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Preface

On behalf of the University of the Balearic Islands, the working group AGOP of EUSFLAT and the Organizing Committee of the Fifth International Summer School on Aggregation Operators, it is my pleasure to welcome the participants of this new edition of AGOP2009.

Many people and institutions have made this Summer School possible. I want to thank all the persons who have assisted me. My recognition to all members of the Scientific Committee, as well as other additional reviewers, that with their effort have contributed to enhance the quality of the accepted papers. I would also recognize the support of EUSFLAT and the sponsors of this event.

I am also thankful to all the distinguished invited participants: G. Beliakov, M. Couceiro, J.-L. Marichal, E. Trillas and V. Torra, for their interest in AGOP2009. Special thanks to my friends T. Calvo and R. Mesiar, as well as to the Dept. of Mathematics and Computer Science of the U.I.B.

I finally hope that you will find the time to enjoy our island and the friendship of mallorquin people.

Gaspar Mayor

Palma, 6th of July 2009

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Summary

We present the uncertain probabilistic ordered weighted averaging (UPOWA) operator. It is an aggregation operator that uses probabilities and OWAs in the same formulation. Moreover, it also uses uncertain information represented with interval numbers in the aggregation process. The main advantage of this aggregation operator is that it is able to use the attitudinal character of the decision maker and the available probabilistic information in an environment where the information is very imprecise and can be assessed with interval numbers. We also compare this new approach with the previous models such as the concept of immediate probabilities and we see its main advantages and how it includes them as special cases. We also develop an application of the new approach in a decision making problem about selection of investments. We see that this model gives more complete information of the decision problem because it is able to deal with decision making problems under uncertainty and under risk in the same formulation.

Keywords: OWA operator, Probabilities, Probabilistic OWA operator, Uncertainty, Interval numbers.

1 INTRODUCTION

The use of probabilities and the ordered weighted averaging (OWA) operator [13] in the same aggregation process is a very useful method for considering the probabilistic information and the attitudinal character of the decision maker in the same formulation. In the literature, there are some studies that have already considered this problem by referring to it as the immediate probability [3-5,16-17]. The main advantage of this approach is the possibility of underestimate or overestimate the probabilistic information according to the degree of orness (or optimism) given in the OWA operator. Thus, we are able to obtain a parameterized family of aggregation operators between the maximum and the minimum. For further reading on the OWA operator, see for example [1-7,9-17]. Note also that there exist in the literature other approaches that uses probabilistic information and OWA operators at the same time such as some decision making methods with Dempster-Shafer belief structure [6,14].

The concept of immediate probability has some limitations. One of the most significant problems, as stated in [5], is that it is not able to unify the probability and the OWA operator considering that sometimes one of them can be more relevant in the aggregation. Therefore, it is necessary to use another approach that it is able to unify both concepts but taking into account that they can be more or less relevant depending on the problem considered. For doing so, in [5] it has been suggested a new aggregation operator that unifies the probability with the OWA operator giving different degrees of importance to each case in the unification depending on the problem studied: the probabilistic ordered weighted averaging (POWA) operator.

The POWA operator is very useful to unify the probability with the OWA operator when using exact numbers in the aggregation process. However, many situations of the real world cannot be assessed with exact numbers because the information is uncertain and very complex. Therefore, it is necessary to use another approach that it is able to assess this situation such as the use of interval numbers. The interval numbers [8] are a very useful technique for representing the uncertainty by considering the best and worst possible results that could happen in the environment and the most possible ones.

Poner más propiedades de los AGOPs al estudiar los OWAs, —

The aim of this paper is to present the uncertain probabilistic OWA (UPOWA) operator. It is an aggregation operator that uses uncertain information given in the form of interval numbers in the POWA operator. Therefore, we are able to assess the POWA operator considering the best and worst results that could happen in the aggregation process. The main advantage of the UPOWA operator is that it provides more complete information to the decision maker by using interval numbers that includes a wide range of results and by using probabilities and OWA operators in the same formulation.

This paper is organized as follows. In Section 2, we briefly review some basic concepts about the interval numbers and the uncertain OWA (UOWA) operator. In Section 3, we present the UPOWA operator. Section 4 concludes the paper summarizing the main findings.

2 PRELIMINARIES

In this Section, we briefly describe the interval numbers and the UOWA operator.

2.1 INTERVAL NUMBERS

The interval numbers [8] are a very useful and simple technique for representing the uncertainty. They have been used in an astonishingly wide range of applications.

The interval numbers can be expressed in different forms. For example, if we assume a 4-tuple (a_1, a_2, a_3, a_4), that is to say, a quadruplet; we could consider that a_1 and a_4 represents the minimum and the maximum of the interval number, and a_2 and a_3, the interval with the highest probability or possibility, depending on the use we want to give to the interval numbers. Note that $a_1 \leq a_2 \leq a_3 \leq a_4$. If $a_1 = a_2 = a_3 = a_4$, then, the interval number is an exact number; if $a_2 = a_3$, it is a 3-tuple known as triplet; and if $a_1 = a_2$ and $a_3 = a_4$, it is a simple 2-tuple interval number.

In the following, we are going to review some basic interval number operations as follows. Let A and B be two triplets, where $A = (a_1, a_2, a_3)$ and $B = (b_1, b_2, b_3)$. Then:

1. $A + B = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$
2. $A - B = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$
3. $A \times k = (k \times a_1, k \times a_2, k \times a_3);\ for\ k > 0$
4. $A \times B = (a_1 \times b_1, a_2 \times b_2, a_3 \times b_3);\ for\ R^+.$
5. $A \div B = (a_1 \div b_3, a_2 \div b_2, a_3 \div b_1);\ for\ R^+.$

Note that other operations could be studied [8] but in this paper we will focus on these ones.

2.2 UOWA OPERATOR

The uncertain OWA (UOWA) operator was introduced by [11]. It is an extension of the OWA operator [1-7,9-18] for uncertain situations where the available information can be assessed with interval numbers. It can be defined as follows:

Definition 1. Let Ω be the set of interval numbers. An UOWA operator of dimension n is a mapping UOWA: $\Omega^n \rightarrow \Omega$ that has an associated weighting vector W of dimension n such that $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, then:

$$UOWA(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \sum_{j=1}^{n} w_j b_j \qquad (1)$$

where b_j is the jth largest of the $\tilde{a}_i$ and $\tilde{a}_i$ is the argument variable represented in the form of interval numbers.

From a generalized perspective of the reordering step it is possible to distinguish between descending (DUOWA) and ascending (AUOWA) orders. Note that in the reordering step of the interval numbers it is necessary to establish a criterion for its comparison. For doing this, we will use the following one. First, we will analyze if there is an order between the interval numbers. That is, if all the values of the interval a are higher than the corresponding values in the interval c. If not, we will calculate an average of the interval number. For example, if $n = 2$, $(a_1 + a_2)$ / 2; if $n = 3$, $(a_1 + 2a_2 + a_3)$ / 4; etc. If there is still a tie, then, we will follow a subjective criterion such as considering only the minimum, the maximum, etc.

Note that if the weighting vector is not normalized, i.e., $W = \sum_{j=1}^{n} w_j \neq 1$, then, the UOWA operator can be expressed as:

$$UOWA(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \frac{1}{W} \sum_{j=1}^{n} w_j b_j \qquad (2)$$

Note also that different families of UOWA operators can be studied by choosing a different weighting vector such as the step-UOWA operator, the window-UOWA, the median-UOWA, the olympic-UOWA, the centered-UOWA, the S-UOWA, etc.

3 UNCERTAIN PROBABILISTIC OWA OPERATOR

The uncertain probabilistic ordered weighted averaging (UPOWA) operator is an extension of the OWA operator for situations where we find probabilistic and uncertain information that can be assessed with interval numbers. It can also be seen as a unification between decision making problems under uncertainty (with OWA operators) and under risk (with probabilities). Its main advantage is that it can unify both concepts considering the degree of importance that they have in the specific problem considered. Thus, we are able to apply this formulation to all the previous models that use probabilities or OWAs. Therefore, we get a more complete approach that it is able to consider a wide range of scenarios and it includes the classical approaches as special cases. Specially, it is worth noting that in decision making problems, this approach is able to include decision making under risk and under uncertainty environments in the same formulation. This approach seems to be complete, at least as an initial real unification between OWA operators and probabilities.

However, it is worth noting that some previous models already considered the possibility of using OWA operators and probabilities in the same formulation. The main model is the concept of immediate probability [3-5,16-17]. Although it seems to be a good approach it is not so complete than the UPOWA because it can unify OWAs and probabilities in the same model but it can not take in consideration the degree of importance of each case in the aggregation process. Before studying the UPOWA, we are going briefly to consider the immediate probabilities with interval numbers (IP-UOWA). For uncertain situations assessed with interval numbers, the immediate probability can be defined as follows.

Definition 2. Let Ω be the set of interval numbers. An IPUOWA operator of dimension n is a mapping IPUOWA: $\Omega^n \rightarrow \Omega$ that has associated a weighting vector W of dimension n such that $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$, according to the following formula:

$$IPUOWA(\tilde{a}_1, ..., \tilde{a}_n) = \sum_{j=1}^{n} \hat{v}_j b_j \tag{3}$$

where b_j is the jth largest of the $\tilde{a}_i$, the $\tilde{a}_i$ are interval numbers and each one has associated a probability $\hat{v}_i$ with $\sum_{i=1}^{n} v_i = 1$ and $v_i \in [0,1]$, $\hat{v}_j = (w_j v_j)/\sum_{j=1}^{n} w_j v_j$ and v_j is the probability v_i ordered according to b_j, that is, according to the jth largest of the $\tilde{a}_i$.

Note that the IPUOWA operator is a good approach for unifying probabilities and OWAs in some particular situations. But it is not always useful, especially in situations where we want to give more importance to the OWA operators or to the probabilities. One way to see why this unification does not seem to be a final model is considering other ways of representing $\hat{v}_j$. For example, we could also use $\hat{v}_j = [w_j + v_j / \sum_{j=1}^{n} (w_j + v_j)]$ or other similar approaches.

Note that other approaches that could be taken into account are the hybrid averaging (HA) [12] and the weighed OWA (WOWA) operator [9]. These models unify the OWA operator with the weighted average (WA). Therefore, they can also be extended for situations with the OWA operator and probabilities assuming that for some situations the WA can be seen as a probability, for example, when we use the WA as a subjective probability. As said before, these an other approaches are useful for some particular situations but they does not seem to be so complete than the UPOWA because they can unify OWAs with probabilities (or with WAs) but they can not unify them giving different degrees of importance to each case. Note that in future research we will also prove that these models can be seen as a special case of a general UPOWA operator (or its respective model with WAs) that uses quasi-arithmetic means. Obviously, it is possible to develop more complex models of the IP-UOWA, the HA (or uncertain HA) and the WOWA that takes into account the degree of importance of the OWAs and the probabilities (or WAs) in the model but they seem to be artificial and not a natural unification as it will be shown below.

In the following, we are going to analyze the UPOWA operator. It can be defined as follows.

Definition 3. Let Ω be the set of interval numbers. An UPOWA operator of dimension n is a mapping UPOWA: $\Omega^n \rightarrow \Omega$ that has associated a weighting vector W of dimension n such that $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$, according to the following formula:

$$UPOWA(\tilde{a}_1, ..., \tilde{a}_n) = \sum_{j=1}^{n} \hat{v}_j b_j. \tag{4}$$

where b_j is the jth largest of the $\tilde{a}_i$, the $\tilde{a}_i$ are interval numbers and each one has an associated probability v_i with $\sum_{i=1}^{n} v_i = 1$ and $v_i \in [0,1]$, $\hat{v}_j = \beta w_j + (1-\beta) v_j$ with $\beta \in [0,1]$ and v_j is the probability v_i ordered according to b_j, that is, according to the jth largest of the $\tilde{a}_i$.

Note that it is also possible to formulate the UPOWA operator separating the part that strictly affects the OWA operator and the part that affects the probabilities. This representation is useful to see both models in the same formulation but it does not seem to be as a unique equation that unifies both models.

Definition 4. Let Ω be the set of interval numbers.

An UPOWA operator is a mapping UPOWA: $\Omega^n \times \Omega^n \rightarrow \Omega$ of dimension n, if it has associated a weighting vector W, with $\sum_{j=1}^{n} w_j = 1$ and $w_j \in [0,1]$ and a probabilistic vector V, with $\sum_{i=1}^{n} v_i = 1$ and $v_i \in [0,1]$, such that:

$$UPOWA(\tilde{a}_1, ..., \tilde{a}_n) = \beta \sum_{j=1}^{n} w_j b_j + (1-\beta) \sum_{i=1}^{n} v_i \hat{a}_i \tag{5}$$

where b_j is the jth largest of the arguments $\tilde{a}_i$, the $\tilde{a}_i$ are interval numbers and $\beta \in [0,1]$.

In the following, we are going to give a simple example of how to aggregate with the UPOWA operator. We consider the aggregation with both definitions.

Example 1. Assume the following arguments in an aggregation process: ([20, 30], [40, 50], [50, 60], [30, 40]). Assume the following weighting vector W = (0.2, 0.2, 0.2, 0.4) and the following probabilistic weighting vector P = (0.4, 0.3, 0.2, 0.1). Note that the probabilistic information has a degree of importance of 60% while the weighting vector W a degree of 40%. If we want to aggregate this information by using the UPOWA operator, we will get the following. The aggregation can be solved either with the Eq. (4) or (5). With Eq. (4) we calculate the new weighting vector as:

$$\hat{v}_1 = 0.4 \times 0.2 + 0.6 \times 0.2 = 0.2,$$

$$\hat{v}_2 = 0.4 \times 0.2 + 0.6 \times 0.3 = 0.26,$$

$$\hat{v}_3 = 0.4 \times 0.2 + 0.6 \times 0.1 = 0.14,$$

$$\hat{v}_4 = 0.4 \times 0.4 + 0.6 \times 0.4 = 0.4.$$

And then, we calculate the aggregation process as follows:

$UPOWA = 0.2 \times [50,60] + 0.26 \times [40,50] + 0.14 \times [30,40] + 0.4 \times [20,30] = [32.6, 42.6]$.

Obviously, we get the same results with both methods.

Note that different types of interval numbers could be used in the aggregation such as 2-tuples, triplets, quadruplets, etc.

When using interval numbers in the UPOWA operator, we have the additional problem of how to reorder the arguments because now we are using interval numbers. Then, in some cases, it is not clear which interval number is higher, so we need to establish an additional criteria for reordering the interval numbers. For simplicity, we recommend the following criteria. For 2-tuples, calculate the arithmetic mean of the interval: $(a_1 + a_2)$ / 2. For 3-tuples and more, calculate a weighted average that gives more importance to the central values. That is, $(a_1 + 2a_2 + a_3)$ / 4. And so on. In the case of tie, we will select the interval with the lowest increment (a_2 - a_1). For 3-tuples and more we will select the interval with the highest central value.

From a generalized perspective of the reordering step, it is possible to distinguish between the descending UPOWA (DUPOWA) and the ascending UPOWA (AUPOWA) operator by using $w_j = w^*_{n-j+1}$, where w_j is the jth weight of the DUPOWA and w^*_{n-j+1} the jth weight of the AUPOWA operator.

If B is a vector corresponding to the ordered arguments b_j, we shall call this the ordered argument vector and W^T is the transpose of the weighting vector, then, the UPOWA operator can be expressed as:

$$UPOWA(\tilde{a}_1, ..., \tilde{a}_n) = W^T B \tag{6}$$

Note that if the weighting vector is not normalized, i.e., $W = \sum_{j=1}^{n} w_j \neq 1$, then, the UPOWA operator can be expressed as:

$$UPOWA(\tilde{a}_1, ..., \tilde{a}_n) = \frac{1}{W} \sum_{j=1}^{n} \hat{v}_j b_j \tag{7}$$

The UPOWA is monotonic, commutative, bounded and idempotent. It is monotonic because if $\tilde{a}_i \geq \tilde{u}_i$, for all $\tilde{a}_i$, then, $UPOWA(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) \geq UPOWA(\tilde{u}_1, \tilde{u}_2, ..., \tilde{u}_n)$. It is commutative because any permutation of the arguments has the same evaluation. That is, $UPOWA(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = UPOWA(\tilde{u}_1, \tilde{u}_2, ..., \tilde{u}_n)$, where $(\tilde{u}_1, \tilde{u}_2, ..., \tilde{u}_n)$ is any permutation of the arguments $(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n)$. It is bounded because the UPOWA aggregation is delimitated by the minimum and the maximum: $Min\{\tilde{a}_i\} \leq UPOWA(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) \leq Max\{\tilde{a}_i\}$. It is idempotent because if $\tilde{a}_i = \tilde{a}$, for all $\tilde{a}_i$, then, $UPOWA(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \tilde{a}$.

Note that it is also possible to consider that the weights and the probabilities of the UPOWA operator are also interval numbers. For doing so, we recommend, for example, to follow the methodology explained in [19] about dealing with imprecise probabilities. Note that if either the weights (or probabilities) and the arguments are interval numbers, then, the operations with interval numbers are a bit more complex than those explained in Section 2.1. For more information on these operations, refer, e.g., to [8].

4 TYPES OF UPOWA OPERATORS

First of all we are going to consider the two main cases of the UPOWA operator that are found by analyzing the coefficient β. Basically, if $\beta = 0$, then, we get the probabilistic approach and if $\beta = 1$, the UOWA operator.

By choosing a different manifestation of the weighting vector in the UPOWA operator, we are able to obtain different types of aggregation operators. For example, we can obtain the uncertain probabilistic maximum, the uncertain probabilistic minimum, the uncertain probabilistic average and the uncertain probabilistic weighted average.

Remark 1. The uncertain probabilistic maximum is found when $w_1 = 1$ and $w_j = 0$ for all $j \neq 1$. The probabilistic minimum is formed when $w_n = 1$ and $w_j = 0$ for all $j \neq n$.

Remark 2. More generally, the step-UPOWA is formed when $w_k = 1$ and $w_j = 0$ for all $j \neq k$. Note that if $k = 1$, the step-UPOWA is transformed into the uncertain probabilistic maximum, and if $k = n$, the step-UPOWA becomes the uncertain probabilistic minimum operator.

Remark 3. The uncertain probabilistic average is obtained when $w_j = 1/n$ for all j, and the uncertain probabilistic weighted average is obtained when the ordered position of i is the same as the ordered position of j.

Remark 4. For the median-UPOWA, if n is odd we assign $w_{(n+1)/2} = 1$ and $w_{j*} = 0$ for all others. If n is even we assign for example, $w_{n/2} = w_{(n/2)+1} = 0.5$ and $w_{j*} = 0$ for all others.

Remark 5. The olympic-UPOWA is generated when $w_1 = w_n = 0$, and for all others $w_{j*} = 1/(n-2)$. Note that it is possible to develop a general form of the olympic-UPOWA by considering that $w_j = 0$ for $j = 1, 2, ..., k, n, n-1, ..., n-k+1$, and for all others $w_{j*} = 1/(n-2k)$, where $k < n/2$. Note that if $k = 1$, then this general form becomes the usual olympic-UPOWA. If $k = (n-1)/2$, then, this general form becomes the median-UPOWA aggregation. That is, if n is odd, we assign $w_{(n+1)/2} = 1$, and $w_{j*} = 0$ for all other values. If n is even, we assign, for example, $w_{n/2} = w_{(n/2)+1} = 0.5$ and $w_{j*} = 0$ for all other values.

Remark 6. Note that it is also possible to develop the contrary case of the general olympic-UPOWA operator. In this case, $w_j = (1/2k)$ for $j = 1, 2, ..., k, n, n-1, ..., n-k+1$, and $w_j = 0$, for all other values, where $k < n/2$. Note that if $k = 1$, then we obtain the contrary case for the median-UPOWA.

Remark 7. Another type of aggregation that could be used is the E-Z UPOWA weights. In this case, we should distinguish between two classes. In the first class, we assign $w_{j*} = (1/q)$ for $j^* = 1$ to q and $w_{j*} = 0$ for $j^* > q$, and in the second class, we assign $w_{j*} = 0$ for $j^* = 1$ to $n-q$ and $w_{j*} = (1/q)$ for $j^* = n-q+1$ to n.

Remark 8. Another interesting family is the S-UPOWA operator. It can be subdivided into three classes: the "or-like", the "and-like" and the generalized S-UPOWA operators. The generalized S-UPOWA operator is obtained if $w_1 = (1/n)(1-(\alpha+\beta)) + \alpha$, $w_n = (1/n)(1-(\alpha+\beta))+\beta$, and $w_j = (1/n)(1-(\alpha+\beta))$ for $j = 2$ to $n-1$, where $\alpha, \beta \in [0, 1]$ and $\alpha + \beta \leq 1$. Note that if $\alpha = 0$, the generalized S-UPOWA operator becomes the "and-like" S-UPOWA operator, and if $\beta = 0$, it becomes the "or-like" S-UPOWA operator.

Remark 9. Another family of aggregation operator that could be used is the centered-UPOWA operator. We can define an UPOWA operator as a centered aggregation operator if it is symmetric, strongly decaying and inclusive. Note that these properties have to be accomplished for the weighting vector W of the UOWA operator but not necessarily for the weighting vector P of the probabilities. It is symmetric if $w_j = w_{j+n-1}$. It is strongly decaying when $i < j \leq (n+1)/2$ then $w_i < w_j$ and when $i > j \geq (n+1)/2$ then $w_i < w_j$. It is inclusive if $w_j > 0$. Note that it is possible to consider a softening of the second condition by using $w_i \leq w_j$ instead of $w_i < w_j$, then, we get the softly decaying centered-UPOWA operator. And if we remove the third condition, we get the non-inclusive centered-UPOWA operator.

Remark 10. A further interesting type is the non-monotonic-UPOWA operator. It is obtained when at least one of the weights w_j is lower than 0 and $\sum_{j=1}^{n} w_j = 1$.

Note that a key aspect of this operator is that it does not always achieve monotonicity. Therefore, this particular case is not strictly an UPOWA operator. However, we can see it as a particular family of operators that is not monotonic but has strong similarities with the UPOWA operator.

Remark 11. Note that other families of UPOWA operators could be used following the recent literature about different methods for obtaining OWA weights. Some of these methods are explained in [1-2,4-5,7,15].

5 CONCLUSIONS

We have presented the UPOWA operator. It is an aggregation operator that unifies the OWA operator and the probability in the same formulation and in an uncertain environment that can be assessed with interval numbers. The main advantage of this new model is that it is able to unify the probability and the OWA operator giving different degrees of importance to them. Moreover, by using interval numbers, we are able to provide more complete information to the decision maker because we represent the environment considering the best and worst result that could occur under uncertainty. We have compared this approach with the concept of immediate probability and we have seen how the UPOWA operator is able to overcome the main limitations of the immediate probability.

In future research, we expect to develop further extensions of the POWA operator by using other techniques for representing the uncertainty (fuzzy numbers, linguistic variables, etc.) and other variables such as inducing orders, generalized means, distance measures, etc. We will also extend this analysis to a similar one when using weighted averages instead of probabilities.

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