A penalized consensus model for GDM problems with multi-granular linguistic information

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Abstract

In this paper, the consensus process in group decision making (GDM) problems with multi-granular linguistic information is addressed. Consensus reaching process is an iterative discussion process where a group of experts try to achieve a high level of agreement before making a decision. During the consensus process an expert can decide not to change his/her opinions like a strategy to impose his/her preferences. The aim of this paper is to propose a penalization mechanism in order to minimize the effects of this type of behavior. The mechanism takes into account the expert's weight or importance when the "group" opinion is calculated. The expert's importance is updated during the consensus process according to expert's behavior.

Keywords: Consensus, multi-granular linguistic information, group decision making, penalization.

1 Introduction

A GDM problem may be defined as a decision situation where a group of individuals or decision makers (e.g., experts, judges, ...) should choose the best alternative/s to solve the problem among a set of possible alternatives. Different approaches based on fuzzy methods have been proposed in the literature to deal with this kind of decision making [4, 12]. Usually GDM problems have been solved carrying out *Selection Processes* where it is obtained a solution set of alternatives from the preferences given by the experts [1]. However it may happen that some experts consider that their preferences have not been taken into account to obtain the final solution, and hence they do not agree with it. To avoid this situation, it is suitable to carry out a consensus process (see Figure 1) where the experts discuss and change their preferences in order to reach a sufficient agreement before applying the selection process [2, 5].

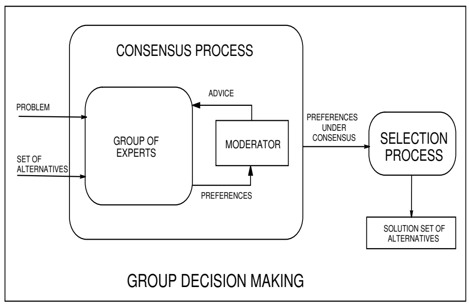

Figure 1: Resolution process of a GDM problem

In this kind of decision problems, traditionally the experts have provided their preferences by means of quantitative assessments, however, due to the increase of the complexity of the social-economic environments where the uncertainty is present, a linguistic approach may be more suitable. The use of linguistic assessments in decision making is appropriate when

i) a decision should be made under time pressure and lack of data [8], ii) many of the attributes are intangibles or they can not be assess by means of quantitative values [10] and iii) an expert has limited his/her capacity of information processing [6]. For example when the "design" or "comfort" of a car is being evaluated, linguistic terms like "fast", "very fast", "slow" could be preferred instead of numerical values [9]. The uncertainty that arises when we try to evaluate aspects of qualitative nature has been successfully addressed by the Fuzzy Linguistic Approach and the concept of linguistic variable [15]. A linguistic variable is defined as a variable whose values are not numbers but words or sentences in a natural or an appropriate artificial language.

In GDM problems it is possible that the experts belong to different research areas each other and therefore they have different degree of knowledge about the problem. In such cases, it seems logical the experts can use different linguistic term sets to express their preferences. A GDM problem under these circumstances is considered as a problem defined in a multi-granular linguistic context [10], being this the kind of problem tackled in this contribution.

In the literature, several approaches to automate the consensus reaching process have been proposed [3, 7, 13]. In all them, an operation to get the group opinion is carried out. This group or collective preference is obtained by aggregating the individual preferences. Usually the consensus processes are carried out into impartiality environments [14] where all the experts' preferences are considered with the same importance. However, may happen expert's objective is not to achieve a real agreement but to impose his/her own individual opinions in order to increase his/her personal interests or benefits. Indeed, an expert can decide not to change the preferences like a strategy to enhance his/her preferences. In order to prevent this type of malicious behavior into the consensus reaching process, a penalization mechanism could be used.

In this work, we propose an initial approach to a penalization mechanism for consensus reaching processes defined into multi-granular linguistic context. This mechanism will be incorporated to the model presented in [11]. The purpose of this mechanism is to prevent that any expert tries to manipulate the consensus reaching process by imposing his/her opinions. To do so, we propose to modify the computation of the collective preferences using an importance degree for each expert. This value will be updated during the consensus reaching process according to the experts follow the recommended changes by the model in order to achieve the agreement.

This contribution is organized as follows. In the Section 2 we briefly review the GDM problems with multi-granular information and the consensus reaching model. In the Section 3 the preferences penalization mechanism is set out. Section 4 shows the mechanism performance by means of example and in the Section 5 we draw some conclusions.

2 Preliminaries

2.1 A Multi-granular Linguistic GDM Problem

Let us focus on GDM problems defined on multi-granular linguistic contexts. A GDM problem may be defined as a decision making process where two or more experts, $E = \{e_1, e_2, \ldots, e_m\}$ $(m \geq 2)$, try to choose the best alternative/s from a set of alternatives $X = \{x_1, x_2, \ldots, x_n\}$ $(n \geq 2)$. An usual preference structure used by the experts to give their opinions is the preference relation, $P_{e_i} \subset X \times X$, where each value p_i^{jk} of the matrix represents the preference of alternative x_j over the alternative x_k according the expert e_i [1].

Into a linguistic context the experts use linguistic terms to assess their preferences, $\mu_{P_{e_i}} : X \times X \rightarrow S$, where $S = \{s_0, s_1, \ldots, s_g\}$ is an appropriate linguistic term set characterized by its cardinality or granularity, $\#(S) = g+1$. The granularity represents the discrimination level among different degrees of uncertainty. Moreover, S must have the following properties [10]:

1. The set S is ordered: $s_i \geq s_j$, if $i \geq$ j.

2. There is the negation operator: $Neg(s_i) = s_j$ such that $j = g - i$.

The semantics of the terms is represented by means of fuzzy numbers defined on the [0,1] interval. One way to characterize a fuzzy number is using a representation based on parameters of its membership function [15]. For example, the following semantics, depicted in Figure 2, can be assigned to a set of seven terms via triangular fuzzy numbers:

$P = Perfect = (.83, 1, 1)$ $VH = Very\_High = (.67, .83, 1)$
$H = High = (.5, .67, .83)$ $M = Medium = (.33, .5, .67)$
$L = Low = (.17, .33, .5)$ $VL = Very\_Low = (0, .17, .33)$
$N = None = (0, 0, .17)$.

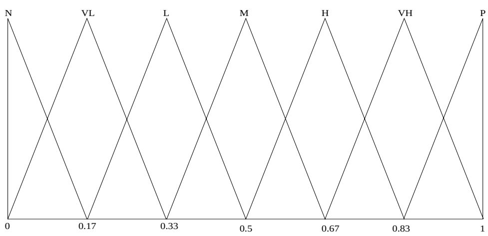

Figure 2: A set of seven linguistic terms

The ideal situation of a linguistic GDM problems would be one where all the experts use the same linguistic term set S to provide their preferences. However, in some cases, experts may belong to distinct research areas and have different knowledge levels about the alternatives. In consequence, preferences will be expressed using linguistic term sets with different granularity, which means that appropriate tools to manage and model these multi-granular linguistic information become essential [10].

In this paper, we deal with multi-granular linguistic GDM problems, where each expert e_i may give his/her preferences using preference relations $\mathbf{P_{e_i}} = (p_i^{jk})$, $p_i^{jk} \in S_i$, and each $S_i = \{s_0^i, \ldots, s_g^i\}$ has different cardinality.

2.2 Consensus Model for Multi-granular Linguistic GDM Problems

Here, we briefly describe the consensus model proposed for GDM problems with multi-granular linguistic information. For a further review see [3, 11].

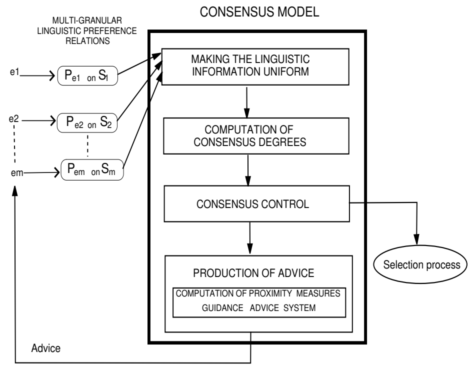

Figure 3: Consensus model with multi-granular linguistic information

This model is composed of four phases (see Figure 3):

1. *Making the linguistic information uniform.* In this phase, the multi-granular linguistic preferences are unified into a single linguistic domain in order to might work with them. To unify the multi-granular linguistic information, transformation functions are defined.

2. *Computation of consensus degrees.* In this phase, the model computes the consensus degrees among the experts. The agreement is obtained at level of pairs of alternatives, alternatives and preference relations.

3. *Consensus control.* In this phase the level of agreement reached is checked. As result, the model decides to continue or to finish the consensus reaching process.

4. *Production of advice.* In this phase, proximity measures to identify the furthest experts' preferences are calculated. To

get them, first one the model obtains a group opinion by aggregating the individual preferences. As we shall sequently see, in this contribution we propose to modify the aggregation operation by incorporating a preferences penalization strategy. Afterward, the model runs a guidance advice system based on a set of direction rules to recommend the changes in the experts' preferences. By means of a feedback mechanism, the experts must apply these suggestions to make their preferences closer and so to increase the level of agreement in the following consensus round.

3 Preferences Penalization Mechanism

As we said before, the purpose of the consensus reaching process is to achieve an agreement before making a decision. This process consists of several rounds where the experts discuss and change their preferences according to suggestions given by a moderator, in our case by the guidance advice system. In any discussion process whose objective is to achieve the consensus, experts should be willing to change their preferences. However, it may happen some experts keep in mind to impose their preferences and then they decide not to modify their opinions. In order to prevent this type of malicious behavior, we have incorporated in the model a penalization mechanism.

In the literature different penalization strategies have been proposed. For instance, to penalize each expert according to the score given about the preferences, in particular about those that expert considers as unacceptable [14]. Another strategy is to apply a constant penalization value which does not depend on the assessments given.

In view of so possibilities, in this contribution we have decided to apply an increasing penalization mechanism. That is, if an expert does not apply the changes of preferences suggested by the model, then his/her preferences will be penalized. The penalization value will be increased in each round in which the expert does not follow the recommendations given by the consensus model. Indeed, whether an expert refuses to change their preferences several times, we could think the expert does not want to reach an agreement but to impose their preferences. In such a case, the preferences of that expert will not be considered into the consensus process from that moment.

To carry out the penalization mechanism, initially the model assigns each expert the same weight or importance. The expert's importance value, called I_i^{rd} (rd represents the current round number), will be decreased when the expert does not change his/her preferences according to model's suggestions. So, in the first consensus round all the experts have the same importance, i.e., $I_i^1 = \frac{1}{m}$, $i = 1\ldots, m$, while from the second round I_i^{rd} will take a value according to the following expression:

$$I_i^{rd} = I_i^{rd-1} - \frac{I_i^{rd-1} \cdot nr}{max\_times} \qquad (1)$$

where nr is the number of times that e_i has refused to make the changes and max_times is the maximum number of times that an expert can make it. The max_times value will be fixed in advance.

In order to understand the penalization mechanism performance, it is necessary to review some features of the consensus model proposed in [11]:

i) Due to we deal with multi-granular linguistic information, in the unification phase, all the experts' preferences are transformed into fuzzy sets, $\tilde{p}_i^{lk}$, defined on a single domain denoted as $S_T = \{s_0, \ldots, s_g\}$ [10],

$$\tilde{p}_i^{lk} = \tau_{S_i S_T}(p_i^{lk}) = \{(s_h, \alpha_{ih}^{lk}) \mid h = 0, \ldots, g\}$$
$$\alpha_{ih}^{lk} = \max_y min\{\mu_{p_i^{lk}}(y), \mu_{c_h}(y)\}.$$

where at least $\exists\ \alpha_{ih}^{lk} > 0$ and $\forall\ \alpha_{ih}^{lk} \in [0, 1]$.

To simplify the representation, we shall only use the membership degrees to denote each fuzzy set $\tilde{p}_i^{lk}$,

$$\tilde{p}_i^{lk} = (\alpha_{i0}^{lk}, \ldots, \alpha_{ig}^{lk})$$

ii) Assuming all experts have the same importance, a collective preference relation is computed, $\tilde{\mathbf{P}}_{\mathbf{e_c}} = (\tilde{p}_c^{lk})$, by aggregating all the uniformed individual preferences relations $\{\tilde{\mathbf{P}}_{\mathbf{e_1}}, \ldots, \tilde{\mathbf{P}}_{\mathbf{e_m}}\}$,

$$\tilde{p}_c^{lk} = \psi(\tilde{p}_1^{lk}, \ldots, \tilde{p}_m^{lk})$$

where

$$\tilde{p}_c^{lk} = (\alpha_{c0}^{lk}, \ldots, \alpha_{cg}^{lk})$$

and

$$\alpha_{cj}^{lk} = \psi(\alpha_{1j}^{lk}, \ldots, \alpha_{mj}^{lk}),$$

using as aggregation operator ψ the arithmetic mean. Collective preference relation represents the group opinion and it is used to identify the furthest experts' preferences.

iii) The model computes the centre of gravity of the information contained in each fuzzy set $\tilde{p}_i^{lk} = (\alpha_{i0}^{lk}, \ldots, \alpha_{ig}^{lk})$, called central value:

$$cv(\tilde{p}_i^{lk}) = \frac{\sum_{h=0}^{g} h \cdot \alpha_{ih}^{lk}}{\sum_{h=0}^{g} \alpha_{ih}^{lk}}, \qquad (2)$$

being h the position of α_{ih}^{lk} into the fuzzy set

Here, in this contribution, we propose to substitute the current aggregation operator by another one which considers the experts' importance in each consensus round. So, when an expert is more important than another, this importance should be reflected on the collective preference value obtained from the aggregation operation. To do so, we suggest to use as aggregation operator a weighted average which takes into account the expert's importance degree, I_i^{rd}, in each round. Now, given that $p_c^{lk} = (\alpha_{c0}^{lk}, \ldots, \alpha_{cg}^{lk})$ is a fuzzy set, each α_{cj}^{lk} will be calculated as:

$$\alpha_{cj}^{lk} = \frac{\sum_{i=1}^{m} I_i^{rd} \cdot \alpha_{ij}^{lk}}{\sum_{i=1}^{m} I_i^{rd}} \qquad (3)$$

4 Penalization Mechanism Application

In this section we show the penalization mechanism performance. To do so, we shall use the example proposed in [3]. A investment company wats to invest a sum of money among four possible industrial sectors:

- Car industry: x_1
- Food company: x_2
- Computer company: x_3
- Arms industry: x_4

Four experts from different departments are consulted. Each expert uses a linguistic term set with a different semantic (see Table 1) to express his/her preferences:

- e_1 and e_2 provide their preferences by using a linguistic term set of granularity 5, **C**.
- e_3 provides preferences using a linguistic term set of granularity 9, **A**.
- e_4 provides preferences using a linguistic term set of granularity 7, **B**.

Set A	**Set B**	**Set C**
$a_0 = (0, 0, 0.13)$	$b_0 = (0, 0, 0.17)$	$c_0 = (0, 0, 0.25)$
$a_1 = (0, 0.13, 0.25)$	$b_1 = (0, 0.17, 0.33)$	$c_1 = (0, 0.25, 0.5)$
$a_2 = (0.12, 0.25, 0.38)$	$b_2 = (0.17, 0.33, 0.5)$	$c_2 = (0.25, 0.5, 0.75)$
$a_3 = (0.25, 0.38, 0.5)$	$b_3 = (0.33, 0.5, 0.67)$	$c_3 = (0.5, 0.75, 1)$
$a_4 = (0.38, 0.5, 0.63)$	$b_4 = (0.5, 0.67, 0.83)$	$c_4 = (0.75, 1, 1)$
$a_5 = (0.5, 0.63, 0.75)$	$b_5 = (0.67, 0.83, 1)$	
$a_6 = (0.63, 0.75, 0.88)$	$b_6 = (0.83, 1, 1)$	
$a_7 = (0.75, 0.88, 1)$		
$a_8 = (0.88, 1, 1)$		

Table 1: Semantics of the linguistic term sets

The initial preferences given by the experts are:

$$\mathbf{P_{e_1}} = \begin{pmatrix} - & c_0 & c_0 & c_2 \\ c_4 & - & c_3 & c_4 \\ c_3 & c_0 & - & c_1 \\ c_2 & c_1 & c_3 & - \end{pmatrix} \quad \mathbf{P_{e_2}} = \begin{pmatrix} - & c_2 & c_0 & c_4 \\ c_1 & - & c_1 & c_1 \\ c_3 & c_3 & - & c_1 \\ c_0 & c_4 & c_3 & - \end{pmatrix}$$

$$\mathbf{P_{e_3}} = \begin{pmatrix} - & a_1 & a_4 & a_3 \\ a_5 & - & a_8 & a_4 \\ a_4 & a_1 & - & a_2 \\ a_5 & a_5 & a_7 & - \end{pmatrix} \quad \mathbf{P_{e_4}} = \begin{pmatrix} - & b_0 & b_4 & b_5 \\ b_6 & - & b_1 & b_6 \\ b_3 & b_4 & - & b_2 \\ b_0 & b_1 & b_4 & - \end{pmatrix}$$

First one we shall see the results returned by the model without applying the penalization mechanism and afterwards applying it. In the example, we only focus on the second consensus round, by distinguishing three cases:

- Case 1. The experts follow the recommendations given by the model. In the first round the expert e_2 and e_4 give the following assessments:

$$p_2^{21} = c_1, \quad p_4^{21} = b_6$$

Once the preferences are transformed into fuzzy sets, the central values according to (2) are computed:

$$cv(\tilde{p}_2^{21}) = 2.02, \quad cv(\tilde{p}_4^{21}) = 7.5$$

At the end of the first round the model recommends to change such preferences:

$$\begin{array}{l} p_2^{21} = c_1 \Rightarrow c_2 \ \ (Increase) \\ p_4^{21} = b_6 \Rightarrow b_5 \ \ (Decrease) \end{array}$$

The collective preference obtained in the second round after carrying out the aggregation operation by using the arithmetic mean is:

$$\tilde{p}_c^{21} = (0, 0, 0.08, 0.17, 0.25, 0.5, 0.35, 0.38, 0.36)$$

being its central value $cv(\tilde{p}_c^{21}) = 5.65$. The proximities at level of pairs of alternatives on those preferences are:

$$pp_2^{21} = 0.8, \quad pp_4^{21} = 0.88$$

- Case 2. The experts do not follow the recommendations given by the model. Let us suppose now that e_4 does not change the assessment given on p_4^{21} with the purpose of manipulating the consensus process. Applying the same aggregation operation than in the first case, the collective value obtained is:

$$\tilde{p}_c^{21} = (0, 0, 0.08, 0.17, 0.25, 0.42, 0.2, 0.3, 0.5)$$

being in this case the central value of fuzzy set $cv(\tilde{p}_c^{21}) = 6.07$, and the proximity measures:

$$pp_2^{21} = 0.74 \quad pp_4^{21} = 0.82$$

- Case 3. The model applies the penalization mechanism. The model detects that e_4 has not changed the value given to p_4^{21} and it decides to penalize this assessment. We shall fix a $max\_times = 4$. The importance of the experts e_1, e_2 and e_3 in the second round will be 1/4, while for e_4 according to (1) is:

$$I_4^2 = I_4^1 - \frac{I_4^1 \cdot 1}{4} = 1/4 - \frac{1/4 \cdot 1}{4} = 0.187$$

The collective preference is calculated now by considering the experts' importance (3),

$$\tilde{p}_c^{21} = (0, 0, 0.09, 0.18, 0.27, 0.45, 0.2, 0.29, 0.47),$$

being its central value $cv(\tilde{p}_c^{21}) = 5.66$ and the proximities

$$pp_2^{21} = 0.79, \quad pp_4^{21} = 0.77$$

From the results, we can deduce that:

a) If e_4 does not change p_4^{21}, e_4 achieves to make the collective preference closer to his/her preference. In the case 1, $cv(\tilde{p}_c^{21}) = 5.65$, in case 2, $cv(\tilde{p}_c^{21}) = 6.07$ and expert's preference $cv(\tilde{p}_4^{21}) = 7.5$. In addition, we can observe as in the case 2 the expert e_2 is far away now and predictably the model will recommend him/her to change this assessment again.

b) The $cv(\tilde{p}_c^{21}) = 5.66$ in the case 3 and $cv(\tilde{p}_c^{21}) = 5.65$ in the case 1 are very similar, therefore, penalization mechanism achieves that e_4 does not impose his/her preferences in the collective opinion.

c) In the case 3, the proximity of the e_2 is hardly affected by the penalization mechanism, however it does not happen the same for the e_4 whose proximity is penalized and therefore the model will recommend to change it again in the following round.

5 Conclusions

In this contribution we have proposed a penalization mechanism for a consensus reaching model with multi-granular linguistic information. This mechanism minimizes the undesirable effects that a manipulation strategy of the consensus process carried out by a malicious expert can generate. To do so, the mechanism penalizes the malicious expert's preferences during the aggregation operation.

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