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A preliminary study of the effects of different aggregation operators on consensus processes

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***Abstract*—Searching for consensus in group decision making is a process in which experts change their preferences in order to achieve a minimum agreement before making a decision. Computing the consensus degree among experts and the group collective opinion by aggregating experts' opinions are two main tasks in a consensus reaching process. In this contribution we have studied the effects of different aggregation operators on the consensus processes. In particular, we have analyzed the obtained outcomes by three different aggregation operators: arithmetic mean, OWA with the linguistic quantifier "most" and Dependent OWA. Finally, some preliminary conclusions about the obtained results and the influence of these aggregation operations on consensus processes are drawn.**

***Keywords*-Consensus; group decision making; aggregation operators;**

I. Introduction

A group decision making (GDM) problem is defined as a decision making problem where several decision makers (judges, experts, ...) attempt to achieve a common solution about a set of alternatives. Usually, this type of problem has been tackled in the literature by carrying out a selection process whose result is a set with the best alternative(s) to solve the problem [1]. However, this solution set might be rejected by some experts whether they consider that their opinions have not been taken into account properly. So, carrying out a consensus reaching process in order to reach a minimum agreement before applying the selection process could be advisable.

A consensus reaching process may be seen as an iterative process composed by several rounds where experts express, discuss and modify their preferences in order to achieve a good agreement. Normally, this process is guided by the figure of a moderator, who helps experts to make their preferences closer to each other. Several approaches have been proposed to model consensus reaching processes, among them, we would like to highlight our proposals to manage heterogeneous information [2], [3]. In any consensus reaching process, two important tasks are to compute the level of agreement and the collective opinion of all experts. Usually, both tasks are accomplished aggregating the individual experts' preferences. Many aggregation operators may be found in the literature, from simple arithmetic mean to fuzzy aggregation operators [4]. Note the ordered weighted averaging (OWA) operators family [5] and its later extensions which have been successfully applied in many areas related to decision making.

In this contribution we study the behaviour of a consensus model according to different aggregation operators: arithmetic mean, OWA guided by the quantifier linguistic "most" [6] and Dependent OWA [7]. These operators have been chosen because we consider that the aggregation technique is appropriate for consensus processes. So, the arithmetic mean takes into account all preferences in the same way, $OWA(\text{“}most\text{”})$ considers the preferences of the majority of the experts (extreme values are penalized) and DOWA assigns weights to experts' preferences according to their distances regarding the central value of all preferences. Our aim is to study effects of these operators on the development of the consensus process. Moreover, we propose computing the consensus degree by using a typical OWA operator with different "orness" degrees. In this way, we can approach the consensus process from different points of view (optimistic, pessimistic or neutral) in the same line as advocated by Yager in [8], [9]. This preliminary study has allowed us to establish a prior assumptions on the relationship between the consensus processes and these aggregation operations.

This work is set out as follows. In Section 2, we summarise the theoretical basic of aggregation operators used in this study: OWA and DOWA operators. In Section 3, a brief revision of the consensus model is carried out. In Section 4, we show an example and the main results obtained as well as some assumptions concerning the consensus process and aggregation operators. Finally, in Section 5 we draw our conclusions.

II. OWA and DOWA Aggregation Operators

A. Ordered Weighted Averaging Operator

The ordered weighted averaging operator was proposed by Yager [8] to aggregate human judgments by using a weighting vector not associated directly to a particular

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judgment but rather to an ordered position of the set of judgments.

Definition: An OWA operator of dimension n is a mapping $R^n \rightarrow R$, which has an associated weighting vector $W = [w_1, w_2, \ldots, w_n]$, such that $w_i \in [0, 1]$ and $\sum_{i=1}^{n} w_i = 1$, whose mathematical expression is,

$$OWA(a_1, \ldots, a_n) = \sum_{i=1}^{n} w_i b_i \quad (1)$$

being b_i the ith largest element of the arguments set $(a_1, \ldots, a_n)$ ordered in descending order. The OWA operators family has the flexibility to use and entire range of weighting vectors to reproduce "or" and "and" operations (i.e. maximum and minimum) according to experts' attitudes concerning aggregation (optimistic or pessimistic). So, a measurement to evaluate the degree of "orness" [8] has been defined as:

$$orness(W) = \frac{1}{n-1} \sum_{i=1}^{n} (n-i) w_i, \quad (2)$$

with $orness(W) \in [0, 1]$. This measurement estimates how similar is the OWA operator to pure "or" operator. When $W = [1, 0, \ldots, 0]$, the $orness(W) = 1$ and the OWA behavior is similar to "or" operator. In this case the optimism is maximum. However, with $W = [0, 0, \ldots, 1]$, the $orness(W) = 0$ and the OWA behavior is similar to "and" operator and the pessimism is maximum. This interpretation of the "orness" will be used by the model to compute the level of agreement as we will explain in the section 3.1

A very interesting approach about OWA operators arises when the linguistic quantifiers proposed by Zadeh [10] are used to compute the weighting vector W [5]. A linguistic quantifier is defined as a function $Q : [0, 1] \rightarrow [0, 1]$, such that $Q(0) = 0$, $Q(1) = 1$, and if $x > y$ then $Q(x) \geq Q(y)$. An OWA aggregation guided by a linguistic quantifier Q is defined as [6]:

$$\Phi_Q(a_1, \ldots, a_n) = \sum_{i=1}^{n} w_i bi \quad (3)$$

being b_i the ith largest element of a_j, with,

$$w_i = Q\Big(\frac{i}{n}\Big) - Q\Big(\frac{i-1}{n}\Big), \quad i = 1, \ldots, n. \quad (4)$$

This approach has been broadly used in fuzzy group decision making to aggregate experts' preferences, particulary to represent the concept of fuzzy majority [11], [12], [13]. In this contribution we use an OWA operator guided by the linguistic quantifier "most" defined as:

$$Q(x) = \begin{cases} 0 & x \leq a \\ \frac{x-a}{b-a} & a < x < b \\ 1 & x \geq b \end{cases} \quad (5)$$

to compute the collective preference.

B. Dependent OWA Operator

The dependent OWA operator (DOWA) was introduced by Xu [7] as a new approach to OWA aggregations where the estimation of weights depends on the aggregated arguments. This operator was proposed as a possible solution to some of the drawbacks of typical OWA aggregation, for instance, its bad behavior to deal with very high or low values. DOWA computes the weight of the arguments according to its distance regarding the center of all arguments (i.e. arithmetic mean). This centralized interpretation has also been adopted in the centered OWA operator [14], where weights are high close to arithmetic mean and decrease towards extreme ones.

The weights of the DOWA operator are obtained according to the following expressions: Let $(a_1, \ldots, a_n)$ be a set of arguments and φ the arithmetic mean of the set. The weights of the weighting vector $W = [w_1, w_2, \ldots, w_n]$ are computed as:

$$w_j = \frac{s(a_j, \varphi)}{\sum_{i=1}^{n} s(a_i, \varphi)}, \; j = 1, 2, \ldots, n \quad (6)$$

where $s(\cdot)$ is the similarity between any argument a_j and the average value φ,

$$s(a_j, \varphi) = 1 - \frac{\mid a_j - \varphi \mid}{\sum_{i=1}^{n} \mid a_i - \varphi \mid}. \quad (7)$$

Note that at least one preference must to be different among all experts' preferences. Finally, DOWA operator is defined as:

$$DOWA(a_1, \ldots, a_n) = \sum_{i=1}^{n} w_i a_i.$$

Here, the weight of each argument is proportional to its magnitude into the set of arguments. Therefore, the typical reordering step of OWA operators is irrelevant for the DOWA operator.

III. Consensus Model Description

A GDM problem is classically defined as a decision situation where given a set of alternatives $X = \{x_1, x_2, \ldots, x_n\}$ $(n \geq 2)$, a group of experts, $E = \{e_1, e_2, \ldots, e_m\}$ $(m \geq 2)$, try find out the best alternative/s to solve the decision problem. In this contribution we assume that preferences are provided by experts by means of reciprocal preference relations [15], $\mathbf{P_{e_i}} = [p_i^{lk}]$, $l, k \in \{1, \ldots, n\}$, with $p_i^{lk} = \mu_{\mathbf{P_{e_i}}}(x_l, x_k)$ assessed in the unit interval $[0, 1]$ and $p_i^{lk} + p_i^{kl} = 1$. The preference p_i^{lk} is interpreted as the preference degree of the alternative x_l over x_k according to the expert e_i.

The consensus model used in this contribution consists of the following phases (see Figure 1):

1) **Computing the consensus degree from a neutral, optimistic or pessimistic point of view**. In this phase the level of agreement among all experts, called cr, is computed by aggregating their preferences. Firstly, for

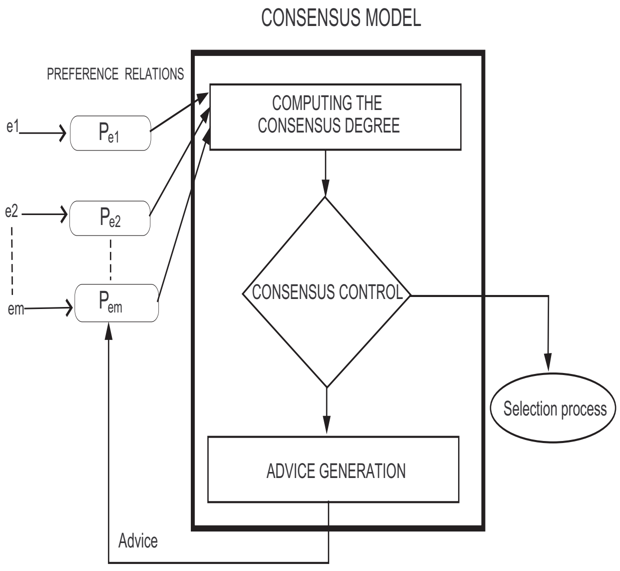

Figure 1. Consensus model

each pair of experts e_i, e_j ($i < j$), a similarity matrix, $SM_{ij} = [sm_{ij}^{lk}]$, is obtained as,

$$sm_{ij}^{lk} = 1- \mid p_i^{lk} - p_j^{lk} \mid \quad (8)$$

being sm_{ij}^{lk} the similarity between the experts e_i and e_j on the pair of alternatives (x_l, x_k). Secondly, a consensus matrix, $CM = [cm^{lk}]$, is calculated by aggregating at level of pairs all similarity matrices:

$$cm^{lk} = \phi(sm_{12}^{lk}, sm_{13}^{lk}, \ldots, sm_{1m}^{lk}, sm_{23}^{lk}, \ldots, sm_{2m}^{lk}, \ldots, sm_{(m-1)m}^{lk}), \text{ for } l, k \in \{1, \ldots, n\}$$

This aggregation operation can be approached from two points of view, i) "neutral", where all experts' preferences have the same importance or weight and the arithmetic mean could be used as aggregation operator, or ii) "oriented", where experts' preferences have different weights. In this second case, we could use a typical OWA operator and apply the optimistic or pessimistic interpretation proposed by Yager in [8], [9]. In [9] Yager defines the term "Attitudinal Character" to refer to "orness" of the weighting vector and so to represent the attitude of a decision maker in a decision making problem. In this contribution, we use this approach to compute the level of agreement among the experts. So, a consensus process may be considered optimistic when the similarity is strengthened and pessimistic otherwise. The optimism or pessimism degree can be controlled taking into account the weighting vector W and its $orness$. An $orness(W) > 0.5$ represents an optimistic aggregation, an $orness(W) < 0.5$ represents a pessimistic aggregation and an $orness(W) = 0.5$ is the neutrality. Many approaches have been suggested for determining the vector W, here we have decided to provide directly the weights vector W according to the optimism or pessimism degree required.

Finally, the agreement is approached from three different points of view:

- Pairs of alternatives, where cp^{lk} measures the agreement on the pair of alternatives (x_l, x_k) (these values are the same that the consensus matrix values),

$$cp^{lk} = cm^{lk}, \quad \forall l, k = 1, \ldots, n \quad \wedge \quad l \neq k.$$

- Alternatives, where ca^l measures the agreement on the alternative x_l,

$$ca^l = \frac{\sum_{k=1,\ l \neq k}^{n} (cp^{lk} + cp^{kl})}{2(n-1)}.$$

- Preference relation, where cr measures the global agreement among all experts,

$$cr = \frac{\sum_{l=1}^{n} ca^l}{n}.$$

2) **Consensus control.** In this phase the level of agreement is checked. If cr is greater or equal than a given consensus threshold γ fixed in advance, then the consensus reaching process should end. Otherwise, the consensus process keeps going. The consensus threshold γ is the desired minimum consensus before starting the selection process. In addition, the model uses a parameter called "Maxrounds" in order to guarantee the end of the consensus process.
3) **Advice generation.** In the last phase, the model suggests how to change the experts' preferences in order to increase the level of agreement. To do that, two tasks are carried out:
   a) *Computing the collective preference and experts' proximity values.* A preference collective $\mathbf{P_{e_c}} = [p_c^{lk}]$ is calculated by aggregating all experts' preference relations $\{\mathbf{P_{e_1}}, \ldots, \mathbf{P_{e_m}}\}$ at level of pairs:

$$p_c^{lk} = \psi(p_1^{lk}, \ldots, p_m^{lk})$$

   being ψ an aggregation operator. As we said before, in this contribution we study the behavior of the consensus model according to different aggregation operators (arithmetic mean, OWA with linguistic quantifier "most" and DOWA.) The results and their interpretation are described in the next section.
   Moreover computing $\mathbf{P_{e_c}}$, in this phase the proximity between preferences of the expert e_i and

the collective preference on each pair (x_l, x_k) is computed as,

$$pp_i^{lk} = 1 - \mid p_i^{lk} - p_c^{lk} \mid .$$

These proximity values are used to identify the furthest experts' preferences and to suggest the opinion changes.

b) *Direction changes.* Firstly, the preferences to be modified are chosen. We propose to change those preferences whose level of agreement is lower than the global consensus, $PREFECHANGE = \{(l,k) \mid cp^{lk} < cr,\ l,k = 1,...,n\}$. Next, the furthest experts from collective preference on the pairs (x_l, x_k) with $(l,k) \in PREFECHANGE$, will be required to modify their assessments. To do that, a proximity threshold $\overline{pp}^{lk}$ for that pair is computed by aggregating all experts' proximity:

$$\overline{pp}^{lk} = \psi(pp_1^{lk}, \ldots, pp_m^{lk}).$$

So, each expert e_i whose $pp_i^{lk} < \overline{pp}^{lk}$ should modify the preference. It is important to highlight that in order to maintain the coherence of the model, the aggregation operator ψ used to compute the collective preferences and the proximity thresholds $\overline{pp}^{lk}$ is the same one. Once experts' preferences have been identified, the opinion changes are suggested according to the following "direction rules" [3]:

DR.1. If $(p_i^{lk} - p_c^{lk}) < 0$, then the expert e_i should increase the assessment associated to the pair of alternatives (x_l, x_k).

DR.2. If $(p_i^{lk} - p_c^{lk}) > 0$, then the expert e_i should decrease the assessment associated to the pair of alternatives (x_l, x_k).

DR.3. If $(p_i^{lk} - p_c^{lk}) = 0$, then the expert e_i should not modify the assessment associated to the pair of alternatives (x_l, x_k).

Note: The model only suggests the direction of the changes but not the range, but it is easy to see that too abrupt changes could produce a misbehavior of the model.

IV. Example and Results

To study the effects of these aggregation operators on consensus processes, we have analyzed the behavior of the consensus model with different examples and in the majority of the cases, we have obtained the same conclusions that we present here. To clarify these assumptions, we show the outcomes obtained in a particular case and under following conditions:

i) Four experts participate in the consensus process with the following reciprocal preference relations:

$$\mathbf{P_{e_1}} = \begin{pmatrix} - & 0.8 & 0.2 & 0.6 \\ 0.2 & - & 0.6 & 0.5 \\ 0.8 & 0.4 & - & 0.7 \\ 0.4 & 0.5 & 0.3 & - \end{pmatrix}$$

$$\mathbf{P_{e_2}} = \begin{pmatrix} - & 0.5 & 0.5 & 0.2 \\ 0.5 & - & 0.2 & 0 \\ 0.5 & 0.8 & - & 0.1 \\ 0.8 & 1 & 0.9 & - \end{pmatrix}$$

$$\mathbf{P_{e_3}} = \begin{pmatrix} - & 0 & 0.4 & 0.9 \\ 1 & - & 0.8 & 0.7 \\ 0.6 & 0.2 & - & 0.6 \\ 0.1 & 0.3 & 0.4 & - \end{pmatrix}$$

$$\mathbf{P_{e_4}} = \begin{pmatrix} - & 0.4 & 0 & 0.3 \\ 0.6 & - & 0.9 & 0.8 \\ 1 & 0.1 & - & 0.4 \\ 0.7 & 0.2 & 0.6 & - \end{pmatrix}$$

ii) The minimum consensus threshold γ required to end the process is 0.8.

iii) The maximum number of rounds is 10.

iv) For the $OWA_{\text{“}most\text{”}}$ operator, we use the weighting vector $W = [0, 0.4, 0.5, 0.1]$ obtained from (4) and with values $a = 0.3$ and $b = 0.8$ in (5).

v) According to the suggestions, the assessments will be increased or decreased in 0.1 or -0.1 range.

A brief summary of the main results according to each point of view is shown in the following tables:

a) Neutral consensus. We use the weighting vector $W_n = [1/6, 1/6, 1/6, 1/6, 1/6, 1/6]$ with $orness(W_n) = 0.5$, to compute the consensus degree from the six similarity matrices, Table I.

Table I
NEUTRAL CONSENSUS.

Round number	**Arith. mean**	**OWA$_{\text{“most”}}$**	**DOWA**
First			
Consensus degree	0,625	0,625	0,625
Number of changes	16	13	16
Second			
Consensus degree	0,69	0,679	0,692
Number of changes	14	7	10
Third			
Consensus degree	0,744	0,708	0,733
Number of changes	16	8	10
Fourth			
Consensus degree	0,8	0,792	0,808
Number of changes	End	7	End
Fifth			
Consensus degree		0,825	
Number of changes		End	

b) Optimistic consensus, with weighting vector $W_o = [0.4, 0.3, 0.1, 0.1, 0.1, 0]$ and $orness(W_o) = 0.74$, Table II.

Table II
OPTIMISTIC CONSENSUS.

Round number	Arith. mean	$\mathbf{OWA}_{\text{“most”}}$	DOWA
First			
Consensus degree	0,767	0,767	0,767
Number of changes	12	10	12
Second			
Consensus degree	0,803	0,795	0,803
Number of changes	End	8	End
Third			
Consensus degree		0,81	
Number of changes		End	

c) Pessimistic consensus, with weighting vector $W_p = [0, 0.1, 0.1, 0.1, 0.3, 0.4]$ and $orness(W_p) = 0.26$, Table III.

Table III
PESSIMISTIC CONSENSUS.

Round number	Arith. mean	$\mathbf{OWA}_{\text{“most”}}$	DOWA
First			
Consensus degree	0,47	0,47	0,47
Number of changes	16	13	16
Second			
Consensus degree	0,558	0,546	0,558
Number of changes	10	7	6
Third			
Consensus degree	0,6	0,59	0,593
Number of changes	16	8	10
Fourth			
Consensus degree	0,68	0,64	0,657
Number of changes	12	12	6
Fifth			
Consensus degree	0.75	0,705	0.693
Number of changes	28	8	6
Sixth			
Consensus degree	0.88	0,753	0.732
Number of changes	End	16	12
Seventh			
Consensus degree		0,82	0.78
Number of changes		End	8
Eight			
Consensus degree			0.83
Number of changes			End

Analysing these data we can highlight the following results:

- Independently of consensus scenarios, in the majority of the cases, the best agreement evolution is reached with the arithmetic mean operator and the worst one with the $OWA_{\text{“}most\text{”}}$ operator. We could consider DOWA returns intermediate values between both of them.
- If we apply an optimistic scenario, the consensus is achieved soon. Therefore this scenario is the most suitable when a decision have to be made quickly. It seems logical that a pessimistic scenario could imply better consensus degrees, but we have verified this is not true and moreover the number of changes is too high.
- Regarding the changes suggested, all operators have the same behavior. So, the three operators recommend to change the same experts although with different number of changes. These differences are due to the different ways to compute the collective preferences and proximity thresholds.
- Now we focus on a specific preference, for instance the pair (x_4, x_2) of the neutral scenario. We have the following initial preferences,

$$p_1^{42} = 0.5, p_2^{42} = 1, p_3^{42} = 0.3, p_4^{42} = 0.2.$$

  The preferences are very different and therefore the level of agreement is not high enough, $cp^{42} = 0.56 < cr = 0.625$. This implies that the preferences on that pair should be changed. The collective value according to aggregation operators are,

$$arith.mean, pc^{42} = 0.5$$
$$OWA_{\text{“}most\text{”}}, pc^{42} = 0.4$$
$$DOWA, pc^{42} = 0.46$$

  and all the operators recommend to change the expert e_2. In this case we can see as the influence of expert $e_2 = 1$ on collective value is bigger with arithmetic mean than with the $OWA_{\text{“}most\text{”}}$, therefore $OWA_{\text{“}most\text{”}}$ penalizes a little the extreme values. This suggests us that this operator may be useful to deal with possible manipulations of the consensus process if some experts try to manipulate their preferences in order to impose their particular opinions, but this assumption would require a deeper study.

V. CONCLUSIONS

In this contribution we have studied the influence of some aggregation operators on the consensus process. We have applied the aggregation operators: arithmetic mean, $OWA_{\text{“}most\text{”}}$ and DOWA, on a consensus reaching model approached from three different points of view, neutral, optimistic and pessimistic. Once the results have been analyzed, we may say that the behaviour of the aggregation operators in each consensus scenario is very similar, although the consensus is reached more quickly by using the arithmetic mean. Regarding the consensus scenarios, we can use one or other according to the necessity of reaching the agreement quickly. As future works, we propose us to continue this research line and to apply other aggregation operators on the consensus model.

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