LINGUISTIC DECISION MAKING USING DEMPSTER-SHAFER THEORY OF EVIDENCE

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Abstract: - We study different approaches for decision making with linguistic information. We describe some basic linguistic aggregation operators. We then analyze the problem of linguistic decision making with Dempster-Shafer (D-S) belief structure. We suggest the use of different types of linguistic aggregation operators in the D-S framework such as the linguistic ordered weighted averaging (LOWA) operator and the linguistic hybrid averaging (LHA) operator. Finally, we develop an illustrative example where we can see the different results obtained by using different types of linguistic aggregation operators.

Keywords: - Linguistic decision making, Dempster-Shafer theory of evidence, Linguistic aggregation operators, Linguistic OWA (LOWA) operator.

1 INTRODUCTION

The Dempster-Shafer (D-S) theory of evidence was introduced by Dempster in (1967; 1968) and by Shafer in (1976). Since its appearance, this theory has been used in a lot of situations (Srivastava and Mock, 2002; Yager et al., 1994). It provides a unifying framework for representing uncertainty because it includes as special cases the situations of risk and ignorance. The difference between their works is that each one associated a different semantics in the theory although their ideas were practically the same. Dempster was interested in a probabilistic framework while Shafer was more oriented to belief measurement. The two fundamental measures of the theory developed by Shafer (1976), belief and plausibility, were previously studied by Dempster. He referred to them as upper and lower probabilities.

Usually, when using the D-S theory in decision making it is considered that the available information is numerical (Engemann, et al., 1996; Merigó and Casanovas, 2006; 2007; Yager, 1992a; 2004). However, this may not be the real situation found in the decision making problem. Sometimes, the available information is vague or imprecise and it is not possible to analyze it with numerical values. Therefore, it is necessary to use another approach such as a qualitative one that uses linguistic assessments. In this paper, we will study the decision making problem with D-S belief structure using linguistic information. In order to develop the linguistic approach we will follow the ideas of (Herrera and Martínez, 2000a; 2000b; 2001; Xu, 2004a; 2004b; 2006). We will focus on the approach developed by Xu (2004a; 2004b; 2006) where it is implicitly assumed high levels of uncertainty and it is not possible to specify the linguistic values with the 2-tuples linguistic representation model (Herrera and Martínez, 2000a; 2000b; 2001). By using a continuous linguistic term set (Xu, 2004a; 2004b) we will be able to establish an order of the alternatives in the decision making problem without losing the information given in the aggregation step.

In order to aggregate the linguistic information we will use different types of linguistic aggregation operators. The reason for using various types of linguistic aggregation operators is that we want to show that the linguistic decision making problem with D-S theory can be modelled in different ways depending on the interests of the decision maker. We will use the linguistic ordered weighted averaging (LOWA) (Herrera and Herrera-Viedma, 1997; Herrera et al., 1995;1996; Xu, 2004a) operator because it provides a parameterized family of linguistic aggregation operators that include the maximum, the minimum, the linguistic average (LA) and the linguistic weighted average (LWA), among others. The LOWA operator is an extension of the traditional ordered weighted averaging (OWA) operator (Calvo et al., 2002; Merigó, 2007; Yager, 1988; Yager and Kacprzyk, 1997) for the cases where the information is given in the form of linguistic values. We should note that we will use the LOWA operator developed by Xu in (2004a) that it is also known as the extended ordered weighted averaging (EOWA) operator. Apart from the LOWA operator, we will use the linguistic hybrid averaging (LHA) operator (Xu,, 2006) because this operator uses in the same formulation the LWA and the LOWA operator. For all these types of linguistic aggregation operators we will develop different families of operators that could be used in the linguistic decision making problem with D-S belief structure such as the step-LOWA operator, the window-LOWA operator, the centered-LOWA operator, the E-Z LOWA weights, the LOWA median, etc (Merigó, 2007). Note that these families are based on the original version developed for the OWA operator (Calvo et al., 2002; Merigó, 2007; Xu, 2005; Yager, 1988; 1992b; 1993; Yager and Kacprzyk, 1997).

In order to do so, this paper is organized as follows. In Section 2 we briefly review some basic concepts to be used throughout the paper such as the linguistic approach and some linguistic aggregation operators. Section 3 develops the new approach about using linguistic information in decision making with D-S theory of evidence. Section 4 gives an illustrative example about the use of the proposed scheme. Finally, Section 5 summarizes the main conclusions found in the paper.

2 PRELIMINARIES

In this Section, we briefly describe the linguistic approach and some basic linguistic aggregation operators that we will use throughout the paper.

2.1 Linguistic Approach

Usually, people are used to work in a quantitative setting, where the information is expressed by means of numerical values. However, many aspects of the real world cannot be assessed in a quantitative form. Instead, it is possible to use a qualitative one, i.e., with vague or imprecise knowledge. In this case, a better approach may be the use of linguistic assessments instead of numerical values. The linguistic approach represents qualitative aspects as linguistic values by means of linguistic variables (Zadeh, 1975).

We have to select the appropriate linguistic descriptors for the term set and their semantics. One possibility for generating the linguistic term set consists in directly supplying the term set by considering all terms distributed on a scale on which a total order is defined (Herrera and Herrera-Viedma, 1997; Yager, 1995). For example, a set of seven terms *S* could be given as follows:

$$S = \{s_1 = N, s_2 = VL, s_3 = L, s_4 = M, s_5 = H, s_6 = VH, s_7 = P\}$$

Note that *N = None*, *VL = Very low*, *L = Low*, *M = Medium*, *H = High*, *VH = Very high*, *P = Perfect*. Usually, in these cases, it is required that in the linguistic term set there exists:

1. A negation operator: $\text{Neg}(s_i) = s_j$ such that $j = g+1-i$.
2. The set is ordered: $s_i \leq s_j$ if and only if $i \leq j$.
3. Max operator: $\text{Max}(s_i, s_j) = s_i$ if $s_i \geq s_j$.
4. Min operator: $\text{Min}(s_i, s_j) = s_i$ if $s_i \leq s_j$.

Different approaches have been developed for dealing with linguistic information such as (Bonissone, 1982; Herrera and Herrera-Viedma, 1997; Yager, 1995). In this paper, we will follow the ideas of (Herrera and Martínez, 2000a; 2000b; 2001; Xu, 2004a; 2004b; 2006). Then, in order to preserve all the given information, we extend the discrete linguistic term set S to a continuous linguistic term set $\hat{S} = \{s_\alpha \mid s_1 < s_\alpha \leq s_t, \alpha \in [1, t]\}$, where, if $s_\alpha \in S$, we call s_α the original linguistic term, otherwise, we call s_α the virtual linguistic term.

Consider any two linguistic terms $s_\alpha, s_\beta \in \hat{S}$, and $\mu, \mu_1, \mu_2 \in [0, 1]$, we define some operational laws as follows (Xu, 2004a; 2004b):

1. $\mu s_\alpha = s_{\mu\alpha}$.
2. $s_\alpha \oplus s_\beta = s_\beta \oplus s_\alpha = s_{\alpha+\beta}$.
3. $(s_\alpha)^\mu = s_{\alpha^\mu}$.
4. $s_\alpha \otimes s_\beta = s_\beta \otimes s_\alpha = s_{\alpha\beta}$.

2.2 Linguistic Aggregation Operators

In the literature, we find a wide range of linguistic aggregation operators (Delgado et al., 1993; Herrera and Herrera-Viedma, 1997; Herrera et al., 1995;1996; Herrera and Martínez, 2000; Xu, 2004a; 2004b; 2006). In this study, we will consider the linguistic ordered weighted averaging (LOWA) operator and the linguistic hybrid averaging (LHA) operator, with their particular cases that include among others the linguistic average (LA) and the linguistic weighted average (LWA). Note that we follow the ideas developed by Xu in (2004a, 2004b; 2006). Then, we should point out that the LOWA operator we are going to use is also known as the extended OWA (EOWA) operator (2004a).

Definition 1. A LOWA operator of dimension n is a mapping LOWA: $\hat{S}^n \rightarrow \hat{S}$, which has an associated weighting vector W such that $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, then:

$$\text{LOWA}(s_{\alpha_1}, s_{\alpha_2}, \ldots, s_{\alpha_n}) = \sum_{j=1}^{n} w_j s_{\beta_j} \tag{1}$$

where s_{β_j} is the jth largest of the s_{α_i}.

From a generalized perspective of the reordering step, we can distinguish between the descending LOWA (DLOWA) and the ascending LOWA (ALOWA) operator. The weights of these operators are related by $w_j = w^*_{n+1-j}$, where w_j is the jth weight of the DLOWA (or LOWA) operator and w^*_{n+1-j} the jth weight of the ALOWA operator. Note that the ALOWA operator is known in other studies as the inverse LOWA (I-LOWA) operator (Herrera and Herrera-Viedma, 1997).

The LOWA operator provides a parameterized family of aggregation operators that includes as special cases the LA and the linguistic weighted average (LWA). The LA is obtained when all the weights w_j are equal for all j. The LWA is obtained if the ordered position of the s_{β_j} is the same than the ordered position of the s_{α_i}.

In this type of operator it is possible to use different measures for characterizing the weighting vector W by using the same measures that it has been used for the OWA operator (Calvo et al., 2002; Merigó, 2007; Xu, 2005; Yager, 1988; 1992b; 1993; Yager and Kacprzyk, 1997) such as the attitudinal character or the measure of dispersion.

Definition 2. A LHA operator of dimension n is a mapping LHA: $\hat{S}^n \rightarrow \hat{S}$, which has an associated weighting vector W such that $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, then:

$$\text{LHA}(s_{\alpha_1}, s_{\alpha_2}, \ldots, s_{\alpha_n}) = \sum_{j=1}^{n} w_j s_{\beta_j} \tag{2}$$

where s_{β_j} is the jth largest of the linguistic weighted argument $\hat{s}_{\alpha_i}$ ($\hat{s}_{\alpha_i} = n\omega_i s_{\alpha_i}$, $i = 1,2,\ldots,n$), $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is the exponential weighting vector of the s_{α_i}, with $\omega_j \in [0, 1]$ and the sum of the weights is 1.

In this case, we can also distinguish between the descending LHA (DLHA) and the ascending LHA (ALHA) operator. Note that in this case they are also related by $w_j = w^*_{n+1-j}$, where w_j is the jth weight of the LHA (or DLHA) operator and w^*_{n+1-j} the jth weight of the ALHA operator.

By using a different manifestation of the weighting vectors, we are able to obtain different families of LHA operators. For example, the LWA is obtained when the all the weights w_j are $1/n$, for all j. The LOWA operator is obtained when all the weights ω_i are $1/n$, for all i.

In the following, we are going to develop different types of linguistic geometric operators. Note that these operators are extensions of the OWG operator (Chiclana et al., 2000; Xu and Da, 2002) by using linguistic variables.

Definition 3. A LOWG operator of dimension n is a mapping LOWG: $\hat{S}^{+n} \rightarrow \hat{S}^{+}$ which has an associated weighting vector W such that $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, then:

$$\text{LOWG}(s_{\alpha_1}, s_{\alpha_2}, \ldots, s_{\alpha_n}) = \prod_{j=1}^{n} (s_{\beta_j})^{w_j} \tag{3}$$

where s_{β_j} is the jth largest of the s_{α_i}.

From a generalized perspective of the reordering step, we have to distinguish between the descending LOWG (DLOWG) operator and the ascending LOWG (ALOWG) operator (Xu and Da, 2002). Note that this operator is commutative, monotonic, bounded and idempotent.

The LOWG operator provides a parameterized family of aggregation operators that includes as special cases the LGA and the linguistic weighted geometric average (LWGA). The LGA is obtained when all the weights w_j are equal for all j. The LWGA is obtained if the ordered position of the s_{β_j} is the same than the ordered position of the s_{α_i}.

Definition 4. A LHGA operator of dimension n is a mapping LHGA: $\hat{S}^{+^n} \rightarrow \hat{S}^{+}$ which has an associated weighting vector W such that $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, then:

$$\text{LHGA}(s_{\alpha_1}, s_{\alpha_2}, \ldots, s_{\alpha_n}) = \prod_{j=1}^{n} (s_{\beta_j})^{w_j} \quad (4)$$

where s_{β_j} is the jth largest of the linguistic weighted argument $\hat{s}_{\alpha_i}$ ($\hat{s}_{\alpha_i} = (s_{\alpha_i})^{n\omega_i}$, $i = 1,2,\ldots,n$), $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is the exponential weighting vector of the s_{α_i}, with $\omega_j \in [0, 1]$ and the sum of the weights is 1.

In this case, it is also possible to distinguish between the descending LHGA (DLHGA) operator and the ascending LHGA (ALHGA) operator.

By using a different manifestation of the weighting vectors, we are able to obtain different families of LHGA operators. For example, the LWGA is obtained when the all the weights w_j are $1/n$, for all j. The LOWA operator is obtained when all the weights ω_i are $1/n$, for all i.

Finally, we should note that other types of linguistic aggregation operators could be developed for the analysis. But in this paper, we will focus on the ones explained above.

3 LINGUISTIC DECISION MAKING USING DEMPSTER-SHAFER THEORY

3.1 Introduction

The D-S theory of evidence was introduced by Dempster in (1967; 1968) and by Shafer in (1976). Since then, a lot of new developments have been developed about it such as (Srivastava and Mock, 2002, Yager et al., 1994). This type of formulation provides a unifying framework for representing uncertainty as it can include the cases of risk and ignorance as special situations of this framework. Obviously, the case of certainty is also included in this generalization as it can be seen as a particular situation of risk or ignorance. Apart from these traditional cases, the D-S framework allows to represent various other forms of information a decision maker may have about the states of nature.

Definition 5. A D-S belief structure defined on a space X consists of a collection of n nonnull subsets of X, B_j for $j = 1,\ldots,n$, called focal elements and a mapping m, called the basic probability assignment, defined as, m: $2^X \rightarrow [0, 1]$ such that:

(1) $m(B_j) \in [0, 1]$.
(2) $\sum_{j=1}^{n} m(B_j) = 1$.
(3) $m(A) = 0$, $\forall A \neq B_j$.

As we said before, the cases of risk and ignorance are included as special cases of belief structure in the D-S framework. For the case of risk, a belief structure is called Bayesian belief structure (Shafer, 1976) if it consists of n focal elements such that $B_j = \{x_j\}$, where each focal element is a singleton. Then, we can see that we are in a situation of decision making under risk environment as $m(B_j) = P_j$ = Prob $\{x_j\}$.

For the case of ignorance, the belief structure consists in only one focal element B, where $m(B)$ essentially is the decision making under ignorance environment as this focal element comprises all the states of nature. Thus, $m(B) = 1$. Other special cases of belief structures such as the consonant belief structure or the simple support function are studied in (Shafer, 1976).

Two important evidential functions associated with these belief structures are the measures of plausibility and belief (Shafer, 1976).

3.2 Linguistic Aggregation Operators in Dempster-Shafer Framework

The problem of decision making with D-S belief structures has been studied by different authors such as (Engemann, et al., 1996; Merigó and Casanovas, 2006; 2007; Yager, 1992a; 2004). In (1992a), Yager proposed a more generalized methodology by using the OWA operator. In these papers, the available information was supposed to be numerical. However, many decision making problems cannot be assessed with numerical values because the knowledge of the decision maker is vague or imprecise. Then, a better approach may be the use of linguistic assessments instead of numerical ones.

In order to develop the decision making process with linguistic variables, we need to aggregate the linguistic information. For doing this, we will use the operational laws and the different aggregation operators commented in Section 2. First, we will study the process to follow when using the LOWA operator in decision making with D-S theory of evidence. The procedure can be summarized as follows.

Assume we have a decision problem in which we have a collection of alternatives $\{A_1, \ldots, A_q\}$ with states of nature $\{N_1, \ldots, N_n\}$. S_{ij} is the linguistic payoff to the decision maker if he selects alternative A_i and the state of nature is N_j. The knowledge of the state of nature is captured in terms of a belief structure m with focal elements $B_1, \ldots, B_r$ and associated with each of these focal elements is a weight $m(B_k)$. The objective of the problem is to select the alternative which best satisfies the linguistic payoff to the decision maker. In order to do so, we should follow the following steps:

Step 1: Calculate the linguistic payoff matrix.

Step 2: Determine the belief function m about the states of nature and the decision makers degree of optimism α. Note that for the LOWA operator we use the same measure than (Yager, 1988).

Step 3: Calculate the collection of weights, w, to be used in the LOWA aggregation for each different cardinality of focal elements.

Step 4: Determine the linguistic payoff collection, M_{ik}, if we select alternative A_i and the focal element B_k occurs, for all the values of i and k. Hence $M_{ik} = \{S_{ij} \mid N_j \in B_k\}$.

Step 5: Calculate the linguistic aggregated payoff, V_{ik} = LOWA(M_{ik}), using Eq. (1), for all the values of i and k. Note that it is possible to use for each focal element a different type of LOWA operator. That is, for each focal element we can use a different weighting vector W.

Step 6: For each alternative, calculate the generalized linguistic expected value, S_i, where:

$$S_i = \sum_{k=1}^{r} V_{ik} \; m(B_k) \qquad (5)$$

Step 7: Select the alternative with the largest S_i as the optimal. Note that it is possible to establish an order of the results obtained.

Analyzing the aggregation steps, we can formulate in one equation the whole aggregation process as:

$$S_i = \sum_{k=1}^{r} m(B_k) LOWA(M_{ik}) \tag{6}$$

As we can see, the focal weights are aggregating the results obtained by using the LOWA operator.

Another interesting issue to comment is that in some cases, we could prefer to aggregate with the ALOWA operator in the D-S decision process instead of the LOWA operator. As it has been explained for the OWA aggregation in (Merigó and Casanovas, 2006), the main motivation for this is that we have to make a distinction when dealing with situations where the highest linguistic argument is the best result and situations where the smallest linguistic argument is the best result.

Then, if we use the ALOWA operator in decision making with D-S belief structures, we should make the following changes in the decision process.

In *Step* 2-3, when calculating the collection of weights, w, to be used in the ALOWA aggregation for each different cardinality of focal elements, we should consider that now the attitudinal character $\alpha(W)$ is defined in ascending order.

In *Step* 5, when calculating the aggregated payoff, we should use V_{ik} = ALOWA(M_{ik}), for all the values of i and k.

In *Step* 7, we should select the alternative with the lowest S_i as the optimal because the best result is the one which predicts the lowest expected values. Note that it is also possible to use the ALOWA operator in situations where the highest value is the best result. But as we already use the DLOWA operator in these situations, it is better to use the ALOWA operator in other situations in order to coordinate both aggregations.

By choosing a different manifestation of the weighting vector in the LOWA operator, we are able to obtain different types of aggregation operators in the decision process with D-S framework. For example, we can obtain the maximum, the minimum, the LA, the Hurwicz linguistic criteria, the LWA and the LOWA operator. Note that these operators can be obtained by using the LOWA or the ALOWA operator. These two parameterized families of aggregation operators are related by $w_j = w^*_{n+1-j}$, where w_j is the jth weight of the LOWA (or DLOWA) operator and w^*_{n+1-j} the jth weight of the ALOWA operator. Other families of aggregation operators could be obtained with the LOWA operator by using a different manifestation in the weighting vector such as the step-LOWA, window-LOWA, olympic-LOWA, E-Z LOWA, the median-LOWA, the weighted median-LOWA, the S-LOWA, the centered-LOWA, the maximal entropy LOWA (MELOWA), etc. For more information on these families, see (Merigó, 2007).

In some situations, we could prefer to use another type of linguistic aggregation operator in the D-S decision process such as the LHA operator. The main advantage of this operator is that it uses in the same aggregation the characteristics of the LWA and the characteristics of the LOWA operator. Then, if we introduce this operator in decision making with D-S belief structures, we are able to develop a unifying framework that includes in the same formulation probabilities, LWAs and LOWAs.

In order to use this type of aggregation operator in D-S framework, we should make the following changes to the decision process explained above for the LOWA operator.

In *Step* 3, when calculating the collection of weights, *w*, to be used in the LHA aggregation for each different cardinality of focal elements, we should consider that now we have to define two weighting vectors. Note that these two weighting vectors are used for combining in the same aggregation the LWA and the LOWA operator.

In *Step* 5, when calculating the linguistic aggregated payoff, we should use V_{ik} = LHA(M_{ik}), using Eq. (2) for all the values of *i* and *k*.

In this case, we could also formulate in one equation the whole aggregation process as follows.

$$S_i = \sum_{k=1}^{r} m(B_k) LHA(M_{ik}) \quad (7)$$

As we can see, the focal weights are aggregating the results obtained by using the LHA operator, which combines in the same aggregation the LWA and the LOWA operator. Note that if all the weights ω_i are 1/*n*, for all *i*, then, Eq. (7) is transformed in Eq. (6).

In this case, it is also possible to find situations where it is better to use an ascending order in the aggregation. Then, we will use the ALHA operator in the decision process with D-S theory. We should note that the main differences against the LHA operator is that now, in *Step 3* we should use an ascending order in the collection of weights and in *Step 5* we should use V_{ik} = ALHA(M_{ik}).

When aggregating the collection of linguistic payoffs of each focal element with the LHA operator, it is also possible to use a wide range of families of LHA operators. For example, we could obtain the maximum, the minimum, the Hurwicz linguistic criteria, the LA, the LWA, the median-LHA, the step-LHA, the window-LHA, the E-Z LHA, the S-LHA, the centered-LHA, etc. For more information on these families, see (Merigó, 2007).

3.3 Linguistic Geometric Operators in Dempster-Shafer Framework

Another alternative for decision making with D-S theory is the use of linguistic geometric aggregation operators. The reason for using linguistic geometric operators appears because there are situations where the decision maker may prefer to use geometric operators instead of the traditional averaging ones. The first model that considered the use of geometric operators in D-S framework was suggested in (Merigó and Casanovas, 2006) for situations with numerical information. In this Section we will develop a similar approach, but now we will assume that the available information is expressed with linguistic variables.

The process to follow when using linguistic geometric operators (Xu, 2004a; 2004b) in decision making with D-S theory of evidence is very similar to the previous methods commented in Section 3.2. First, we will consider the use of the LOWG operator, that it is based on the OWG operator (Chiclana et al., 2000; Xu and Da, 2002) in D-S framework. Assuming that we use the same variables as it has been explained in Section 3.2 when using the LOWA operator we could summarize the procedure as follows.

Step 1: Calculate the linguistic payoff matrix.
Step 2: Calculate the belief function *m* about the states of nature and the decision makers degree of optimism.

Step 3: Calculate the collection of weights, *w*, to be used in the LOWG aggregation for each different cardinality of focal elements.

Step 4: Determine the payoff collection, M_{ik}, if we select alternative A_i and the focal element B_k occurs, for all the values of *i* and *k*. Hence $M_{ik} = \{S_{ij} \mid N_j \in B_k\}$.

Step 5: Calculate the linguistic aggregated payoff, V_{ik} = LOWG(M_{ik}), using Eq. (3), for all the values of *i* and *k*.

Step 6: For each alternative, calculate the generalized linguistic expected value, S_i, where:

$$S_i = \sum_{k=1}^{r} V_{ik}\ m(B_k) \tag{8}$$

Step 7: Select the alternative with the largest S_i as the optimal. Note that in a situation of costs or similar, we should select the alternative with the lowest S_i.

Analyzing the aggregation in *Step 6* and *Step 7*, we can formulate in one equation the whole aggregation process as:

$$S_i = \sum_{k=1}^{r} m(B_k) LOWG(M_{ik}) \tag{9}$$

As we can see, the focal weights are aggregating the results obtained by using the LOWG operator.

From a generalized perspective of the reordering step we can distinguish between the DLOWG and the ALOWG operator. As the definition of the DLOWG operator is the same than the LOWG operator, its use in D-S belief structure is also the same. The reason for using ALOWG operators is because sometimes it is better to use an ascending order in the aggregation. For example, we could use it in situations where the lowest linguistic value is the best result and we want to start the reordering step from this best result.

The procedure to follow if we use the ALOWG operator in the aggregation step is the same than the procedure used for the LOWG or DLOWG operator with the difference that now we use ascending orders in the aggregation.

As it has been explained in Section 3.2, sometimes, the decision maker may prefer to use another type of linguistic geometric operator such as the LHGA operator. Its main advantage is that it uses the LWGA and the LOWG operator in the same aggregation process. In order to use this type of aggregation operator in D-S framework, we just need to replace the LOWA operator by this linguistic geometric operator and adequate the rest of characteristics to it. That is, considering that now we have two weighting vectors and the attitudinal character of the decision maker is different.

Different families of linguistic geometric operators can be obtained in the decision process with D-S theory by using a different manifestation of the weighting vector of the linguistic geometric aggregation operators commented above. For example, we could use the linguistic maximum, the linguistic minimum, the Hurwicz linguistic geometric criteria, the LGA or the LWGA. Other families that could be used in the weighting vector are the step-LOWG operator, the window-LOWG operator, the olympic LOWG, the E-Z LOWG weights, the LOWG median, the weighted LOWG median, the S-LOWG operator, the centered LOWG operator, etc. Note that similar families could also be obtained with the LHGA operator. For more information on these families, see (Merigó, 2007).

4 ILLUSTRATIVE EXAMPLE

In the following, we are going to develop an illustrative example in order to understand the procedures commented above. We will analyze a decision making problem with D-S belief structure. We will use different types of linguistic aggregation operators such as the LA, the LWA, the LOWA, the ALOWA, the LHA and the ALHA operator. Note that we assume for all the cases a situation where the highest value is the best result.

Step 1: Assume an investment company has five possible investments and they want to select the alternative that better adapts to his interests.

1) A_1 is a car company.
2) A_2 is a food company.
3) A_3 is a computer company.
4) A_4 is a chemical company.
5) A_5 is a TV company.

Depending on different uncertain situations that could happen in the future the experts of the investment company establishes the payoff matrix. As the future states of nature are very imprecise, the experts cannot determine numerical values in the payoff matrix. Instead, they use linguistic variables to calculate the future benefits of the companies depending on the state of nature that happens in the future. They establish the following linguistic scale.

S = {s_1 = *Extremely low*, s_2 = *Very low*, s_3 = *Low*, s_4 = *Medium*, s_5 = *High*, s_6 = *Very high*, s_7 = *Extremely high*}.

The possible results depending on the state of nature that happens in the future are shown in Table 1.

Table 1: Linguistic payoff matrix

	N_1	N_2	N_3	N_4	N_5	N_6	N_7	N_8
A_1	S_2	S_5	S_6	S_4	S_3	S_7	S_2	S_1
A_2	S_3	S_4	S_4	S_4	S_7	S_6	S_1	S_2
A_3	S_2	S_4	S_5	S_3	S_4	S_4	S_5	S_3
A_4	S_4	S_3	S_1	S_1	S_7	S_4	S_2	S_7
A_5	S_3	S_4	S_6	S_2	S_1	S_6	S_7	S_2

Step 2: Although the information is very imprecise, the experts have obtained some empirical and historical data that has permitted them to establish some probabilistic information about which state of nature will happen in the future. This information is represented by the following belief structure.

Focal element
$B_1 = \{N_2, N_3, N_4, N_5\} = 0.3$
$B_2 = \{N_1, N_3, N_7, N_8\} = 0.3$
$B_3 = \{N_1, N_4, N_5, N_6, N_7\} = 0.4$

Step 3: Assume we have used one of the existing methods for determining the LOWA weights and we have obtained $W_4 = (0.2, 0.2, 0.3, 0.3)$ and $W_5 = (0.1, 0.2, 0.2, 0.2, 0.3)$. For the weighting vector to be used in the hybrid aggregations we assume $\omega_4 = (0.2, 0.2, 0.3, 0.3)$ and $\omega_5 = (0.1, 0.2, 0.2, 0.2, 0.3)$.

Step 4: Calculate the payoff collection, M_{ik}, if we select alternative A_i and the focal element B_k occurs, for all the values of i and k.

A_1: $M_{11} = \langle S_5, S_6, S_4, S_3\rangle$; $M_{12} = \langle S_2, S_6, S_2, S_1\rangle$; $M_{13} = \langle S_2, S_4, S_3, S_7, S_2\rangle$.
A_2: $M_{21} = \langle S_4, S_4, S_4, S_7\rangle$; $M_{22} = \langle S_3, S_4, S_1, S_2\rangle$; $M_{23} = \langle S_3, S_4, S_7, S_6, S_1\rangle$.
A_3: $M_{31} = \langle S_4, S_5, S_3, S_4\rangle$; $M_{32} = \langle S_2, S_5, S_5, S_3\rangle$; $M_{33} = \langle S_2, S_3, S_4, S_4, S_5\rangle$.
A_4: $M_{41} = \langle S_3, S_1, S_1, S_7\rangle$; $M_{42} = \langle S_4, S_1, S_2, S_7\rangle$; $M_{43} = \langle S_4, S_1, S_7, S_4, S_2\rangle$.
A_5: $M_{51} = \langle S_4, S_6, S_2, S_1\rangle$; $M_{52} = \langle S_3, S_6, S_7, S_2\rangle$; $M_{53} = \langle S_3, S_2, S_1, S_6, S_7\rangle$.

Step 5: Calculate the aggregated linguistic payoff, V_{ik}, using Eq. (1) for the LOWA, the ALOWA, the LA and the LWA, and using Eq. (2) for the LHA and the ALHA, The results are shown in Table 2.

Table 2: Aggregated payoff for the linguistic aggregation operators

	LA	*LWA*	*LOWA*	*ALOWA*	*LHA*	*ALHA*
M_{11}	$S_{4.5}$	$S_{4.3}$	$S_{4.3}$	$S_{4.7}$	$S_{4.2}$	$S_{4.4}$
M_{12}	$S_{2.75}$	$S_{2.5}$	$S_{2.5}$	S_3	$S_{2.28}$	$S_{2.72}$
M_{13}	$S_{3.6}$	$S_{3.6}$	$S_{3.1}$	$S_{4.1}$	$S_{2.4}$	$S_{3.36}$
M_{21}	$S_{4.75}$	$S_{4.9}$	$S_{4.6}$	$S_{4.9}$	$S_{4.56}$	$S_{5.24}$
M_{22}	$S_{2.5}$	$S_{2.3}$	$S_{2.3}$	$S_{2.7}$	$S_{2.2}$	$S_{2.4}$
M_{23}	$S_{4.2}$	S_4	$S_{3.6}$	$S_{4.8}$	$S_{2.76}$	$S_{3.64}$
M_{31}	S_4	$S_{3.9}$	$S_{3.9}$	$S_{4.1}$	$S_{3.8}$	S_4
M_{32}	$S_{3.75}$	$S_{3.8}$	$S_{3.5}$	S_4	$S_{3.56}$	$S_{4.04}$
M_{33}	$S_{3.6}$	$S_{3.9}$	$S_{3.3}$	$S_{3.9}$	$S_{2.6}$	$S_{3.64}$
M_{41}	S_3	$S_{3.2}$	$S_{2.6}$	$S_{3.4}$	$S_{2.76}$	$S_{3.64}$
M_{42}	$S_{3.5}$	$S_{3.7}$	$S_{3.1}$	$S_{3.9}$	$S_{3.28}$	$S_{4.12}$
M_{43}	$S_{3.6}$	$S_{3.4}$	S_3	$S_{4.2}$	$S_{2.24}$	$S_{3.2}$
M_{51}	$S_{3.25}$	$S_{2.9}$	$S_{2.9}$	$S_{3.6}$	$S_{2.68}$	$S_{3.12}$
M_{52}	$S_{4.5}$	$S_{4.5}$	$S_{4.1}$	$S_{4.9}$	$S_{4.08}$	$S_{4.92}$
M_{53}	$S_{3.8}$	$S_{4.2}$	$S_{3.2}$	$S_{4.4}$	$S_{2.6}$	$S_{4.12}$

Step 6: For each alternative, calculate the generalized expected value, C_i, using Eq. (6) - (7) for the LA, the LWA, the LOWA, the ALOWA, the LHA and the ALHA operator. The results are shown in Table 3.

Table 3: Generalized linguistic expected value for the linguistic aggregation operators

	LA	*LWA*	*LOWA*	*ALOWA*	*LHA*	*ALHA*
A_1	$S_{3.615}$	$S_{3.48}$	$S_{3.28}$	$S_{4.25}$	$S_{2.904}$	$S_{3.48}$
A_2	$S_{3.855}$	$S_{3.76}$	$S_{3.51}$	$S_{4.2}$	$S_{3.132}$	$S_{3.748}$
A_3	$S_{3.765}$	$S_{3.87}$	$S_{3.54}$	$S_{3.99}$	$S_{3.248}$	$S_{3.868}$
A_4	$S_{3.39}$	$S_{3.43}$	$S_{2.91}$	$S_{3.87}$	$S_{2.708}$	$S_{3.608}$
A_5	$S_{3.845}$	$S_{3.9}$	$S_{3.38}$	$S_{4.31}$	$S_{3.068}$	$S_{4.06}$

Step 7: Select the best alternative for each aggregation operator. That is, select the investment with the highest linguistic expected value. As we can see, with the LA we will select alternative A_2. With the LOWA and the LHA, operator, we will select alternative A_3. Finally, we will select alternative A_5 with the LWA, the ALOWA and the ALHA operator.

If we establish an order for the investments, a typical situation if we want to select more than one alternative, we can see that each aggregation gives us a different order of the investments. Note that $\succ$ means *preferred to*. The results are shown in table 4.

Table 4: Ordering of the investments

	Ordering		*Ordering*
LA	$A_2 \succ A_5 \succ A_3 \succ A_1 \succ A_4$	*ALOWA*	$A_5 \succ A_1 \succ A_2 \succ A_3 \succ A_4$
LWA	$A_5 \succ A_3 \succ A_2 \succ A_1 \succ A_4$	*LHA*	$A_3 \succ A_2 \succ A_5 \succ A_1 \succ A_4$
LOWA	$A_3 \succ A_2 \succ A_5 \succ A_1 \succ A_4$	*ALHA*	$A_5 \succ A_3 \succ A_2 \succ A_4 \succ A_1$

As we can see, depending on the linguistic aggregation operator used, the ordering of the investments will be different.

5 CONCLUSIONS

We have studied the D-S theory of evidence in situations of decision making with linguistic information. First, we have reviewed some basic concepts about using linguistic information. We have considered different types of linguistic aggregation operators such as the LOWA operator and the LHA operator.

Next, we have developed the new approach about using linguistic information in decision making with D-S belief structure. We have developed two general cases. In the first case, we have used different types of linguistic aggregation operators in the aggregation step of the D-S framework. In the second case, we have developed a similar approach by using linguistic geometric operators. In both cases, we have considered different families that could be used in the analysis such as the step-LOWA, the window-LOWA, the E-Z LOWA, the centered LOWA, etc.

Finally, we have shown an illustrative example about the new approach developed in the paper. We have developed the example considering a wide range of linguistic aggregation operators.

References

Bonissone, P.P. (1982). "A Fuzzy Sets Based Linguistic Approach: Theory and Applications", in: M.M. Gupta and E. Sanchez, Eds., *Approximate Reasoning in Decision Analysis*, North-Holland, pp. 329-339.

Calvo, T., Mayor, G., Mesiar, R. (2002). *Aggregation Operators: New Trends and Applications*, Physica-Verlag, New York.

Chiclana, F., Herrera, F., Herrera-Viedma, E. (2000). "The ordered weighted geometric operator: Properties and application", in: *Proc. 8th International Conference on Information Processing and Management of Uncertainty in Knowledge-Based Systems*, Madrid, Spain, pp. 985-991.

Delgado, M., Verdegay, J.L., Vila, M.A. (1993). "On aggregation operations of linguistic labels", *International Journal of Intelligent Systems*, 8, pp. 351-370.

Dempster, A.P. (1967). "Upper and lower probabilities induced by a multi-valued mapping", *Annals of Mathematical Statistics*, 38, pp. 325-339.

Dempster, A.P. (1968). "A generalization of Bayesian inference", *Journal of the Royal Statistical Society*, B 30, pp. 205-247.

Engemann, K.J., Miller, H.E., Yager, R.R. (1996). "Decision making with belief structures: an application in risk management", *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, 4, pp. 1-26.
Herrera, F., Herrera-Viedma, E. (1997). "Aggregation operators for linguistic weighted information", *IEEE Transactions on Systems, Man and Cybernetics*, B 27, pp. 646-655.
Herrera, F., Herrera-Viedma, E., Verdegay, J.L. (1995). "A Sequential Selection Process in Group Decision Making with a Linguistic Assessment Approach", *Information Sciences*, 85, pp. 223-239.
Herrera, F., Herrera-Viedma, E., Verdegay, J.L. (1996). "Direct approach processes in group decision making using linguistic OWA Operators", *Fuzzy Sets and Systems*, 79, pp. 175-190.
Herrera, F., Martínez, L. (2000). "A 2-tuple Fuzzy Linguistic Representation Model for Computing with Words", *IEEE Transactions on Fuzzy Systems*, 8, pp. 746-752.
Herrera, F., Martínez, L. (2000). "An Approach for Combining Numerical and Linguistic Information based on the 2-tuple fuzzy linguistic representation model in Decision Making", *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, 8, pp. 539-562.
Herrera, F., Martínez, L. (2001). "The 2-tuple Linguistic Computational Model. Advantages of its linguistic description, accuracy and consistency", *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, 9, pp. 33-48.
Merigó, J.M. (2007) *Nuevas extensiones a los operadores OWA y su aplicación en los métodos de decisión empresarial*, Unpublished thesis, Department of Business Administration, University of Barcelona.
Merigó, J.M., Casanovas, M. (2006). "Ordered weighted geometric operators in decision making with Dempster-Shafer belief structure", in: *Proc. 13th Congress of the International Association for Fuzzy Set Management and Economy (SIGEF)*, Hammamet, Tunisia, pp.709-727.
Merigó, J.M., Casanovas, M. (2007). "Decision making with Dempster-Shafer theory using induced aggregation operators", in: *Proc. 4th International Summer School on Aggregation Operators (AGOP)*, Ghent, Belgium, pp.95-100.
Shafer, G. (1976). *Mathematical theory of evidence*, Princeton University Press, Princeton, NJ.
Srivastava, R.P., Mock, T. (2002). *Belief Functions in Business Decisions*, Physica-Verlag, Heidelberg.
Xu, Z.S. (2004a). "EOWA and EOWG operators for aggregating linguistic labels based on linguistic preference relations", *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, 12, pp. 791-810.
Xu, Z.S. (2004b). "A method based on linguistic aggregation operators for group decision making with linguistic preference relations", *Information Sciences*, 166, pp. 19-30.
Xu, Z.S. (2005). "An Overview of Methods for Determining OWA Weights", *International Journal of Intelligent Systems*, 20, pp. 843-865.
Xu, Z.S. (2006). "A Note on Linguistic Hybrid Arithmetic Averaging Operator in Multiple Attribute Group Decision Making with Linguistic Information", *Group Decision and Negotiation*, 15, pp. 593-604.
Xu, Z.S., Da, Q.L. (2002). "The Ordered Weighted Geometric Averaging Operators", *International Journal of Intelligent Systems*, 17, pp. 709-716.
Yager, R.R. (1988). "On Ordered Weighted Averaging Aggregation Operators in Multi-Criteria Decision Making", *IEEE Transactions on Systems, Man and Cybernetics*, B 18, pp. 183-190.
Yager, R.R. (1992a). "Decision Making Under Dempster-Shafer Uncertainties", *International Journal of General Systems*, 20, pp. 233-245.
Yager, R.R. (1992b). "On generalized measures of realization in uncertain environments", *Theory and Decision*, 33, pp. 41-69.
Yager, R.R. (1993). "Families of OWA operators", *Fuzzy Sets and Systems*, 59, pp. 125-148.
Yager, R.R. (1995). "An Approach to Ordinal Decision Making", *International Journal of Approximate Reasoning*, 12, pp. 237-261.

Yager, R.R. (2004). "Uncertainty modeling and decision support", *Reliability Engineering and System Safety*, 85, pp. 341-354.

Yager, R.R., Fedrizzi, M., Kacprzyk, J. (1994). *Advances in the Dempster-Shafer theory of evidence*, John Wiley & Sons, New York.

Yager, R.R., Kacprzyk, J. (1997). *The Ordered Weighted Averaging Operators: Theory and Applications*, Kluwer Academic Publishers, Norwell, MA.

Zadeh, L.A. (1975). "The Concept of a Linguistic Variable and its application to Approximate Reasoning. Part 1", *Information Sciences*, 8, pp. 199-249, "Part 2", *Information Sciences*, 8, pp. 301-357, "Part 3", *Information Sciences*, 9, pp. 43-80.