A Decision Making Model Based on Dempster-Shafer Theory and Linguistic Hybrid Aggregation Operators

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Abstract

The solving processes for decision making problems based on the use of the Dempster-Shafer (D-S) theory can be accomplished in different ways according to the necessities of each single problem. In this contribution we present a decision making scheme based on the D-S defined in a linguistic framework and then, we propose the use of an hybrid averaging operator (2-THA) that use the 2-tuple linguistic representation model. By using the 2-THA in D-S theory, we obtain a new aggregation operator: the belief structure - 2-THA (BS-2-THA) operator. We study some of its main properties and then show an illustrative example of the new approach in a decision making problem.

1. Introduction

The Dempster-Shafer (D-S) theory of evidence [2,10] provides a unifying framework for representing uncertainty as it includes situations of certainty, risk and uncertainty in the same formulation. Since its appearance, it has been used in a wide range of applications [3,8-9,15,18]. Usually, the use of the D-S theory in decision making, considers that the available information is numerical. However, this may not be always the situation in real decision making problems. Sometimes, the available information is vague or imprecise and it is not possible to analyze it with numerical values. Then, a better approach may be the use of linguistic assessments [19]. In the literature, we find a wide range of approaches for dealing with linguistic information such as [1,4-6,11-12,19]. In this paper, we will use the linguistic 2-tuple representation model [5-6] in order to accomplish processes of computing with words without loss of information.

The objective of this contribution is to introduce the use of an hybrid aggregation operator in a linguistic decision making model based on the D-S theory. This operator uses the 2-tuple linguistic representation model and hybridizes the weighted average (WA) and the ordered weighted averaging (OWA) operators [14,16-17] in a similar way to the hybrid averaging (HA) operator [13]. We shall refer to our aggregation operator as the 2-tuple hybrid averaging (2-THA) operator. It will allow to assess probabilities, WAs and OWAs in the same formulation. Then, the whole aggregation process will be referred as the belief structure – 2-THA (BS-2-THA) operator and different families of the BS-2-THA, will be pointed out. Finally, an application of the new approach in a decision making problem about selection of strategies is presented in order to show the applicability of the 2-THA operator. We will compare it with two of its main particular cases: the 2-TWA and the 2-TOWA. Then, we will see the different results obtained by using these aggregation operators. This paper is organized as follows. In Section 2, we review some basic concepts about the 2-tuple linguistic representa-ion model, the 2-THA operator and the D-S theory of evidence. In Section 3, we introduce the new approach about using 2-THA operators in decision making with D-S belief structure. Section 4 presents an illustrative example of the new model and Section 5 summarizes the main conclusions found in the paper.

2. Preliminaries

In this Section, we briefly comment some basic preliminaries to be used throughout the paper such as the 2-tuple linguistic representation model, the 2-THA operator and the D-S belief structure.

2.1. The 2-tuple linguistic representation model

In [5], Herrera and Martínez developed a fuzzy linguistic representation model, which represents the linguistic information with a pair of values called 2-

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tuple, (s, α), where s is a linguistic label and α is a numerical value that represents the value of the symbolic translation. With this model, it is possible to accomplish CW processes without loss of information, solving one of the main limitations of the previous linguistic computational models [1,4].

Definition 1. *Let β be the result of an aggregation of the indexes of a set of labels assessed in the linguistic label set $S = \{s_0, s_1, \ldots, s_g\}$, i.e., the result of a symbolic aggregation operation. $\beta \in [0, g]$, being $g + 1$ the cardinality of S. Let $i = round(\beta)$ and $\alpha = \beta - i$ be two values, such that, $i \in [0, g]$ and $\alpha \in [-0.5, 0.5)$, then α is called a symbolic translation.*

Note that the 2-tuple (s_i, α) that expresses the equivalent information to β is obtained with the following function:

$$\Delta : [0, g] \to S \times [-0.5, 0.5),$$

$$\Delta(\beta) = \begin{cases} s_i & i = round(\beta), \\ \alpha = \beta - i & \alpha \in [-0.5, 0.5). \end{cases} \quad (1)$$

where round is the usual round operation, s_i has the closest index label to β and α is the value of the symbolic translation. For further information on the 2-tuple linguistic representation model, see [5-6,11].

2.2. The 2-tuple hybrid averaging operator

Among the wide range of 2-tuple linguistic aggregation operators available, in this paper, we will focus on the 2-THA operator. It is an extension of the HA operator [13] for situations where the available information is assessed with the 2-tuple linguistic representation model. We are then able to consider linguistic information, the 2-TWA and the 2-TOWA operator in the same problem. It can be defined as follows.

Definition 2. *Let $\hat{S}$ be the set of the 2-tuples. A 2-THA operator of dimension n is a mapping f: $\hat{S}^n \to \hat{S}$, which has an associated weighting vector W such that $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, then:*

$$f((s_1, \alpha_1), (s_2, \alpha_2), \ldots, (s_n, \alpha_n)) = \Delta\left(\sum_{j=1}^{n} w_j \beta_j^*\right) \quad (2)$$

where β_j^* is the jth largest of the linguistic weighted argument β'_i ($\beta'_i = n\omega_i\beta_i$, $i = 1,2,\ldots,n$), $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is the exponential weighting vector of the s_{α_i}, with $\omega_j \in [0, 1]$ and the sum of the weights is 1. Note that β_i is represented in the definition with the 2-tuples (s_i, α_i).

Note that it is possible to distinguish between descending (2-TDHA) and ascending (2-TAHA) orders. Note also that the weights of these operators are related by $w_j = w^*_{n+1-j}$, where w_j is the jth weight of the 2-TDHA (or 2-THA) operator and w^*_{n+1-j} the jth weight of the 2-TAHA operator.

And it is also possible to study a wide range of families of 2-THA operators such as the olympic-2-THA, the S-2-THA, centered-2-THA, etc. For further information, see [7].

2.3. The Dempster-Shafer theory of evidence

The D-S theory provides a unifying framework for representing uncertainty as it can include the situations of risk and ignorance as special cases. Note that the case of certainty is also included as it can be seen as a particular case of risk and ignorance.

Definition 3. *A D-S belief structure defined on a space X consists of a collection of n nonnull subsets of X, B_j for $j = 1,\ldots,n$, called focal elements and a mapping m, called the basic probability assignment, defined as, m: $2^X \to [0, 1]$ such that:*

$$\begin{aligned} &(1) \quad m(B_j) \in [0, 1]. \\ &(2) \quad \textstyle\sum_{j=1}^{n} m(B_j) = 1. \qquad (3) \\ &(3) \quad m(A) = 0, \ \forall A \neq B_j. \end{aligned}$$

As said before, the cases of risk and ignorance are included as special cases of belief structure in the D-S framework. For the case of risk, a belief structure is called Bayesian belief structure if it consists of n focal elements such that $B_j = \{x_j\}$, where each focal element is a singleton. Then, we can see that we are in a situation of decision making under risk environment as $m(B_j) = P_j = \text{Prob}\ \{x_j\}$.

The case of ignorance is found when the belief structure consists in only one focal element B, where $m(B)$ essentially is the decision making under ignorance environment as this focal element comprises all the states of nature. Thus, $m(B) = 1$. Other special cases of belief structures such as the consonant belief structure are studied in [10]. Note that two important evidential functions associated with these belief structures are the measures of plausibility and belief.

3. Linguistic decision making with D-S theory and the 2-THA operator

In this Section, we propose a new approach in decision making with D-S belief structure by using hybrid aggregation operators and the 2-tuple linguistic representation model in decision making with D-S belief structure. First, we will present the decision process to follow in these situations. Next, we will study the new aggregation operator: the BS-2-THA operator. Finally, we will analyze different families of 2-THA operators that could be used in the aggregation.

3.1. Decision making approach

The use of D-S framework in decision making has been studied by different authors such as [3,8,15]. In [15], Yager suggested the use of the OWA operator in decision making with D-S framework in order to provide a more general formulation. In all these papers, it is assumed that the available information is numerical. However, in the real life we may find different situations that can not be assessed with numerical variables. Then, it is necessary to use another approach such as the use of linguistic assessments. In [9], it was suggested the use of different types of linguistic aggregation operators to assess the problem. In this contribution, we will use the 2-tuple linguistic representation model. In order to aggregate the information we will consider the 2-THA operator because it is a more general operator than the LOWA because it uses the LWA and the LOWA in the same formulation. The approach can be summarized as follows.

Assume we have a linguistic decision problem in which we have a collection of alternatives $\{A_1, \ldots, A_q\}$ with states of nature $\{N_1, \ldots, N_n\}$. (s_{ih}, α_{ih}) is the 2-tuple linguistic payoff to the decision maker if he selects alternative A_i and the state of nature is N_h. The knowledge of the state of nature is captured in terms of a belief structure m with focal elements $B_1, \ldots, B_r$ and associated with each of these focal elements is a weight $m(B_k)$. The objective of the problem is to select the alternative which best satisfies the payoff to the decision maker. In order to do so, we should follow the following steps:

Step 1: Calculate the 2-tuple linguistic payoff matrix.

Step 2: Calculate the belief function m about the states of nature and the decision makers degree of optimism based on the measure explained in [7,14].

Step 3: Calculate the collection of weights, w, to be used in the 2-THA aggregation for each different cardinality of focal elements.

Step 4: Determine the 2-tuple linguistic payoff collection, M_{ik}, if we select alternative A_i and the focal element B_k occurs, for all the values of i and k. Hence $M_{ik} = \{(s_{ih}, \alpha_{ih}) \mid N_h \in B_k\}$.

Step 5: Calculate the linguistic aggregated payoff, V_{ik} = 2-THA(M_{ik}), using Eq. (2), for all the values of i and k. Note that it is possible to use for each focal element a different type of 2-THA operator.

Step 6: For each alternative, calculate the generalized expected value, (s_i, α_i), where:

$$(s_i, \alpha_i) = \sum_{k=1}^{r} V_{ik}\, m(B_k) \tag{4}$$

Step 7: Select the alternative with the largest (s_i, α_i) as the optimal.

Remark 1: Sometimes, it could be better to use an ascending order in the aggregation of the 2-THA operator. Then, we will use the 2-TAHA operator.

Remark 2: Note that another possibility could be the use of linguistic geometric aggregation operators, linguistic generalized means, linguistic quasi-arithmetic means, etc.

3.2. The belief structure 2-THA

Analyzing the aggregation in *Steps* 6 and 7, it is possible to formulate in one equation the whole aggregation process. Then, the result obtained is that the focal weights are aggregating the results obtained by using the 2-THA operator. We will call this aggregation process the belief structure - 2-THA (BS-2-THA) aggregation and it can be defined as follows.

Definition 4. *A BS-2-THA operator is defined by*

$$(s_i, \alpha_i) = \sum_{k=1}^{r} \sum_{j_k=1}^{q_k} m(B_k)\, w_{j_k} \beta^*_{j_k} \tag{5}$$

where w_{j_k} *is the weighting vector of the kth focal element such that* $\sum_{j=1}^{n} w_{j_k} = 1$ *and* $w_{j_k} \in [0,1]$, $\beta^*_{j_k}$ *is the* j_k*th largest of the* (s_{ik}, α_{ik}), (s_{ik}, α_{ik}) *is the argument variable and* $m(B_k)$ *is the basic probability assignment. Note that* q_k *refers to the cardinality of each focal element and r is the total number of focal elements.*

The BS-2-THA operator accomplishes some typical properties of the mean operators such as commutativity, monotonicity and idempotency and it provides a wide range of special cases. Note that it is not bounded by the minimum and the maximum because for some exceptional situations, the hybrid aggregations may be higher than the maximum and lower than the minimum[7,9].

3.3. Families of 2-THA operators in belief structures

Different types of 2-THA operators can be used in the aggregation process by using a different manifestation of the weighting vector. For example, it is possible to use the 2-tuple hybrid maximum, the 2-tuple hybrid minimum, the 2-tuple average (2-TA), the 2-TWA and the 2-TOWA operator.

The 2-tuple hybrid maximum is obtained if $w_1 = 1$ and $w_{j*} = 0$, for all $j \neq 1$. The 2-tuple hybrid minimum is obtained if $w_n = 1$ and $w_j = 0$, for all $j \neq n$. The 2-TA is found when $w_j = 1/n$, for all j and $\omega_i = 1/n$, for all i. The 2-TWA is obtained if $w_j = 1/n$, for all j. And the 2-TOWA if $\omega_i = 1/n$, for all i.

Other families of 2-THA operators could be used such as the window-2-THA, the olympic-2-THA, the step-2-THA, the median-2-THA, the S-2-THA, the centered-2-THA, etc. For further information on these and other families, see for example [7, 16].

Remark 3: When $w_j = 1/m$ for $k \leq j \leq k + m - 1$ and $w_j = 0$ for $j > k + m$ and $j < k$, we are using the window-2-THA operator. Note that k and m must be positive integers such that $k + m - 1 \leq n$. Also note that if $m = k = 1$, the window-2-THA is transformed in the linguistic maximum. If $m = 1$, $k = n$, the window-2-THA becomes the linguistic minimum. And if $m = n$ and $k = 1$, the window-2-THA is transformed in the 2-TA.

Remark 4: The olympic-2-THA operator is found if $w_1 = w_n = 0$, and for all others $w_j = 1/(n - 2)$. Note that the window-2-THA can be seen as a generalization of this case when $m = n - 2$ and $k = 2$.

Remark 5: The median-2-THA operator can also be used in this case. If n is odd we assign $w_{(n+1)/2} = 1$ and $w_{j*} = 0$ for all others. If n is even we assign for example, $w_{n/2} = w_{(n/2)+1} = 0.5$ and $w_{j*} = 0$ for all others. Note that if n is even, it is possible to use other methods such as the weighted average.

Remark 6: The step-2-THA operator is found when $w_k = 1$ and $w_j = 0$, for all $j \neq k$. Note that the median-2-THA can be seen as a particular case of this situation when the number of arguments is odd and $k = n/2$.

Remark 7: A further interesting family is the S-2-THA operator. In this case, we can distinguish between three types: the "orlike", the "andlike", and the "generalized" S-2-THA operator. Summarizing, we can say that the generalized S-2-THA operator is obtained when $w_1 = (1/n)(1 - (\alpha + \beta)) + \alpha$, $w_n = (1/n)(1 - (\alpha + \beta)) + \beta$, and $w_j = (1/n)(1 - (\alpha + \beta))$ for all $j = 2$ to $n - 1$ where $\alpha, \beta \in [0, 1]$ and $\alpha + \beta \leq 1$. Note that if $\alpha = 0$, we get the andlike S-2-THA and if $\beta = 0$, the orlike S-2-THA. Also note that if $\alpha + \beta = 1$, we get the 2-tuple hybrid Hurwicz criteria.

Finally, if we assume that all the focal elements use the same weighting vector, then, we can refer to these families as the BS-olympic-2-THA, the BS-S-2-THA, the BS-median-2-THA, the BS-centered-2-THA, etc.

4. Illustrative example

In the following, we are going to develop an illustrative example of the new approach. We will consider a decision making problem where a company is looking for its optimal strategy in an expansion process.

We will use the 2-THA operator and two of its main particular cases: the 2-TWA and the 2-TOWA operator. The main reason for considering these three cases is that they are the most complete ones. The 2-TWA expresses the subjectivity of the decision maker, the 2-TOWA the attitudinal character and the 2-THA includes both cases in the same formulation. However, in a more complete analysis, it would be useful to consider other particular cases in order to provide more information to the decision maker.

Assume a company that operates in Europe and North America is planning an expansion policy to another continent. They consider three possible alternatives to follow.

1) A_1: Expansion to the Asian market.
2) A_2: Expansion to the South American market.
3) A_3: Expansion to the African market.

Table 1. Linguistic payoff matrix.

	N_1	N_2	N_3	N_4	N_5	N_6
A_1	$(S_3, 0.2)$	$(S_4, -0.3)$	$(S_5, 0.1)$	S_6	S_1	$(S_4, 0.2)$
A_2	$(S_4, 0.2)$	$(S_2, 0.1)$	S_4	$(S_3, -0.2)$	S_5	$(S_3, 0.1)$
A_3	$(S_2, 0.4)$	$(S_5, 0.2)$	$(S_1, 0.4)$	S_3	$(S_4, 0.3)$	$(S_6, 0.2)$

In order to evaluate these strategies, the group of experts of the company considers that the key factor is the economic situation for the next year. After careful analysis, the experts have considered six possible situations that could happen in the future: S_1 = Very bad, S_2 = Bad, S_3 = Regular-Bad, S_4 = Regular-Good, S_5 = Good, S_6 = Very good.

The experts of the company establish the payoff matrix. As the environment is very uncertain, they use linguistic information to assess the information. The results are shown in Table 1.

After careful analysis of the information, the experts have obtained some probabilistic information about which state of nature will happen in the future. This information is represented by the following belief structure about the states of nature.

Focal element

$B_1 = \{N_2, N_3, N_4\} = 0.2$

$B_2 = \{N_1, N_6\} = 0.3$

$B_3 = \{N_1, N_4, N_5\} = 0.5$

The experts establish the following weighting vectors for the 2-TOWA operator.

Weighting vector

$W_2 = (0.3, 0.7)$

$W_3 = (0.3, 0.3, 0.4)$

For the 2-THA, they assume the following weighting vector: $\omega = (0.1, 0.1, 0.1, 0.2, 0.2, 0.3)$. Note that when using only two or three arguments, we have to normalize the weights affected. For example, for the first focal element we will use: $\omega_1 = (0.25, 0.25, 0.5)$, for the second one: $\omega_2 = (0.25, 0.75)$ and for the third one: $\omega_3 = (0.2, 0.4, 0.4)$.

With this information, we can obtain the linguistic aggregated payoffs of the focal elements. Once we have the aggregated results, we have to calculate the linguistic generalized expected value. The results are shown in Tables 2 and 3, respectively.

Table 2. Linguistic aggregated results.

	2-TWA	2-TOWA	2-THA
V_{11}	$(S_5, 0.2)$	$(S_5, -0.19)$	$(S_5, -0.04)$
V_{12}	$(S_4, -0.05)$	$(S_3, 0.5)$	$(S_8, -0.1)$
V_{13}	$(S_3, 0.44)$	$(S_3, 0.16)$	$(S_3, 0.21)$
V_{21}	$(S_3, -0.07)$	$(S_3, -0.12)$	$(S_3, -0.21)$
V_{22}	$(S_3, 0.37)$	$(S_3, 0.43)$	$(S_7, -0.25)$
V_{23}	$(S_4, -0.04)$	$(S_4, -0.12)$	$(S_4, -0.18)$
V_{31}	$(S_3, 0.15)$	$(S_3, 0.02)$	$(S_3, -0.06)$
V_{32}	$(S_5, 0.25)$	$(S_4, -0.46)$	$(S_{10}, 0.5)$
V_{33}	$(S_3, 0.4)$	$(S_3, 0.15)$	$(S_3, 0.2)$

Table 3. Linguistic generalized expected value.

	2-TWA	2-TOWA	2-THA
A_1	$(S_4, -0.05)$	$(S_4, -0.4)$	$(S_5, -0.03)$
A_2	$(S_3, 0.18)$	$(S_4, -0.45)$	$(S_4, 0.49)$
A_3	$(S_4, -0.09)$	$(S_3, 0.24)$	$(S_5, 0.34)$

As we can see, depending on the linguistic aggregation operator used, the results and decisions may be different. In this case, our optimal choice is A_1 with the 2-TWA and the 2-TOWA, and A_3 with the 2-THA.

Another possibility is to consider an ordering of the strategies. Note that this is very useful when the decision maker wants to consider more than one alternative. The results are shown in Table 4.

Table 4. Ordering of the strategies.

	Ordering
BS-2-TWA	$A_1 \succ A_3 \succ A_2$
BS-2-TOWA	$A_1 \succ A_2 \succ A_3$
BS-2-THA	$A_3 \succ A_1 \succ A_2$

As we can see, depending on the linguistic aggregation operator used, the results and decisions may be different.

5. Conclusions

We have presented a new approach for decision making with D-S theory by using hybrid aggregations in the 2-tuple linguistic representation model. The main advantage of this approach is the possibility of using in the same framework, probabilities, WAs and OWAs, and uncertain information represented with linguistic variables. Then, the model is able to assess linguistic information in situations where we can use probabilistic information and the attitudinal character of the decision maker. We have analyzed some of its main properties. We have also developed an application of the new approach in a decision making problem about selection of strategies. We have seen that depending on the type of 2-THA operator used, the results may lead to different decisions.

In future research, we expect to develop further extensions to this approach by adding new characteristics in the problem such as the use of order inducing variables, generalized means, etc.

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