})$, being $index(s_h^i) = h$. The range of this central value is the closed interval $[0, g]$.

The closer $s(\tilde{p}_i^{lk}, \tilde{p}_j^{lk})$ to 1 the more similar preferences p_i^{lk} and p_j^{lk} are, while the closer $s(\tilde{p}_i^{lk}, \tilde{p}_j^{lk})$ to 0 the more distant p_i^{lk} and p_j^{lk} are.

Once we have defined the function to evaluate the similarity, the consensus degree is computed according to the following steps:

1. First, the central values of all fuzzy sets are calculated:

$$cv_i^{lk};\ \forall\ i = 1, \ldots, m;\ \ l, k = 1, \ldots, n\ \wedge\ l \neq k. \tag{9.3}$$

2. Afterwards, for each pair of experts e_i and e_j $(i < j)$, a *similarity matrix* $SM_{ij} = \left(sm_{ij}^{lk}\right)$ is calculated, where

$$sm_{ij}^{lk} = s(\tilde{p}_i^{lk}, \tilde{p}_j^{lk}). \tag{9.4}$$

3. Finally a *consensus matrix*, CM, is obtained by aggregating all the similarity matrices

$$CM = \begin{pmatrix} cm^{11} & \cdots & cm^{1n} \\ \vdots & \ddots & \vdots \\ cm^{n1} & \cdots & cm^{nn} \end{pmatrix}.$$

This aggregation is carried out at the level of pairs of alternatives:

$$cm^{lk} = \phi(sm_{ij}^{lk});\ i, j = 1, \ldots, m\ \ \wedge \forall\ l, k = 1, \ldots, n\ \wedge\ i < j.$$

In our case, we use the arithmetic mean as the aggregation function ϕ, although, different aggregation operators could be used according to the particular properties we want to implement.

Interpretation of the Consensus Degree

The consensus degree is analyzed in three different levels: pairs of alternatives, alternatives and relations. In this way, we can know in a precise way the level of agreement in each pair and so to identify the pairs as well as the alternatives in which there exists greater disagreement.

Level 1. *Consensus on pairs of alternatives.* The consensus degree on a pair of alternatives (x_l, x_k), called cp^{lk}, measures the consensus degree amongst all the experts on that pair. In our case, this is expressed by the element (l, k) of the consensus matrix CM, i.e.,

$$cp^{lk} = cm^{lk},\ \ \forall l, k = 1, \ldots, n\ \ \wedge\ \ l \neq k.$$

Values of cp^{lk} close to 1 mean a greater agreement. This measure will allow the identification of those pairs of alternatives with a poor level of agreement.

Level 2. *Consensus on alternatives.* The consensus degree on an alternative x_l, called ca^l, measures the consensus degree amongst all the experts on that alternative. It is calculated as the average of each row l of the consensus matrix CM, i.e.,

$$ca^l = \frac{\sum_{k=1,\ l\neq k}^{n} cp^{lk}}{n-1}. \tag{9.5}$$

These values are used to propose the modification of preferences associated to those alternatives with a consensus degree lower than a minimal consensus threshold γ, i.e., $ca^l < \gamma$.

Level 3. *Consensus on relations or global consensus.* The consensus degree on relations, called cr, measures the global consensus degree amongst the experts' preferences. It is computed as the average of all the consensus degrees on the alternatives, i.e.,

$$cr = \frac{\sum_{l=1}^{n} ca^l}{n}. \tag{9.6}$$

The CSS uses this value to check the level of agreement achieved in each round, so if cr is closer to 1, the level of agreement is high, while if cr is closer to 0, the level of agreement is low.

9.3.3 Checking the Agreement

In this phase the CSS controls the level of agreement achieved in the current consensus round. Before applying the CSS model, a minimum consensus threshold, $\gamma \in [0, 1]$, is fixed, which will depend on the particular problem we are dealing with. When the consequences of the decision are of a transcendent importance, the minimum level of consensus required to make that decision should be logically high, for example $\gamma = 0.8$ or higher. At the other extreme, when the consequences are not so transcendental (but are still important) and it is urgent to obtain a solution of the problem, a fewer consensus threshold near to 0.5 could be required.

In any case, independently of the value γ, when the global consensus cr reaches γ, the CSS will stop and the selection process will be applied to obtain the solution. However, there is the possibility that the global consensus will not converge to consensus threshold and the process will get block. In order to avoid this circumstance, the model incorporates a parameter, $Maxcycles$, to limit the number of consensus rounds to carry out. The performance of this phase is shown in Fig. 9.9.

9.3.4 Generating Advice

When the agreement is not big enough, $cr < \gamma$, experts should modify their preferences in order to make them closer and increase the consensus in the next consensus round. To do so, we will use proximity measures to identify the furthest experts' preferences from the collective opinion. Once these preferences have been identified, a guided advance generator is in charge of suggesting how to change them in order to increase the consensus in the next consensus round. Both processes are presented in detail following.

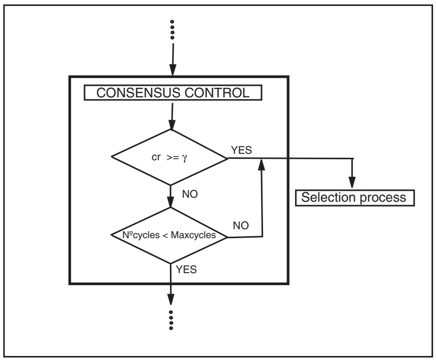

Fig. 9.9. Consensus control

Computing Proximity Measure

The proximity measure evaluates the distance between the individual experts' preferences and the collective preference. To calculate it, firstly we need to obtain a collective preference relations $\tilde{\mathbf{P}}_c$,

$$\tilde{\mathbf{P}}_{\mathbf{c}} = \begin{pmatrix} \tilde{p}_c^{11} & \cdots & \tilde{p}_c^{1n} \\ \vdots & \ddots & \vdots \\ \tilde{p}_c^{n1} & \cdots & \tilde{p}_c^{nn} \end{pmatrix}$$

which represents the group's opinion. $\tilde{\mathbf{P}}_{\mathbf{c}}$ is calculated by aggregating the set of (uniformed) individual preference relations $\{\tilde{\mathbf{P}}_{\mathbf{e_1}}, \ldots, \tilde{\mathbf{P}}_{\mathbf{e_m}}\}$:

$$\tilde{p}_c^{lk} = \psi(\tilde{p}_1^{lk}, \ldots, \tilde{p}_m^{lk}) = (\alpha_{c0}^{lk}, \ldots, \alpha_{cg}^{lk}),$$

where

$$\alpha_{cj}^{lk} = \psi(\alpha_{1j}^{lk}, \ldots, \alpha_{mj}^{lk})$$

being ψ an "aggregation operator".

Once the CSS has obtained the collective preference relation, it computes a proximity matrix, PM_i, for each expert e_i,

$$PM_i = \begin{pmatrix} pm_i^{11} & \cdots & pm_i^{1n} \\ \vdots & \ddots & \vdots \\ pm_i^{n1} & \cdots & pm_i^{nn} \end{pmatrix}.$$

To evaluate the proximity between each expert's individual preferences, $\tilde{\mathbf{P}}_{\mathbf{e_i}}$, and collective preferences, $\tilde{\mathbf{P}}_{\mathbf{c}}$, we use the similarity function defined in expression (9.1),

$$pm_i^{lk} = s(\tilde{p}_i^{lk}, \tilde{p}_c^{lk}).$$

These matrices contain the necessary information to know the position of the preferences of each expert with regards to the group's position.

Interpretation Proximity Measures

From the proximity matrices we can also know the proximity of the preferences of each expert at level of pairs of alternatives, alternatives and relations. In this way it is easy to identify the furthest experts on those assessments where the consensus is not enough:

Level 1. *Proximity on pairs of alternatives.* Given an expert e_i, his/her proximity measure on a pair of alternatives, (x_l, x_k), called pp_i^{lk}, measures the proximity between his/her preference value and the collective's one on that pair. In our case, this value coincides with the element (l, k) of the proximity matrix PM_i, i.e.,

$$pp_i^{lk} = pm_i^{lk}, \quad \forall l, k = 1, \ldots, n \quad \wedge \quad l \neq k.$$

Level 2. *Proximity on alternatives.* Given an expert e_i, his/her proximity measure on an alternative x_l, called pa^l, measures the proximity between his/her preference values on that alternative and the collective's ones. It is computed as the average of the proximities on pairs of alternatives of x_l

$$pa_i^l = \frac{\sum_{k=1, k\neq l}^{n} pp_i^{lk}}{n-1}. \tag{9.7}$$

Level 3. *Proximity on the relation.* Given an expert e_i, his/her proximity measure on the relation, called pr_i, measures the global proximity between his/her preference values on all alternatives and the collective's one. It is computed as the average of all proximity on alternative values, i.e.,

$$pr_i = \frac{\sum_{l=1}^{n} pa_i^l}{n}. \tag{9.8}$$

If pr_i is close to 1 then e_i contributes positively to the consensus, while if pr_i is close to 0 then e_i has a negative contribution to consensus.

Guided Advice Generator

The goal of the guided advice generator is to identify the furthest experts' preferences and suggest how to change them in order to increase the consensus.

To achieve this purpose the guided advice generator uses two types of advice rules: identification rules and direction rules.

Identification Rules (IR)

These rules identify what experts, alternatives and pairs of alternatives should be changed. In this way, the model only focuses on the preferences in disagreement and will not recommend to change those preferences where the agreement is enough. The model uses three rules:

1. *An identification rule of experts.* It identifies those experts that should change some of their preferences values. Previously, we should have decided the number or % of experts (ne) that should modify their preferences. The choice of the value of ne may depend on the type of problem and/or the amount of time available to achieve the consensus. If a quick achievement of consensus is desired, then the value of ne might be high (for example $ne = 75\%$), while if ne is low (for example $ne = 25\%$) then more time will be needed to reach the consensus. Once decided the number of experts, the ne experts with the lowest proximity values must change their preferences. This set of experts is denoted as $EXPCH$. Therefore, the identification rule of experts is the following:
   IR.1. $\forall e_i \in E \cap EXPCH$, then e_i must change his/her preferences, being
   $$EXPCH = \{e_{\sigma(1)}, \ldots, e_{\sigma(ne)}\},$$
   where σ is a permutation over the set of proximities on the relation defined as $pr_{\sigma(j)} \leq pr_{\sigma(i)}\ \forall\ j \leq i$.
2. *An identification rule of alternatives.* It identifies those alternatives where there is not enough consensus and therefore they should be changed. This set of alternatives is denoted as ALT and is composed of those alternatives whose consensus degree ca^l is lower than the consensus threshold γ, i.e.,
   $$ALT = \{x_l \in X |\ ca^l < \gamma\}.$$
   The identification rule of alternatives is the following:
   IR.2. $\forall e_i \in EXPCH$, e_i should change some assessments associated to the pairs that belong to the alternative x_l, such that, $x_l \in ALT$.
3. *An identification rule of pairs of alternatives.* It identifies those particular pairs of alternatives (x_l, x_k) of the alternatives in disagreement $x_l \in ALT$ that should be changed. This set of pairs of alternatives is denoted as $PALT_i$. To do this, we use the proximity measures on pairs of alternatives, being the identification rule the following:
   IR.3. $\forall(x_l \in ALT\ \wedge\ e_i \in EXPCH)$, if $(x_l, x_k) \in PALT_i$ then e_i should change p_i^{lk}, being $PALT_i$ the set of pairs of alternatives (x_l, x_k) whose proximity values at level of pairs, pp_i^{lk}, are fewer that a minimum proximity threshold, β, i.e.,
   $$PALT_i = \{(x_l, x_k) \mid x_l \in ALT\ \wedge\ e_i \in EXPCH\ \wedge\ pp_i^{lk} < \beta\}.$$
   Clearly, the greater β the greater the number of changes needed.

Direction Rules (DR)

Once the model has identified the pairs of alternatives to be changed, $(x_l, x_k) \in PALT_i$, it uses a set of direction rules to suggest how to change the current assessments in order to increase the agreement in the next consensus round. Taking into account that $\tilde{p}_i^{lk}$ is a fuzzy set, the guided advice generator defines two direction parameters: *ml* or main and *sl* or secondary. These parameters represent the value and position of the two highest membership values of the expert's preference $\left(\tilde{p}_i^{lk}(ml_{pos}), \tilde{p}_i^{lk}(ml_{val}), \tilde{p}_i^{lk}(sl_{pos}), \tilde{p}_i^{lk}(sl_{val})\right)$ and the collective preference $\left(\tilde{p}_c^{lk}(ml_{pos}),\ \tilde{p}_c^{lk}(ml_{val}), \tilde{p}_c^{lk}(sl_{pos}), \tilde{p}_c^{lk}(sl_{val})\right)$. The rules compare the position and value of the parameters *ml* and *sl* of the expert's preference and collective preference. According to the result of this comparison, the advice generator suggests increase o decrease the expert's current assessment.

These parameters are used by the following direction rules:

DR.1. If $\tilde{p}_i^{lk}(ml_{pos}) > \tilde{p}_c^{lk}(ml_{pos})$ then the expert e_i should decrease the assessment associated to the pair of alternatives (x_l, x_k).
DR.2. If $\tilde{p}_i^{lk}(ml_{pos}) < \tilde{p}_c^{lk}(ml_{pos})$ then the expert e_i should increase the assessment associated to the pair of alternatives (x_l, x_k).
DR.3. If $\tilde{p}_i^{lk}(ml_{pos}) = \tilde{p}_c^{lk}(ml_{pos})$ then rules DR.1, and DR.2 are applied using the membership values of the main labels, $\tilde{p}_i^{lk}(ml_{val})$ and $\tilde{p}_c^{lk}(ml_{val})$.
DR.4. If $\left(\tilde{p}_i^{lk}(ml_{pos}) = \tilde{p}_c^{lk}(ml_{pos}), \tilde{p}_i^{lk}(ml_{val}) = \tilde{p}_c^{lk}(ml_{val})\right)$, then rules DR.1, DR.2, and DR.3 are applied using the position and membership values of the secondary labels *sl*.

Example 4 *Given the expert's preference,* $\tilde{p}_1^{12} = (\underline{\boldsymbol{1}}, \underline{\boldsymbol{0.67}}, 0.33, 0, 0, 0, 0, 0, 0)$, *and the collective preference* $\tilde{p}_c^{12} = (\underline{\boldsymbol{0.38}}, 0.28, 0.14, 0.17, \underline{\boldsymbol{0.3}}, 0.27, 0.19, 0.11, 0.13)$, *their direction parameters are respectively:*

$$\tilde{p}_1^{12}(ml_{pos}) = 0, \quad \tilde{p}_1^{12}(ml_{val}) = 1, \qquad \tilde{p}_1^{12}(sl_{pos}) = 1, \quad \tilde{p}_1^{12}(sl_{val}) = 0.67,$$

$$\tilde{p}_c^{12}(ml_{pos}) = 0, \quad \tilde{p}_c^{12}(ml_{val}) = 0.38, \qquad \tilde{p}_c^{12}(sl_{pos}) = 4, \quad \tilde{p}_c^{12}(sl_{val}) = 0.3.$$

Finally to note that the consensus reaching process will depend on the size of the group of experts as well as on the size of the set of alternatives. So, when these sizes are small and when opinions are similar, the consensus level required is easier to obtain. However, when the experts opinions are quite different, the number of consensus rounds and the time to reach the wanted agreement will be greater.

9.4 Example of Application of the CSS Model

In this section we show an application example of the proposed CSS model to carry out a consensus reaching process in a real-word problem. We shall only focus on the consensus process, by recommending readers to consult (Delgado et al. 1998; Herrera and Martínez 2000; Herrera et al. 2005) to see how the selection of the best alternative(s) is carried out.

A drink company specializing in sport drinks is planning to launch a new soft drink, but first, it has to choose a taste that is accepted by the majority of the sportsmen. The company is considering four possible tastes:

- Lemon taste: x_1
- Apple taste: x_2
- Orange taste: x_3
- Peach taste: x_4

The management has decided to consult three experts. Experts have to express their preferences about the different tastes or alternatives by using preferences relations and they must reach a high level of agreement before making the decision. Each expert belongs to a different area and expresses his preferences by using a different information domain:

- The expert e_1 belongs to the marketing department and gives his preferences by means of numerical values in $[0,1]$, $\mathbf{P_{e_1}}$.
- The expert e_2 is an elite sportsman and prefers to use linguistic assessments of the linguistic term set S described in section "Fuzzy Linguistic Preference Relations" $\mathbf{P_{e_2}}$.
- The expert e_3 is a specialistic in soft drinks and gives his preferences by means interval-valued preference values in $[0,1]$, $\mathbf{P_{e_3}}$.

Note that the preferences p_i^{ll} do not have been considered because they represent the preference degree of an alternative over itself

$$\mathbf{P_{e_1}} = \begin{pmatrix} - & .5 & .8 & .4 \\ .3 & - & .9 & .3 \\ .3 & .2 & - & .4 \\ .9 & .8 & .5 & - \end{pmatrix}; \ \mathbf{P_{e_2}} = \begin{pmatrix} - & H & VH & M \\ L & - & H & VH \\ VL & N & - & VH \\ L & VL & N & - \end{pmatrix}$$

$$\mathbf{P_{e_3}} = \begin{pmatrix} - & [.7,.8] & [.65,.7] & [.8,.9] \\ [.3,.35] & - & [.6,.7] & [.8,.85] \\ [.3,.35] & [.3,.4] & - & [.7,.9] \\ [.1,.2] & [.2,.4] & [.1,.3] & - \end{pmatrix}.$$

9.4.1 First Round

Once the experts have provided their preferences, the CSS carries out the first round of the consensus reaching process following the phases described in the Sect. 9.3.

Making the Information Uniform

In this phase the heterogeneous information is unified into a common domain S_T. As we said in the Sect. 9.3.1, once an appropriate S_T has been chosen, the model applies different transformation functions τ_{DS_T} to transform each expert's preference into a fuzzy set defined on S_T, obtaining the following fuzzy sets:

$$\tilde{\mathbf{P}}_{\mathbf{e_1}} = \begin{pmatrix} - & (0,0,0,1,0,0,0) & (0,0,0,0,.19,.81,0) & (0,0,.59,.41,0,0,0) \\ (0,.19,.81,0,0,0,0) & - & (0,0,0,0,0,.59,.41) & (0,.19,.81,0,0,0,0) \\ (0,.19,.81,0,0,0,0) & (0,.81,.19,0,0,0,0) & - & (0,0,.59,.41,0,0,0) \\ (0,0,0,0,0,.59,.41) & (0,0,0,0,.19,.81,0) & (0,0,0,1,0,0,0) & - \end{pmatrix}$$

$$\tilde{\mathbf{P}}_{\mathbf{e_2}} = \begin{pmatrix} - & (0,0,0,0,1,0,0) & (0,0,0,0,0,1,0) & (0,0,0,1,0,0,0) \\ (0,0,1,0,0,0,0) & - & (0,0,0,0,1,0,0) & (0,0,0,0,0,1,0) \\ (0,1,0,0,0,0,0) & (1,0,0,0,0,0,0) & - & (0,0,0,0,0,1,0) \\ (0,0,1,0,0,0,0) & (0,1,0,0,0,0,0) & (1,0,0,0,0,0,0) & - \end{pmatrix}$$

$$\tilde{\mathbf{P}}_{\mathbf{e_3}} = \begin{pmatrix} - & (0,0,0,0,.81,.81,0) & (0,0,0,.12,1,.19,0) & (0,0,0,0,.19,1,.41) \\ (0,.19,1,.12,0,0,0) & - & (0,0,0,.41,1,.19,0) & (0,0,0,0,.19,1,.12) \\ (0,.19,1,.12,0,0,0) & (0,.19,1,.41,0,0,0) & - & (0,0,0,0,.81,1,.41) \\ (.41,1,.19,0,0,0,0) & (0,.81,1,.41,0,0,0) & (.41,1,.81,0,0,0,0) & - \end{pmatrix}$$

Computing Consensus Degrees

1. *Central values.* Applying (9.2), the model computes the central values of the fuzzy sets:

$$cv(e_1) = \begin{pmatrix} - & 3 & 4.81 & 2.41 \\ 1.81 & - & 5.41 & 1.81 \\ 1.81 & 1.19 & - & 2.41 \\ 5.41 & 4.81 & 3 & - \end{pmatrix} \quad cv(e_2) = \begin{pmatrix} - & 4 & 5 & 3 \\ 2 & - & 4 & 5 \\ 1 & 0 & - & 5 \\ 2 & 1 & 0 & - \end{pmatrix}$$

$$cv(e_3) = \begin{pmatrix} - & 4.5 & 4 & 5.13 \\ 1.94 & - & 3.86 & 4.94 \\ 1.94 & 2.13 & - & 4.81 \\ 0.86 & 1.81 & 1.18 & - \end{pmatrix}$$

2. *Similarity matrices.* The model computes a similarity matrix between each pair of experts by using the distance function (9.1):

$$SM_{12} = \begin{pmatrix} - & 0.83 & 0.96 & 0.9 \\ 0.96 & - & 0.76 & 0.46 \\ 0.86 & 0.8 & - & 0.56 \\ 0.43 & 0.36 & 0.5 & - \end{pmatrix} \quad SM_{13} = \begin{pmatrix} - & 0.75 & 0.87 & 0.54 \\ 0.97 & - & 0.74 & 0.47 \\ 0.97 & 0.84 & - & 0.59 \\ 0.24 & 0.5 & 0.69 & - \end{pmatrix}$$

$$SM_{23} = \begin{pmatrix} - & 0.91 & 0.84 & 0.64 \\ 0.99 & - & 0.97 & 0.99 \\ 0.84 & 0.64 & - & 0.97 \\ 0.81 & 0.86 & 0.8 & - \end{pmatrix}$$

3. *Consensus matrix.* The model calculates the consensus matrix by aggregating the similarity matrices:

$$CM = \begin{pmatrix} - & 0.83 & 0.89 & 0.69 \\ 0.97 & - & 0.82 & 0.64 \\ 0.89 & 0.76 & - & 0.71 \\ 0.49 & 0.57 & 0.66 & - \end{pmatrix}$$

4. *Consensus degrees.* The model computes the consensus degree at different levels:
   Level 1. *Consensus on pairs of alternatives.* The element (l, k) of CM represents the consensus degree on the pair of alternatives (x_l, x_k).
   Level 2. *Consensus on alternatives.*

$$ca^1 = 0.8,\ ca^2 = 0.81,\ ca^3 = 0.78,\ ca^4 = 0.57$$

   Level 3. *Consensus on the relations or global consensus.*

$$cr = 0.74$$

From these results, we can draw some conclusions:

1. The level of agreement in the pair (21) is very high, $cp^{21} = 0.97$, it means that the assessments given on that pair are very similar. On the contrary, the assessments given on the pair (41) have to be enough different because $cp^{41} = 0.49$ is low.
2. The alternative where the agreement is bigger is x_2, while the alternative with smaller agreement is x_4.
3. The level of global agreement among experts is not bad, $cr = 0.74$, but as we shall see following, it is not enough to finish the consensus process.

Checking the Agreement

In this phase the global consensus value cr is compared with the consensus threshold γ. In this example, we have decided to use a high consensus threshold, $\gamma = 0.8$. As $cr = 0.74 < \gamma$, the current consensus is not big enough to finish the consensus process and therefore the process must continue.

Production of Advice

In this phase the CSS identifies what preferences should be changed and how to carry out these changes.

Computation of Proximity Measures

The model computes the collective preference relation aggregating all individual preference relations using the average as aggregation operator:

1. Computing collective preferences

$$p_c^{12} = (0, 0, 0, 0.33, 0.6, 0.27, 0)$$
$$p_c^{13} = (0, 0, 0, 0.4, 0.39, 0.66, 0)$$
$$p_c^{14} = (0, 0, 0.19, 0.47, 0.06, 0.33, 0.13)$$
$$p_c^{21} = (0, 0.12, 0.93, 0.04, 0, 0, 0)$$
$$p_c^{23} = (0, 0, 0, 0.13, 0.66, 0.26, 0.13)$$
$$p_c^{24} = (0, 0.06, 0.27, 0, 0.06, 0.66, 0.04)$$

$$p_c^{31} = (0, 0.46, 0.6, 0.04, 0, 0, 0)$$
$$p_c^{32} = (0.33, 0.33, 0.39, 0.13, 0, 0, 0)$$
$$p_c^{34} = (0, 0, 0.19, 0.13, 0.27, 0.66, 0.13)$$
$$p_c^{41} = (0.13, 0.33, 0.39, 0, 0, 0.19, 0.13)$$
$$p_c^{42} = (0, 0.6, 0.33, 0.13, 0.06, 0.27, 0)$$
$$p_c^{43} = (0.47, 0.33, 0.27, 0.33, 0, 0, 0)$$

2. Proximity matrices. A proximity matrix for each expert is obtained:

$$PM_1 = \begin{pmatrix} - & 0.84 & 0.95 & 0.77 \\ 0.98 & - & 0.82 & 0.63 \\ 0.96 & 0.98 & - & 0.68 \\ 0.5 & 0.58 & 0.72 & - \end{pmatrix}; \quad PM_2 = \begin{pmatrix} - & 0.99 & 0.92 & 0.86 \\ 0.98 & - & 0.94 & 0.83 \\ 0.89 & 0.78 & - & 0.88 \\ 0.92 & 0.77 & 0.77 & - \end{pmatrix}$$

$$PM_3 = \begin{pmatrix} - & 0.9 & 0.91 & 0.77 \\ 0.99 & - & 0.92 & 0.84 \\ 0.94 & 0.85 & - & 0.91 \\ 0.73 & 0.91 & 0.97 & - \end{pmatrix}$$

3. Proximity measures. The model computes the proximity at different levels:
   Level 1. *Proximity on pairs of alternatives.* These values are equal to values of the proximity matrices.
   Level 2. *Proximity on alternatives.*

x_1	x_2	x_3	x_4
$pa_1^1 = 0.85$	$pa_1^2 = 0.81$	$pa_1^3 = 0.87$	$pa_1^4 = 0.6$
$pa_2^1 = 0.92$	$pa_2^2 = 0.92$	$pa_2^3 = 0.85$	$pa_2^4 = 0.82$
$pa_3^1 = 0.86$	$pa_3^2 = 0.92$	$pa_3^3 = 0.9$	$pa_3^4 = 0.87$

   Level 3. *Proximity on the relation.*

$$pr_1 = 0.78, pr_2 = 0.88, pr_3 = 0.89$$

According to the results, the furthest expert is e_1 and the nearest expert is e_3.

Guided Advice Generator

The model applies the identification rules to identify what preferences have to be changed and the direction rules to suggest how to make the changes.

Identification Rules

1. Set of experts to change their preferences, $EXPCH$. The ranking of the experts according to their proximity is e_3, e_2, e_1, being e_1 the furthest expert. In this example, like we are working with three experts, we will suggest that only one change their assessments, i.e., $ne = 1$:

$$EXPCH = \{e_1\}.$$

2. Set of alternatives whose assessments should be changed, ALT. In this case, as we have fixed a consensus threshold $\gamma = 0.8$, we have:

$$ALT = \{x_l \in X | \; ca^l < 0.8\} = \{x_3, x_4\}.$$

3. Set of pairs of alternatives whose associated assessments should be changed, $PALT_i$. At this point, the model identifies the pairs of alternatives that have to be changed taking into account a proximity threshold $\beta = 0.75$:

$$PALT_1 = \{(x_3, x_4), (x_4, x_1), (x_4, x_2), (x_4, x_3)\}$$

Finally, the list of preference to change is:

$$p_1^{34}, p_1^{41}, p_1^{42}, p_1^{43}$$

Direction Rules

1. Direction parameters.

	$\big(p_i^{lk}(ml_{pos}), p_i^{lk}(ml_{val}), p_i^{lk}(sl_{pos}), p_i^{lk}(sl_{val})\big)$	$\big(p_c^{lk}(ml_{pos}), p_c^{lk}(ml_{val}), p_c^{lk}(sl_{pos}), p_c^{lk}(sl_{val})\big)$
p_1^{34}	$(2, 0.59, 3, 0.41)$	$(5, 0.66, 4, 0.27)$
p_1^{41}	$(5, 0.59, 6, 0.41)$	$(2, 0.39, 1, 0.33)$
p_1^{42}	$(5, 0.81, 4, 0.19)$	$(1, 0.6, 2, 0.33)$
p_1^{43}	$(3, 1, *, 0)$	$(0, 0.47, 2, 0.27)$

(*) means that there are more than one possible secondary label candidates but they do not play any role in the production of recommendations.

2. Application of the direction rules.
   - Given that $p_1^{41}(ml_{pos}) > p_c^{41}(ml_{pos})$, $p_1^{42}(ml_{pos}) > p_c^{42}(ml_{pos})$ and $p_1^{43}(ml_{pos}) > p_c^{43}(ml_{pos})$, expert e_1 is advised to decrease these assessments according to the rule DR1.
   - Given that $p_1^{34}(ml_{pos}) < p_c^{41}(ml_{pos})$ expert e_1 is advised to increase this assessment according to the rule DR2.

9.4.2 Second Round

Following the previous advice given by de CSS model, the expert e_1 changes his preferences. In order to avoid abrupt changes in experts' preferences, we have decided to increase or decrease the current assessments 0.2.

$$\mathbf{P_{e_1}} = \begin{pmatrix} - & .5 & .8 & .4 \\ .3 & - & .9 & .3 \\ .3 & .2 & - & \mathbf{.6} \\ \mathbf{.7} & \mathbf{.6} & \mathbf{.3} & - \end{pmatrix}$$

Due to the CSS carries out the same operations in all rounds of consensus, in the following rounds we only show the results that provide us bigger information about the performance of the model.

Making the Information Uniform

The operations in this phase are the same than in the first round.

Computing Consensus Degree

1. *Consensus matrix.*

$$CM = \begin{pmatrix} - & 0.83 & 0.89 & 0.69 \\ 0.97 & - & 0.82 & 0.64 \\ 0.89 & 0.76 & - & 0.84 \\ 0.56 & 0.71 & 0.79 & - \end{pmatrix}$$

2. *Consensus degrees*. The model computes the consensus degree at different levels:

   Level 1. *Consensus on pairs of alternatives.* Elements (l, k) of the consensus matrix CM.

   Level 2. *Consensus on alternatives.*

$$ca^1 = 0.8,\ ca^2 = 0.81,\ ca^3 = 0.83,\ ca^4 = 0.69$$

   Level 3. *Consensus on the relations or global consensus.*

$$cr = 0.78$$

By comparing the results obtained in the first and second round, we can highlight that:

1. The level of agreement in the pair (41), $cp^{41} = 0.56$, is bigger in the second round than in the first round, $cp^{41} = 0.49$, therefore we can verify that decreasing the value given by the expert e_1 on p_1^{41}, e_1 has been able to bring near his assessment to the assessments given by e_2 and e_3.
2. The level of agreement in the alternatives affected by the changes has increased, therefore the correct direction of the changes have been recommended.

Checking the Agreement

Given that $cr = 0.78 < \gamma = 0.8$, the consensus degree is not big enough yet and the consensus process must continue.

Production of Advice

Computation of Proximity Measure

1. *Proximity measures.* The model computes the proximity at different levels:
   Level 1. *Proximity on pairs of alternatives* for expert e_i are given in PM_i.
   Level 2. *Proximity on alternatives.*

x_1	x_2	x_3	x_4
$pa_1^1 = 0.85$	$pa_1^2 = 0.81$	$pa_1^3 = 0.92$	$pa_1^4 = 0.73$
$pa_2^1 = 0.92$	$pa_2^2 = 0.92$	$pa_2^3 = 0.87$	$pa_2^4 = 0.86$
$pa_3^1 = 0.86$	$pa_3^2 = 0.92$	$pa_3^3 = 0.92$	$pa_3^4 = 0.9$

   Level 3. *Proximity on the relation.*

$$pr_1 = 0.83, pr_2 = 0.89, pr_3 = 0.9$$

Note that although e_1 has been able to bring near his preferences to the collective preference in the second round (from $pr_1 = 0.78$ to $pr_1 = 0.83$), e_1 continues being the furthest experts, and therefore, the CSS model will recommend him to change his preferences again.

Guided Advice Generator

Identification Rules

1. Set of experts to change their preferences, $EXPCH$.

$$EXPCH = \{e_1\}$$

2. Set of alternatives whose assessments should be changed, ALT.

$$ALT = \{x_l \in X |\ ca^l < 0.8\} = \{x_4\}$$

3. Set of pairs of alternatives whose associated assessments should be changed, $PALT_i$.

$$PALT_1 = \{(x_4, x_1), (x_4, x_2)\}$$

   List of preference to change:

$$p_1^{41}, p_1^{42}$$

Direction Rules

1. Direction parameters.

	$(p_i^{lk}(ml_{pos}), p_i^{lk}(ml_{val}), p_i^{lk}(sl_{pos}), p_i^{lk}(sl_{val}))$	$(p_c^{lk}(ml_{pos}), p_c^{lk}(ml_{val}), p_c^{lk}(sl_{pos}), p_c^{lk}(sl_{val}))$
p_1^{41}	$(5, 0.81, 4, 0.19)$	$(2, 0.39, 1, 0.33)$
p_1^{42}	$(4, 0.59, 3, 0.41)$	$(1, 0.6, 2, 0.33)$

2. Application of the direction rules.
   - Due to fact that $p_1^{41}(ml_{pos}) > p_c^{41}(ml_{pos})$ and $p_1^{42}(ml_{pos}) > p_c^{42}(ml_{pos})$, expert e_1 is advised to decrease these assessments according to the rule DR1.

9.4.3 Third Round

Following the advice given in the second round by de CSS, the expert e_1 changes his preferences.

$$\mathbf{P_{e_1}} = \begin{pmatrix} - & .5 & .8 & .4 \\ .3 & - & .9 & .3 \\ .3 & .2 & - & .6 \\ \mathbf{.5} & \mathbf{.4} & .3 & - \end{pmatrix}$$

Making the Information Uniform

The operations in this phase are the same than in the first round.

Computing Consensus Degree

1. *Consensus matrix.*

$$CM = \begin{pmatrix} - & 0.83 & 0.89 & 0.69 \\ 0.97 & - & 0.82 & 0.64 \\ 0.89 & 0.76 & - & 0.84 \\ 0.76 & 0.84 & 0.79 & - \end{pmatrix}$$

2. *Consensus degrees.* The model computes the consensus degree at different levels:

   Level 1. *Consensus on pairs of alternatives.* Elements (l, k) of the consensus matrix CM.

   Level 2. *Consensus on alternatives.*

$$ca^1 = 0.8,\ ca^2 = 0.81,\ ca^3 = 0.83,\ ca^4 = 0.8$$

   Level 3. *Consensus on the relations or global consensus.*

$$cr = 0.81$$

Checking the Agreement

Finally, in the third round the level of agreement is bigger than the consensus threshold, $cr = 81 > \gamma = 0.8$. Therefore, the experts have been able to reach the minimum level of agreement fixed initially and the consensus reaching process should finish. Immediately afterward, a selection process should be run to obtain the final solution of the decision problem.

As summary of this section and according to the results shown in each consensus rounds, if the experts follow the recommendation given by the model, we can affirm that the CSS achieves to increase the level of agreement during the consensus reaching process.

9.5 Conclusions

In this chapter we have proposed a CSS model to automate the consensus processes in GDM problems where the experts use different information domain to provide their opinions. Two main features may be emphasized about this model: (1) it is able to manage consensus processes in problems where experts use numerical, interval-valued or linguistic assessment to express their preferences, and (2) it is able to suggest the changes of preferences that experts should apply in order to reach the wanted consensus. The model can be used to substitute the figure of the moderator, avoiding in this way a possible moderator's partiality during the consensus reaching process.

This CSS model uses a methodology based on transformation functions to unify the heterogeneous information into a common domain. It also defines a similarity function based on central values of the fuzzy sets to compute two kind of measurements: the consensus degree and the proximity values. These calculations are carried out at three different levels: pairs of alternatives, alternatives and relations. Based on both measurements, a guided advice system has been designed to help the experts to identify the preferences where the disagreement is bigger and to suggest how to change such preferences in order to increase the agreement among the experts.

Once the experts have changed their preferences and have achieved a high level of consensus, they are prepared to carry out the process to choose the best alternative(s) to solve the outlined problem.

Acknowledgements

This chapter has been supported by the Research Project TIN2006-02121 and FEDER funds.

References

P.P. Bonissone and K.S. Decker. Selecting uncertainty calculi and granularity: An experiment in trading-off precision and complexity. *Uncertainty in Artificial Intelligence*, pages 217–247. North-Holland, Amsterdam, 1986.

G. Bordogna, M. Fedrizzi, and G. Pasi. A linguistic modeling of consensus in group decision making based on OWA operators. *IEEE Transactions on Systems, Man and Cybernetics, Part A: Systems and Humans*, 27:126–132, 1997.

N. Bryson. Group decision-making and the analytic hierarchy process: Exploring the consensus-relevant information content. *Computers and Operational Research*, 1(23):27–35, 1996.

T.X. Bui. *A Group Decison Support System for Cooperative Multiple Criteria Group Decison-Making*. Springer, Berlin Heidelberg New York, 1987.

C. Carlsson, D. Ehrenberg, P. Eklund, M. Fedrizzi, P. Gustafsson, P. Lindholm, G. Merkuryeva, T. Riissanen, and A.G.S. Ventre. Consensus in distributed soft environments. *European Journal of Operational Research*, 61:165–185, 1992.

M. Delgado, F. Herrera, E. Herrera-Viedma, and L. Martínez. Combinig numerical and linguistic information in group decision making. *Information Sciences*, 107:177–194, 1998.

Z.P. Fan and X. Chen. Consensus measures and adjusting inconsistency of linguistic preference relations in group decision making. *Lecture Notes in Artificial Intelligence*, 3613:130–139, 2005.

J. Fodor and M. Roubens. *Fuzzy Preference Modelling and Multicriteria Decision Support*. Kluwer, Dordrecht, 1994.

F. Herrera and E. Herrera-Viedma. Linguistic decision analysis: Steps for solving decision problems under linguistic information. *Fuzzy Sets and Systems*, 115:67–82, 2000.

F. Herrera and L. Martínez. An approach for combining linguistic and numerical information based on 2-tuple fuzzy representation model in decision-making. *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, 8(5):539–562, 2000.

F. Herrera, E. Herrera-Viedma, and J.L. Verdegay. A model of consensus in group decision making under linguistic assessments. *Fuzzy Sets and Systems*, 79:73–87, 1996.

F. Herrera, L. Martínez, and P.J. Sánchez. Managing non-homogeneous information in group decision making. *European Journal of Operational Research*, 166(1):115–132, 2005.

E. Herrera-Viedma, F. Herrera, and F. Chiclana. A consensus model for multiperson decision making with different preference structures. *IEEE Transactions on Systems, Man and Cybernetics-Part A: Systems and Humans*, 32(3):394–402, 2002.

E. Herrera-Viedma, F. Mata, L. Martínez, F. Chiclana, and L.G. Pérez. Measurements of consensus in multi-granular linguistic group decision making. *Lecture Notes in Artificial Intelligence*, 3131:194–204, 2004.

E. Herrera-Viedma, L. Martínez, F. Mata, and F. Chiclana. A consensus support system model for group decision-making problems with multi-granular linguistic preference relations. *IEEE Transactions on Fuzzy Systems*, 13(5):644–658, 2005.

J. Kacprzyk. Group decision making with a fuzzy linguistic majority. *Fuzzy Sets and Systems*, 18:105–118, 1986.

J. Kacprzyk, M. Fedrizzi, and H. Nurmi. “Soft” degrees of consensus under fuzzy preferences and fuzzy majorities. *Consensus Under Fuzziness*, pages 55–81. Kluwer, Dordrecht, 1997.

S.H. Kim, S.H. Choi, and J.K. Kim. An interactive procedure for multiple attribute group decision making with incomplete information: Range-based approach. *European Journal of Operational Research*, 118:139–152, 1999.

D. Kuchta. Fuzzy capital budgeting. *Fuzzy Sets and Systems*, 111:367–385, 2000.

L.I. Kuncheva. Five measures of consensus in group decision making using fuzzy sets. In *International Conference on Fuzzy Sets and Applications IFSA-95*, pages 141–144, 1991.

S. Kundu. Min-transitivity of fuzzy leftness relationship and its application to decision making. *Fuzzy Sets and Systems*, 86:357–367, 1997.

J.F. Le Téno and B. Mareschal. An interval version of PROMETHEE for the comparison of building products' design with ill-defined data on environmental quality. *European Journal of Operational Research*, 109:522–529, 1998.

E. Levrat, A. Voisin, S. Bombardier, and J. Bremont. Subjective evaluation of car seat comfort with fuzzy set techniques. *International Journal of Intelligent Systems*, 12:891–913, 1997.

M. Roubens. Fuzzy sets and decision analysis. *Fuzzy Sets and Systems*, 90:199–206, 1997.

S. Saint and J.R. Lawson. *Rules for Reaching Consensus. A Modern Approach to Decision Making*. Jossey-Bass, San Francisco, 1994.

E. Szmidt and J. Kacprzyk. A consensus reaching process under intuitionistic fuzzy preference relations. *International Journal of Intelligent System*, 18(7):837–852, 2003.

R.R. Yager. On ordered weighted averaging aggregation operators in multicriteria decision making. *IEEE Transactions on Systems, Man and Cybernetics*, 18:183–190, 1988.

R.R. Yager. An approach to ordinal decision making. *International Journal of Approximate Reasoning*, 12:237–261, 1995.

R.R. Yager. Protocol for negotiations among multiple intelligent agents. *Consensus Under Fuzziness*, pages 165–174. Kluwer, Dordrecht, 1997.

L.A. Zadeh. The concept of a linguistic variable and its applications to approximate reasoning. *Information Sciences, Part I, II, III*, 8, 8, 9:199–249, 301–357, 43–80, 1975.

L.A. Zadeh. Fuzzy logic = computing with words. *IEEE Transactions on Fuzzy Systems*, 4(2):103–111, 1996.

S. Zadrozny. An approach to the consensus reaching support in fuzzy environment. *Consensus Under Fuzziness*, pages 83–109. Kluwer, Dordrecht, 1997.

Q. Zhang, J.C.H. Chen, and P.P. Chong. Decision consolidation: Criteria weight determination using multiple preference formats. *Decision Support Systems*, 38(2):247–258, 2004.