Consensus reaching with different aggregation techniques

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Abstract

The consensus building process in group decision making usually requires experts change their opinions with the aim of maximizing agreement before making a decision. To help experts to make the changes, the collective opinion for the group is needed. This collective opinion is obtained by aggregating the experts' preferences. In this contribution we study influences of different aggregation techniques on a consensus reaching process. Specifically, we analyze results of four different aggregation operators: the arithmetic mean and three operators of the OWA family, OWA with linguistic quantifier "most", Dependent OWA and Clus-DOWA. Finally, some conclusions about effects of these operators on the consensus reaching process are drawn.

1 Introduction

Group decision making problems (GDM problems) can be defined as decision situations where two or more experts try to attain a common solution about a problem taking into account their opinions or preferences. Usually GDM problems have been resolved through selection processes where experts get the best set of alternatives from the preferences expressed by themselves [6]. However it may happen that some experts consider that their preferences were not considered to obtain the solution, and therefore they may disagree with that solution [12]. To avoid this situation, it is advisable to carry out a consensus process where experts discuss and change their preferences to reach enough agreement before making the selection process.

The consensus is an important research area GDM. Several approaches have been proposed in the literature from different points of view [1, 7, 9, 10]. The consensus is defined as a mutual-agreement state among the members of a group where all opinions have been expressed and listened to the satisfaction of all members. The consensus building process is defined as a dynamic and iterative process consisting of several rounds, where experts express and discuss their points of view. Traditionally this process is coordinated by a human moderator, which calculates the consensus among experts in each round using different consensus measures. If the agreement is not enough, the moderator encourages experts to change their preferences further from the group opinion in an effort to make then closer in the next consensus round.

In any consensus reaching process, a very important task is to compute a collective opinion which reflects the opinion of all experts. Usually, the collective preference is obtained by aggregating the individual experts' preferences. Many aggregation operators have been proposed in the literature, from simple arithmetic mean to fuzzy aggregation operators [2]. Note OWA operators family [15] and its later extensions which have been successfully applied in many areas related to decision making.

In this contribution we study the behavior of a consensus model according to different aggregation operators: arithmetic mean, OWA guided by the linguistic quantifier "most", Dependent-OWA (DOWA) operator and the operator called Clus-DOWA based on a clustering technique. These operators have been chosen because we have considered these aggregation techniques are appropriated to represent the concept of collective preference in consensus processes. So, the arithmetic mean takes into account all preferences in the same way; $OWA_{\text{"most"}}$ considers the preferences of the majority of the experts (extreme values are penalized); DOWA assigns weights to experts' preferences according to their distances regarding the central value of all preferences; and Clus-DOWA allows to assign weights taking into account clusters of experts with very similar preferences and therefore with a high level of agreement. Our aim is to study effects of these operators on both the development of the consensus process and obtained results. This contribution is organized as follows. In Section 2, we introduce the theoretical concepts of the aggregation operators used in this study. In Section 3, a brief revision of the consensus model is carried out. In Section 4, we show an example and the main results obtained as well as some assumptions concerning the consensus process and aggregation operators. Finally, in Section 5 we draw our conclusions.

2 Preliminaries

2.1 Ordered Weighted Averaging Operator

The ordered weighted averaging operator was proposed by Yager [14] to aggregate human judgments by using a weighting vector not associated directly to a particular judgment but rather to an ordered position of the set of judgments.

Definition: An OWA operator of dimension n is a mapping $R^n \rightarrow R$, which has an associated weighting vector $W = (w_1, w_2, \ldots, w_n)$, such that $w_i \in [0,1]$ and $\sum_{i=1}^{n} w_i = 1$, whose mathematical expression is,

$$OWA(a_1, \ldots, a_n) = \sum_{i=1}^{n} w_i b_i \qquad (1)$$

where b_i the i-th largest element of the arguments set $(a_1, \ldots, a_n)$ ordered in descending order. The OWA operators family allow to use and entire range of weighting vectors to reproduce "or" and "and" operations (i.e. maximum and minimum) according to experts' attitudes concerning aggregation (optimistic or pessimistic). So, a measurement to evaluate the degree of "orness" [14] has been defined as:

$$orness(W) = \frac{1}{n-1} \sum_{i=1}^{n} (n-i) w_i, \qquad (2)$$

with $orness(W) \in [0,1]$. This measurement estimates how similar is the OWA operator to pure "or" operator. When $W = (1, 0, \ldots, 0)$, the OWA behavior is similar to "or" operator and the $orness(W) = 1$, while with $W = (0, 0, \ldots, 1)$, the OWA behavior is similar to "and" operator and the $orness(W) = 0$.

A very interesting approach about OWA operators arises when the linguistic quantifiers proposed by Zadeh [18] are used to compute the weighting vector W. A linguistic quantifier is defined as a function $Q : [0,1] \rightarrow [0,1]$, such that $Q(0) = 0$, $Q(1) = 1$, and if $x > y$ then $Q(x) \geq Q(y)$. An OWA aggregation guided by a linguistic quantifier Q is defined as [16]:

$$\Phi_Q(a_1, \ldots, a_n) = \sum_{i=1}^{n} w_i bi \qquad (3)$$

where b_i is the ith largest element of a_j, with,

$$w_i = Q\left(\frac{i}{n}\right) - Q\left(\frac{i-1}{n}\right), \quad i = 1, \ldots, n. \quad (4)$$

This approach has widely been used in fuzzy group decision making to aggregate experts' preferences, particulary to represent the concept of fuzzy majority [8, 11]. In this contribution we use an OWA operator guided by the quantifier linguistic "most" to obtain the collective preference,

$$Q(x) = \begin{cases} 0 & x \leq a \\ \frac{x-a}{b-a} & a < x < b \\ 1 & x \geq b \end{cases} \qquad (5)$$

2.2 Dependent OWA Operator

The Dependent OWA operator (DOWA) was introduced by Xu [13] as a new approach to

OWA aggregations where the estimation of weights depends on the aggregated arguments. This operator was proposed as a possible solution to some of the drawbacks of a typical OWA aggregation, i.e, its bad behavior to deal with very high or low values. DOWA computes the weights of the arguments according to its distance regarding the center of all arguments (i.e. arithmetic mean). This centralized interpretation has also been adopted in the centered OWA operator [17], where weights are high close to arithmetic mean and decrease towards extreme ones.

The weights of the DOWA operator are obtained according to the following expressions: Let $(a_1, \ldots, a_n)$ be a set of arguments and φ the arithmetic mean of the set. The weights of the weighting vector $W = (w_1, w_2, \ldots, w_n)$ are computed as:

$$w_j = \frac{s(a_j, \varphi)}{\sum_{i=1}^{n} s(a_i, \varphi)}, \; j = 1, 2, \ldots, n \qquad (6)$$

where $s(\cdot)$ is the similarity between any argument a_j and the average value φ,

$$s(a_j, \varphi) = 1 - \frac{\mid a_j - \varphi \mid}{\sum_{i=1}^{n} \mid a_i - \varphi \mid}. \qquad (7)$$

Note that at least one preference must to be different among all experts' preferences. Finally, DOWA operator is defined as:

$$DOWA(a_1, \ldots, a_n) = \sum_{i=1}^{n} w_i a_i. \qquad (8)$$

Here, the weight of each argument is proportional to its magnitude into the set of arguments. Therefore, the typical reordering step of OWA operators is irrelevant for the DOWA operator.

2.3 Clus-DOWA Operator

In the previous operators, all arguments are aggregated as elements of one unique set, i.e as a great cluster. The difference among operators is the way to compute the weighting vector W. But in a consensus process, where many experts' preferences may be considered, it could be interesting to group experts' preferences according to its similarity before computing the weights. Groups of experts with very similar preferences could be considered as local clusters and the distance of each expert's preference to the nearest cluster could be used to assign its weight. So, preferences very close could have weights higher than others independently their distances to the global center.

In this sense, Boongoen and Shen [3] proposed the cluster-based DOWA operator (Clus-DOWA) as an operator which takes into account the distribution of the preferences in order to assign the weights. So, those values very far from the group center (e.g. arithmetic mean) are not assigned with the lowest weights if they are very similar to their neighbors. This operator is supported on the idea of applying a clustering process in order to identify the local clusters. Each cluster represents a group of preferences very close and therefore with a high level of agreement. Authors propose to use a modified version of the agglomerative hierarchical clustering algorithm [4] depicted in the Figure 1.

Begin
  Initialize $a = n; c = 0; d_i = \{\}; Reference_i = \{\}; i = 1 \ldots n$
  Do
  Find nearest elements (argument or cluster) E_x and E_y
    If E_x and E_y are arguments a_x and a_y
      $d_x = distance(a_x, a_y); Reference_x = a_y;$
      $d_y = distance(a_y, a_x); Reference_y = a_x; a = a - 2$
    If E_x is argument a_x and E_y is cluster Cl_y
      $d_x = distance(a_x, center(Cl_y)); Reference_x = center(Cl_y); a = a - 1$
    If E_x is cluster Cl_x and E_y is argument a_y
      $d_y = distance(a_y, center(Cl_x)); Reference_y = center(Cl_x); a = a - 1$
    Merge E_x and E_y to a new cluster Cl_c, where $c = c + 1$
    Drop E_x and E_y, and include Cl_c in next iteration
  Until $a = 0$
  Return $d_i, Reference_i, i = 1 \ldots n$
End

Figure 1: Agglomerative hierarchical clustering algorithm

where d_i is the distance between each argument a_i and the cluster center called $Reference_i$.

Once all arguments a_i have been assigned to its respective cluster, a reliability value r_i can be computed according to the distance to its nearest cluster d_i,

$$r_i = 1 - \frac{d_i}{\sum_{j=1}^{n} d_j}. \quad (9)$$

The reliability values vector $(r_1, r_2, \ldots, r_n)$ is used to obtain the weighting vector as follows:

$$w_i = \frac{r_i}{\sum_{j=1}^{n} r_j},\ i = 1, 2, \ldots, n. \quad (10)$$

Finally, the Clus-DOWA operator is defined as:

$$Clus - DOWA(a_1, \ldots, a_n) = \sum_{i=1}^{n} w_i a_i. \quad (11)$$

3 Consensus Model Description

A GDM problem is classically defined as a decision situation where given a set of alternatives $X = \{x_1, x_2, \ldots, x_n\}$ $(n \geq 2)$, a group of experts, $E = \{e_1, e_2, \ldots, e_m\}$ $(m \geq 2)$, try find out the best alternative/s to solve the decision problem. In this contribution we assume that preferences are provided by experts by means of reciprocal preference relations [5], $\mathbf{P}_{\mathbf{e}_i} = (p_i^{lk})$, $l, k \in \{1, \ldots, n\}$, with $p_i^{lk} = \mu_{\mathbf{P}_{\mathbf{e}_i}}(x_l, x_k)$ assessed in the unit interval $[0, 1]$. The preference p_i^{lk} is interpreted as the preference degree of the alternative x_l over x_k according to the expert e_i..

Explained in a roughly way, for each round the model computes the consensus degree, and if it not were highly enough, the model identifies those experts farther from the collective opinion. Then it advices them to perform the right changes in order to increase the agreement degree at the next round. More exactly, the model is composed by the following phases (see Figure 2):

1. **Computing the consensus degree.** The level of agreement among all experts' preferences, cr, is computed. Firstly, for each pair of experts e_i, e_j $(i < j)$, a similarity matrix, $SM_{ij} = (sm_{ij}^{lk})$, is obtained as,

$$sm_{ij}^{lk} = 1 - \mid p_i^{lk} - p_j^{lk} \mid \quad (12)$$

where sm_{ij}^{lk} is the similarity between the experts e_i and e_j on the pair of alternatives (x_l, x_k). Secondly, a consensus matrix, $CM = (cm^{lk})$, is calculated by aggregating at level of pairs of alternatives all similarity matrices:

$$cm^{lk} = \phi(sm_{ij}^{lk});\ i, j = 1, \ldots, m\ \wedge \forall\, l, k = 1, \ldots, n\ \wedge\ i < j$$

where the arithmetic mean is used as aggregation operator ϕ.

Finally, the agreement is computed at level of:

- Pairs of alternatives, where cp^{lk} represents the agreement on the pair of alternatives (x_l, x_k). They are obtained from the consensus matrix CM,

$$cp^{lk} = cm^{lk},\ \forall l, k = 1, \ldots, n\ \wedge\ l \neq k.$$

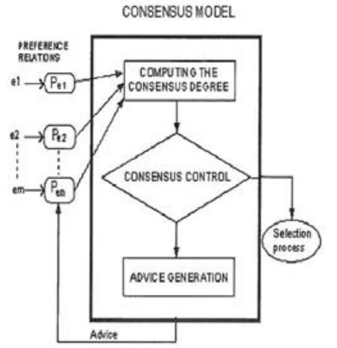

Figure 2: Consensus model

- Alternatives, where ca^l represents the agreement on the alternative x_l,

$$ca^l = \frac{\sum_{k=1,\ l\neq k}^{n}(cp^{lk} + cp^{kl})}{2(n-1)}. \tag{13}$$

- Preference relation, where cr represents the global agreement among all experts,

$$cr = \frac{\sum_{l=1}^{n} ca^l}{n}. \tag{14}$$

2. **Consensus control.** In this phase is checked the agreement cr computed in the previous phase. If cr is greater or equal than a given consensus threshold γ fixed in advance, then the consensus reaching process will succesfully end. Otherwise, the consensus process needs more rounds. The consensus threshold γ is the desired minimum consensus before starting the selection process. In addition, the model uses a parameter called "Maxrounds" containing the maximum number of rounds allowed before giving up the process, in order to guarantee the end of the process.

3. **Advice generation.** In the last phase, the model suggests how to change the experts' preferences in order to increase the level of agreement. To do that, two tasks are carried out:

   (a) *Computing the collective preference and experts' proximity values.* A preference collective $\mathbf{P_{e_c}} = (p_c^{lk})$ is calculated by aggregating all experts' preference relations $\{\mathbf{P_{e_1}}, \ldots, \mathbf{P_{e_m}}\}$ at level of pairs:

$$p_c^{lk} = \psi(p_1^{lk}, \ldots, p_m^{lk}) \tag{15}$$

   where ψ is the aggregation operator. This task is very important in this contribution because here is where we apply the four aggregation operators in order to study their effects on the consensus process. The results as well as their possible interpretation are described in the next section.

   Moreover computing $\mathbf{P_{e_c}}$, the proximity between preferences of the expert e_i and the collective preference on each pair (x_l, x_k) is calculated in this phase,

$$pp_i^{lk} = 1 - \mid p_i^{lk} - p_c^{lk} \mid. \tag{16}$$

   These proximity values are used to identify the furthest experts' preferences and to suggest the opinion changes.

   (b) *Direction changes.* Firstly, the preferences to be modified are chosen. We propose to change those preferences whose level of agreement is lower than the global consensus, i.e., $Prefechange = \{(l,k) \mid cp^{lk} < cr,\ l,k = 1,...,n\}$.

   Next, the furthest experts from collective preference on the pairs (x_l, x_k) with $(l,k) \in Prefechange$, will be required to modify their assessments. To do that, a proximity threshold $\overline{pp}^{lk}$ for that pair is computed by aggregating all experts' proximity:

$$\overline{pp}^{lk} = \psi(pp_1^{lk}, \ldots, pp_m^{lk}). \tag{17}$$

   So, each expert e_i whose $pp_i^{lk} < \overline{pp}^{lk}$ should modify the preference. It is important to highlight that in order to maintain the coherence of the model, the aggregation operator ψ used to compute the collective preferences is also used to compute the proximity thresholds $\overline{pp}^{lk}$. Once experts' preferences have been identified, the opinion changes are suggested according to the following "direction rules" [10]:

   DR.1. If $(p_i^{lk} - p_c^{lk}) < 0$, then the expert e_i should increase the assessment associated to the pair of alternatives (x_l, x_k).

   DR.2. If $(p_i^{lk} - p_c^{lk}) > 0$, then the expert e_i should decrease the assessment associated to the pair of alternatives (x_l, x_k).

DR.3. If $(p_i^{lk} - p_c^{lk}) = 0$, then the expert e_i should not modify the assessment associated to the pair of alternatives (x_l, x_k).

Note: The model only suggests the direction of the changes but not the range, but it is easy to see that too abrupt changes could produce a misbehavior of the model.

4 Example and Results

To study effects of these aggregation operators on the consensus model proposed, the model has been tested with different examples. According to the results obtained, we have deduced some conclusions that we present here. To show these conclusions, we have used an example with the following characteristics:

i) Eight experts, $E = \{e_1, e_2, \ldots, e_8\}$, who use reciprocal preference relations with assessments $p_i^{lk} \in [0,1]$ on four alternatives, $X = \{x_1, x_2, \ldots, x_4\}$. The preference relations have been omitted because the values lack importance and the size of the contribution is also reduced.

ii) The minimum consensus threshold γ required to end the process is 0.8.

iii) The maximum number of rounds is 10.

iv) For the $OWA_{\text{"}most\text{"}}$ operator, we use the weighting vector

$$W = (0, 0, 0.15, 0.15, 0.25, 0.25, 0.1, 0)$$

obtained from (4) and with values $a = 0.3$ and $b = 0.8$ in (5).

v) According to the directions suggested, the assessments will be increased or decreased in 0.1 or -0.1 range.

A brief summary of the main results obtained can be seen in the Table 1.

It is difficult to identify a clear behavior pattern for each operator because results depend on initial preferences of experts. However, we could highlight the following conclusions:

Table 1: Table of results.

Round number	Arith. mean	OWA most	DOWA	Clus DOWA
First				
Consensus degree	0.706	0.706	0.706	0.706
Number of changes	22	20	22	20
Second				
Consensus degree	0.743	0.74	0.743	0.742
Number of changes	24	9	14	14
Third				
Consensus degree	0.778	0.756	0.767	0.765
Number of changes	42	24	24	28
Fourth				
Consensus degree	0.851	0.796	0.808	0.811
Number of changes	End	21	End	End
Fifth				
Consensus degree		0.832		
Number of changes		End		

- Regarding the agreement, the best evolution is generally reached with the arithmetic mean, the worst one with the $OWA_{\text{"}most\text{"}}$ operator. Both DOWA and Clus-DOWA usually return intermediate values.

- Clus-DOWA provides a more reliable set of weights than DOWA, however, clus-DOWA takes a more expensive computation.

- Regarding the changes suggested, we have verified that all operators have a similar behavior. So, all operators recommend to change the same experts although with different number of changes.

- Let us focus on a particular preference, for instance the pair (x_3, x_4), with the initial preferences, $e_1 = 0.9, e_2 = 0.6, e_3 = 0.7, e_4 = 0.2, e_5 = 0.5, e_6 = 0.1, e_7 = 0.1, e_8 = 1$. According to the disparity of assessments, it seems logic that the level of agreement is not high enough, $cp^{34} = 0.575 < cr = 0.706$, and therefore the preferences on that pair should be changed. The collective value according to each aggregation operator is,

$$arith.mean = 0.513$$
$$OWA_{\text{“}most\text{”}} = 0.44$$
$$DOWA = 0.513$$
$$Clus - DOWA = 0.495.$$

In this particular case we can see as the influence of extreme values (e.g., $e_8 = 1$) on the collective value with the arithmetic mean operator is bigger than with the $OWA_{\text{“}most\text{”}}$ because $OWA_{\text{“}most\text{”}}$ penalizes the extreme ones. This suggests us that this operator may be useful to deal with possible manipulations of the consensus processes if some experts provide extreme values as strategy to impose their particular opinions, but this assumption would require a deeper study. Regarding the Clus-DOWA operator, this obtains a value smaller than DOWA because the cluster with the lowest values ($e_4 = 0.2, e_6 = 0.1, e_7 = 0.1$) receives a weight higher than the cluster with the highest values ($e_1 = 0.9, e_8 = 1$).

5 Conclusions

In this contribution we have analyzed the behavior of a consensus reaching model with different aggregation techniques. We have applied four aggregation operators: arithmetic mean, $OWA_{\text{“}most\text{”}}$, DOWA and Clus-DOWA. Each one uses a different strategy to assign the argument weights, in this case the experts' preferences. Once results have been analyzed, we may say that the behavior of the model is very similar independently of the operator used, although in the majority of the cases the arithmetic mean achieves the agreement more quickly. Regarding the number of changes and preferences to be changed, the consensus model recommends to change the same experts but with different number of changes, highlighting the arithmetic mean as the operator with the largest number of changes.

As future works, we want to extend this study on other operators and to define a new aggregation operator for consensus reaching process. Another aspect to explore could be the "quality" of the consensus process respect with not only the number of rounds needed to reach consensus, but the satisfaction degree of the experts that take part in the process (there is an inversely proportional relationship between the satisfaction degree of an expert and the number of changes carried out by him).

Acknowledgement

This work has been supported by the Research Project UJA2009/12/25.

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