TYPE-1 OWA BASED MULTI-GRANULAR CONSENSUS MODEL

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Abstract

In the multi-granular context, different theoretical models have been proposed in the literature to address the consensus reaching process, in which transformation functions were used to unify the multi-granular linguistic information. A drawback of such unification process is that the consensus reaching process loses information because the transformations functions used are not bijective. In this paper we present a type-1 OWA based multi-granular consensus model that makes the unification step unnecessary and therefore does not suffer from the loss of information in the necessary aggregation step of these GDM problems.

Keywords: Linguistic variable, Muti-granular information, Type-1 OWA operator, Consensus.

1 INTRODUCTION

The linguistic approach mostly used nowadays in human decision processes has its origins in the proposal that Zadeh outlined in his 1973 seminal paper [16] and further elaboration in [17]. In essence, this approach relies on the use of the *linguistic variable*, i.e. a variable whose *primary* values are words instead of numbers. The meaning of the primary values, although assumed to be 'subjective and context-dependent,' is to be specified before other terms are generated from them using linguistic hedges in conjunction with the negator and the connective operators [16, 17]. The main purpose of using linguistic values instead of numbers is that linguistic characterizations are, in general, less specific than numerical ones, but much closer to the way that humans express and use their knowledge [5].

A linguistic variable is formally represented by a 5-tuple $\langle L, T(L), U, S, M\rangle$ [17] where (i) L is the name of the variable; (ii) $T(L)$ is a finite term set of (primary) labels or words (a collection of linguistic values); (iii) U is a universe of discourse or base variable; (iv) S is the syntactic rule which generates the terms in $T(L)$; and (v) M is a semantic rule which associates with each linguistic value X its meaning $M(X) : U \to [0,1]$. Usually, $T(L)$ is denoted as L when there is no risk of confusion.

The semantic rule, also known as 'compatibility function' [17], associates with each element of the base variable its compatibility with each linguistic value. This interpretation of the meaning of a linguistic label coincides with that of a fuzzy set, and therefore linguistic labels can be considered and formally represented as fuzzy subsets of their base variable. Therefore, the nature of the base variable will dictate the general method to use when manipulating linguistic values. A non numerical base variable makes the definition of the compatibility function 'difficult to be formalized in explicit terms' [17]. As a result, it turns out to be problematic when implemented at present in computer programmes. Thus, it is fair to say that most, if not all, important linguistic decision models in the literature assume that the base variable is a subset of the set of real numbers, and therefore numeric in nature. Indeed, these linguistic decision models usually start associating the linguistic values (labels) to be used with membership functions (triangular, trapezoidal, Gaussian . . .) to represent their meanings.

Figure 1 (taken from [5]) illustrates a representation of the linguistic variable '*Height*', whose corresponding set of primary linguistic values is $T(Height) = \{Very\ Low, Low, Medium, High, Very\ High\}$. We can see how the semantic rule associates each of the primary linguistic values X to its fuzzy subset $M(X)$ of U. As mentioned above, a crucial aspect that will determine the validity of the linguistic approach is the determination of correct membership functions for the linguistic term set.

The ideal situation in group decision making (GDM) problems within a linguistic context would be one where all the experts use the same linguistic term set to express their preferences about the alternatives. However, in some cases,

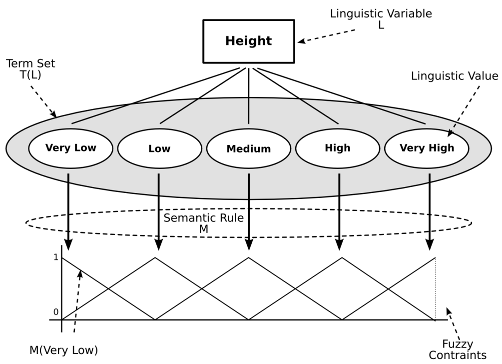

Figure 1: Example of the linguistic variable "Height" [5]

experts may belong to distinct research areas and will, therefore, have different backgrounds and levels of knowledge. A consequence of this is that the expression of preferences will depend on linguistic term sets with different granularity. As a result, the development of adequate tools to manage and model multi-granular linguistic information becomes essential [6, 8, 9].

In this paper we present a consensus model with multi-granular linguistic preferences that follows the recommendation above, i.e. a model that makes use of the whole membership functions used to represent the linguistic terms within the problem and that is built on the use of the extension principle of fuzzy sets [17]. This proposal takes further the model presented in [9], whose methodology was based on transformation functions to unify the multi-granular linguistic information. A drawback of such unification process is that the consensus reaching process loses information because the transformations functions, although made use of the membership functions used to represents the linguistic preference values, are not bijective. Indeed, a different mathematical representation (membership functions) of the linguistic preference values could be sought leading to the same result after the unification process. However, the justification for the necessity of such unification phase was that there were no tools available at that moment to directly compute over linguistic sets of different granularity, while it was a necessary step to manipulate the linguistic sets with different granularity in order to make the computation of both consensus degrees and proximity measures possible [9]. Interestingly, new tools have emerged that allow for the direct manipulation of different elements belonging to linguistic sets of different granularity. In particular, the type-1 OWA operator introduced in [18], and developed by applying the extension principle to Yager's OWA operator [15], has been proved to be a useful tool to aggregate the linguistic opinions or preferences in human decision making with linguistic weights [19]. Thus, our aim is to present a type-1 OWA based multi-granular consensus model that makes the unification step unnecessary and therefore does not suffer from the loss of information in the necessary aggregation step of these GDM problems.

The rest of the paper is set out as follows. Section 2 is devoted to the preliminary concepts needed to present the consensus model: the multi-granular linguistic GDM problem is described in Subsection 2.1, while Subsection 2.2 presents the type-1 OWA operator, its fast implementation and how to derive linguistic weights to be used in type-1 OWA aggregation given a type-2 linguistic quantifier. Section 3 details the architecture of the type-1 OWA based multi-granular consensus model. Finally, in Section 4 we draw our conclusions and suggest further research.

2 PRELIMINARIES

2.1 MULTI-GRANULAR INFORMATION IN GROUP DECISION MAKING

GDM problems are classically described as decision situations where, given a set of feasible alternatives $X = \{x_1, x_2, \ldots, x_n\}$ $(n \geq 2)$, a set of experts $E = \{e_1, e_2, \ldots, e_m\}$ $(m \geq 2)$ try to achieve a common solution together. In a linguistic context, experts' opinions or preferences are mathematically modelled by means of preference relations [13], $\mathbf{P_{e_i}} = (p_i^{lk}),\ l,k \in \{1,\ldots,n\}$, where $p_i^{lk} = \mu_{\mathbf{P_{e_i}}}(x_l, x_k)$ is a fuzzy subset of the unit interval $[0,1]$ and that will represent the expert e_i's preference of the alternative x_l over x_k.

As mentioned above, experts may have different backgrounds and levels of knowledge about the problem to solve and therefore they could prefer to use linguistic term sets with different granularity according to their expertise [6, 7, 14]. The granularity of a linguistic term set should be small enough so as not to impose useless precision levels on experts but big enough to allow a discrimination of the assessments in a limited number of degrees.

In this contribution we deal with GDM problems where each expert e_i may use a distinct linguistic term set with different cardinality and/or semantics $S_i = \{s_0^i, \ldots, s_g^i\}$.

2.2 TYPE-1 OWA OPERATOR

Unlike Yager's OWA operator that aggregates crisp values [15], the type-1 OWA operator is able to aggregate type-1 fuzzy sets with uncertain weights, with these uncertain weights being also modelled as type-1 fuzzy sets. As a generalisation of Yager's OWA operator, and based on the extension principle, a type-1 OWA operator is defined as follows [18]:

Definition 1 *Given n linguistic weights* $\left\{W^i\right\}_{i=1}^n$ *in the form of type-1 fuzzy sets defined on the domain of discourse*

$[0,1]$, *a type-1 OWA operator is a mapping,* Φ,

$$\begin{array}{rcl}\Phi\colon \tilde{P}(\mathbb{R})\times\cdots\tilde{P}(\mathbb{R}) & \longrightarrow & \tilde{P}(\mathbb{R})\\ (A^1,\cdots,A^n) & \mapsto & Y\end{array}$$

such that

$$\mu_Y(y) = \sup_{\substack{\sum_{k=1}^{n}\bar{w}_i a_{\sigma(i)} = y\\ w_i\in U,\, a_i\in X}} \begin{pmatrix}\mu_{W^1}(w_1)\wedge\cdots\wedge\mu_{W^n}(w_n)\\ \wedge\mu_{A^1}(a_1)\wedge\cdots\wedge\mu_{A^n}(a_n)\end{pmatrix} \tag{1}$$

where $\bar{w}_i = \frac{w_i}{\sum_{i=1}^{n} w_i}$; σ *is a permutation function such that* $a_{\sigma(i)} \geq a_{\sigma(i+1)}$, $\forall i = 1,\cdots,n-1$; *and* $\tilde{P}(\mathbb{R})$ *is the set of fuzzy sets on* $\mathbb{R}$.

A Direct Approach to performing type-1 OWA operation was suggested in [18]. However, this approach is computationally expensive, which inevitably curtails further applications of the type-1 OWA operator to real world decision making. So a fast approach to type-1 OWA operations has been developed based on the α-level of fuzzy sets [19].

2.2.1 α-Level Type-1 OWA Operator.

Definition 2 *Given the n linguistic weights* $\{W^i\}_{i=1}^{n}$ *in the form of type-1 fuzzy sets defined on the domain of discourse* $[0,1]$, *then for each* $\alpha\in[0,1]$, *an* α*-level type-1 OWA operator with* α*-level weight sets* $\{W_\alpha^i\}_{i=1}^{n}$ *to aggregate the* α*-level of type-1 fuzzy sets* $\{A^i\}_{i=1}^{n}$ *is given as*

$$\Phi_\alpha\left(A_\alpha^1,\cdots,A_\alpha^n\right) = \left\{ \frac{\sum_{i=1}^{n} w_i a_{\sigma(i)}}{\sum_{i=1}^{n} w_i} \,\middle|\, w_i\in W_\alpha^i,\ a_i\in A_\alpha^i, \forall i \right\} \tag{2}$$

where $W_\alpha^i = \{w|\mu_{W_i}(w)\geq\alpha\}$, $A_\alpha^i = \{x|\mu_{A_i}(x)\geq\alpha\}$, *and* σ *is a permutation function such that* $a_{\sigma(i)}\geq a_{\sigma(i+1)}$, $\forall\, i = 1,\cdots,n-1$.

According to the Representation Theorem of type-1 fuzzy sets, the α-level sets $\Phi_\alpha\left(A_\alpha^1,\cdots,A_\alpha^n\right)$ obtained via Definition 2 can be used to construct the following type-1 fuzzy set on $\mathbb{R}$

$$G = \bigcup_{0<\alpha\leq1} \alpha\Phi_\alpha\left(A_\alpha^1,\cdots,A_\alpha^n\right) \tag{3}$$

with membership function

$$\mu_G(x) = \bigvee_{\alpha : x\in\Phi_\alpha\left(A_\alpha^1,\cdots,A_\alpha^n\right)_\alpha} \alpha \tag{4}$$

2.2.2 Representation Theorem of Type-1 OWA Operators.

The two apparently different aggregation results in (1) and (3) obtained according to Zadeh's Extension Principle and the α-level of type-1 fuzzy sets, respectively, are equivalent as proved in [19]:

Theorem 1 *Given the n linguistic weights* $\{W^i\}_{i=1}^{n}$ *in the form of type-1 fuzzy sets defined on the domain of discourse* $[0,1]$, *and the type-1 fuzzy sets* $A^1,\cdots,A^n$, *then we have that*

$$Y = G$$

where Y is the aggregation result defined in (1) and G is the result defined in (3).

Theorem 1 is called the *Representation Theorem of Type-1 OWA Operators*. Therefore, an effective and practical way of carrying out type-1 OWA operations is to decompose the type-1 OWA aggregation into the α-level type-1 OWA operations and then reconstruct it via the above representation theorem. This α-level approach has been proved to be much faster than the direct approach [19], so it can be used in real time decision making and data mining applications.

2.2.3 α-Level Type-1 OWA of Fuzzy Numbers.

When the linguistic weights and the aggregated sets are fuzzy number, the α-level type-1 OWA operator produces closed intervals [19]:

Theorem 2 *Let* $\{W^i\}_{i=1}^{n}$ *be fuzzy numbers on* $[0,1]$ *and* $\{A^i\}_{i=1}^{n}$ *be fuzzy numbers on* $\mathbb{R}$. *Then for each* $\alpha\in U$, $\Phi_\alpha\left(A_\alpha^1,\cdots,A_\alpha^n\right)$ *is a closed interval.*

Based on this result, the computation of the type-1 OWA output according to (3), G, reduces to compute the left end-points and right end-points of the intervals $\Phi_\alpha\left(A_\alpha^1,\cdots,A_\alpha^n\right)$:

$$\Phi_\alpha\left(A_\alpha^1,\cdots,A_\alpha^n\right)_- \text{ and } \Phi_\alpha\left(A_\alpha^1,\cdots,A_\alpha^n\right)_+,$$

where $A_\alpha^i = [A_{\alpha-}^i, A_{\alpha+}^i], W_\alpha^i = [W_{\alpha-}^i, W_{\alpha+}^i]$.

For the left end-points, we have

$$\Phi_\alpha\left(A_\alpha^1,\cdots,A_\alpha^n\right)_- = \min_{\substack{W_{\alpha-}^i\leq w_i\leq W_{\alpha+}^i\\ A_{\alpha-}^i\leq a_i\leq A_{\alpha+}^i}} \sum_{i=1}^{n} w_i a_{\sigma(i)} \Big/ \sum_{i=1}^{n} w_i \tag{5}$$

while for the right end-points, we have

$$\Phi_\alpha\left(A_\alpha^1,\cdots,A_\alpha^n\right)_+ = \max_{\substack{W_{\alpha-}^i\leq w_i\leq W_{\alpha+}^i\\ A_{\alpha-}^i\leq a_i\leq A_{\alpha+}^i}} \sum_{i=1}^{n} w_i a_{\sigma(i)} \Big/ \sum_{i=1}^{n} w_i \tag{6}$$

It can be seen that (5) and (6) are programming problems. Solutions to these problems, so that the type-1 OWA aggregation operation can be performed efficiently, are available from [19].

3 TYPE-1 OWA BASED MULTI-GRANULAR CONSENSUS MODEL

Consensus reaching processes can be defined as iterative processes composed by several rounds where experts express and discuss about their preferences with the aim to achieve a minimum level of agreement before making a decision. In real-world problems, these processes are guided by a human moderator who is in charged of supervising all process phases and carries out the necessary actions to drive the consensus process toward success.

In the multi-granular context, the different theoretical models proposed in the literature [2, 10, 11] have been proposed to address the consensus reaching process, in which transformation functions were used to unify the multi-granular linguistic information. As pointed our earlier, a drawback of such unification process is that the consensus reaching process loses information because the transformations functions are not bijective. These proposed models did not address the issues of direct manipulation of different elements belonging to linguistic sets of different granularity, specially in the necessary aggregation step of GDM problems, because there were no mathematical tools available at that moment. Interestingly, the introduction of the type-1 OWA operator provides such a needed tool for direct manipulating linguistic sets with different granularity in consensus decision making.

In this section we present, and illustrate in 2, a consensus reaching model based on type-1 OWA operator to address GDM problems defined in multi-granular linguistic contexts. Specifically our proposed model includes the following steps:

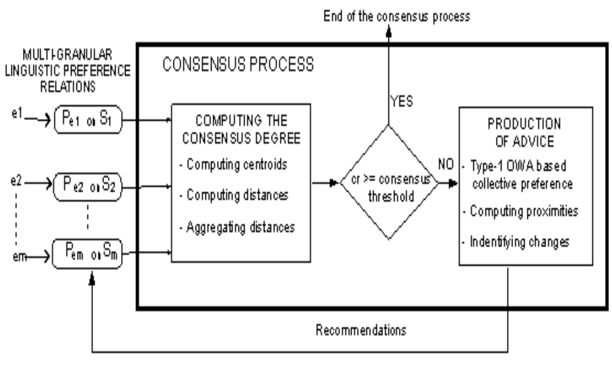

Figure 2: Type-1 OWA based multi-granular consensus model phases

1. **Computing the consensus degree**. The consensus degree will represent the level of agreement among experts, i.e., how close experts' preferences are. In order to evaluate the agreement, the model computes and aggregates the 'distance' representing disagreement between the preferences of each pair of experts on each pair of alternatives. It is well known that the set of fuzzy sets of the unit interval is not totally ordered. Therefore, for our purpose, following the methodology proposed in [4], each fuzzy set will be associated to a numeric representative score, in our case its centroid, and the distance between two fuzzy sets will be set as the distance between their respective scores.

The centroid of a type-1 fuzzy set A in a continuous domain X is calculated as,

$$C_A = \frac{\int_x x \cdot \mu_A(x)dx}{\int_x \mu_A(x_i)}. \tag{7}$$

The centroid when the domain X is discretised into n points is

$$C_A = \frac{\sum_{i=1}^{n} x_i \cdot \mu_A(x_i)}{\sum_{i=1}^{n} \mu_A(x_i)}. \tag{8}$$

The centroid of linguistic label p_i^{lk} will be denoted by c_i^{lk}.

For each pair of experts e_i, e_j $(i < j)$, a similarity matrix, $SM_{ij} = (sm_{ij}^{lk})$, is obtained as

$$sm_{ij}^{lk} = 1 - \mid c_i^{lk} - c_j^{lk} \mid \tag{9}$$

where sm_{ij}^{lk} represent the similarity between the preferences of the experts e_i and e_j on the pair of alternatives (x_l, x_k). The closer sm_{ij}^{lk} to 1 the more similar p_i^{lk} and p_j^{lk} are considered, while the closer sm_{ij}^{lk} to 0 the more distant p_i^{lk} and p_j^{lk} are considered.

A consensus matrix, $CM = (cm^{lk})$, is calculated by aggregating all similarity matrices, where each aggregated pair of alternatives is computed as:

$$\begin{aligned} cm^{lk} \quad = \quad & \theta(sm_{12}^{lk}, sm_{13}^{lk}, \ldots, sm_{1m}^{lk}, \\ & sm_{23}^{lk}, \ldots, sm_{2m}^{lk}, \ \ldots, sm_{(m-1)m}^{lk}) \end{aligned}$$

for $l, k \in \{1, \ldots, n\}$ and where θ is an aggregation operator. Note that different aggregation operators can be used according to different consensus strategies (more details on this can be found in [12]).

The level of agreement is computed at three different levels [9]:

i) Pairs of alternatives,

$$cp^{lk} = cm^{lk}, \ \forall\, l, k = 1, \ldots, n \ \wedge \ l \neq k,$$

where cp^{lk} represents the level of agreement by all experts on the pair of alternatives (x_l, x_k).

ii) Alternatives,

$$ca^l = \frac{\sum_{k=1,\ l\neq k}^{n} cp^{lk}}{(n-1)}, \tag{10}$$

where ca^l represents the level of agreement by all experts on the alternative x_l.

iii) Preference relation,

$$cr = \frac{\sum_{l=1}^{n} ca^l}{n}, \tag{11}$$

where cr represents the global agreement among all experts.

2. **Consensus control.** The agreement cr obtained in the previous phase is checked here against a consensus threshold agreed by the set of expert previous to the application of the consensus process. If cr is greater or equal than this consensus threshold then the consensus reaching process is considered over. Otherwise, the experts need to discuss and attempt to get their opinions closer for their level of agreement to increase and achieve the threshold value set.

3. **Advice generation.** In this phase, the model suggests how experts should change their opinions in order to bring them preferences closer and to increase the level of agreement. Three tasks are carried out in order to achieve this:

(a) *Computing the collective preference and the experts' proximity values.* Firstly, a collective preference $\mathbf{P_{e_c}} = (p_c^{lk})$ is calculated by aggregating all experts' preference relations $\{\mathbf{P_{e_1}}, \ldots, \mathbf{P_{e_m}}\}$ at level of pairs of alternatives. This aggregation is carried out by using the type-1 OWA operator.

$$\mathbf{P_c} = \Phi(\mathbf{P_{e_1}}, \ldots, \mathbf{P_{e_m}}). \tag{12}$$

The proximity between each individual linguistic preference relation and the collective linguistic one is obtained, $PM_i = (pm_i^{lk})$,

$$pp_i^{lk} = 1 - \mid c_i^{lk} - c_c^{lk} \mid \tag{13}$$

where pp_i^{lk} represents the proximity between the preference of expert e_i and the collective one on the pair of alternatives (x_l, x_k). These proximity values are used by the model to identify the furthest individual preference values from the collective ones, and therefore the preference values that should be considered by the group of experts for possible changes.

(b) *Identification of preferences to be changed.* The identification of the preference values to be subject of modification is done by considering both the consensus degrees and the proximity values. The alternatives with a level of agreement by all experts, ca^l, lower than the consensus threshold are identified, and for these alternatives the preference values with a level of agreement by all experts, cp^{lk}, lower than the threshold will be subject of a recommendation of change:

$$SC = \{(l,k) \in \{1,\ldots,n\} \mid \max\{ca^l, cp^{lk}\} < cr\}$$

The recommendation of change will be produced for just those experts that are furthest from the whole group in the identified elements of SC. This is done by comparing, for each $(l,k) \in SC$, pp_i^{lk} with a collective threshold $\overline{pp}^{lk}$ computed by aggregating all individual proximity values:

$$\overline{pp}^{lk} = \theta(pp_1^{lk}, \ldots, pp_m^{lk}). \tag{14}$$

Those experts for which $pp_i^{lk} < \overline{pp}^{lk}$ will be provided with a recommendation of the direction of change of their current preference values associated to $(l,k) \in SC$.

(c) *Direction changes.* For each preference to be changed, the model will suggest increasing or decreasing its current assessment [11]:

DR.1. If $(c_i^{lk} - c_c^{lk}) < 0$, then expert e_i should increase the linguistic assessment associated to the pair of alternatives (x_l, x_k).

DR.2. If $(c_i^{lk} - c_c^{lk}) > 0$, then expert e_i should decrease the linguistic assessment associated to the pair of alternatives (x_l, x_k).

DR.3. If $(c_i^{lk} - c_c^{lk}) = 0$, then expert e_i should not modify the linguistic assessment associated to the pair of alternatives (x_l, x_k).

4 CONCLUSIONS

In this contribution we have presented a consensus modelled for multi-granular linguistic GDM problems that is based on the use of the type-1 OWA operator. This operator allows the direct aggregation of fuzzy sets, therefore making it superfluous to unify the multi-granular information as proposed in previous models. An advantage of the proposed method is that it avoids the loss of information that the mentioned methods incurred. One further issue is to carry out a comparative study of the proposed method with previous ones, in order to ascertain the extent of the influence that the type-1 OWA operators brings about in the measuring of consensus.

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