Applying OWA Operators and RIM Quantifiers to measure Consensus with Large Groups of Experts

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Summary

Consensus based approaches provide a way to drive group decisions which are more accepted and appreciated by decision makers. Different consensus approaches and methodologies have been proposed to conduct these processes successfully, but there still remain some aspects that must be further studied, such as the use and effects of different aggregation operators to measure consensus in the group. This paper shows the application of ordered weighted averaging (OWA) operators based on linear RIM (Regular Increasing Monotone) quantifiers in a consensus support system, and analyzes the impact of using them to measure consensus in decision problems with a large number of experts participating.

Keywords: OWA, GDM, Consensus, Linguistic Quantifier, Attitude, Optimism.

1 INTRODUCTION

Group Decision Making (GDM) problems can be roughly defined as decision situations where two or more decision makers or *experts* try to achieve a common solution to a decision problem, which consists of a set of possible solutions or *alternatives* [6]. Experts must express their individual opinion on each of these alternatives.

Some guiding rules have been proposed to support real situations in GDM problems, for example the majority rule, minority rule, unique person-based decision and unanimity [1]. However, in many situations a problem arises when some experts consider that their individual opinions haven't been sufficiently taken into account, and therefore they disagree with the solution achieved [14], what could imply either a lack of implication in future GDM problems or a behavior against the solution obtained. For this reason, the need for making relevant decisions under *consensus* is becoming increasingly common in a variety of social situations.

Consensus processes imply that experts achieve an agreement about the problem before making the decision, thus yielding a more accepted solution by the organization, society or themselves. Different consensus approaches have been proposed, ranging from a rigid view of consensus as unanimity to more flexible approaches [2, 6, 7]. In these approaches, it is crucial to establish a *consensus measure* to calculate the level of agreement. Consensus measures are, therefore, an indicator to evaluate how far a group of experts' opinions are from unanimity. To achieve this purpose, consensus measures can use different *similarity measures* to calculate closeness among opinions, as well as *aggregation operators* to obtain a collective consensus degree and determine a collective opinion [8].

Different aggregation operators have been applied to some developed *consensus support systems* (CSS), with successful results. However, no practical application of aggregation operators in consensus processes involving *many* experts has been performed, due to the considerable computational cost and scalability required to do this [9]. In this paper, a practical study on the effect of OWA aggregation operators [15] defined by RIM quantifiers [17] is conducted on a multi-agent-based CSS, whose high scalability allows to solve decision problems involving large groups efficiently. The aim of this study is to determine how inherent properties and parameters in RIM quantifier-based OWA operators can be used to express the decision group's desired attitude regarding the consensus problem to address, i.e. how the group can express a desired attitude (optimistic, pessimistic or neutral) by setting OWA and RIM quantifier's parameters properly.

This paper is organized as follows. In Section 2, we briefly review some basic concepts about consensus approaches and OWA operators defined by RIM quantifiers. Section 3 presents the experimental study conducted and the results obtained. Section 4 concludes the paper summarizing the results and commenting some future works.

2 PRELIMINARIES

In this Section, we briefly describe consensus approaches in GDM and OWA operators based on RIM quantifiers.

2.1 CONSENSUS APPROACHES IN GDM

GDM problems are formally defined by:

- A set $X = \{x_1, \ldots, x_n\}$ ($n \geq 2$) of possible *alternatives* to choose as possible solutions to the problem.
- A set $E = \{e_1, \ldots, e_m\}$ ($m \geq 2$) of decision makers or *experts*, who express their judgements on the alternatives in X, having all of them the intention to achieve a common solution to the proposed problem.

Each expert e_i must express his opinions over alternatives in X by means of a preference structure [5]. One of the most usual preference structures, especially appropriate when dealing with uncertain information, is the so-called *fuzzy preference relation* (FPR). The FPR P_i associated to an expert e_i on a set of alternatives X, is characterized by the membership function $\mu_{P_i} : X \times X \longrightarrow [0,1]$ and usually represented as a $n \times n$ matrix as follows:

$$P_i = \begin{pmatrix} p_i^{11} & \ldots & p_i^{1n} \\ \vdots & \ddots & \vdots \\ p_i^{n1} & \ldots & p_i^{nn} \end{pmatrix}$$

where the numerical opinion or *assessment* $p_i^{lk} = \mu_{P_i}(x_l, x_k)$ is the degree of preference of alternative x_l over x_k, so that $p_i^{lk} > 1/2$ indicates that x_l is preferred to x_k, whereas $p_i^{lk} < 1/2$ indicates that x_k is preferred to x_l and $p_i^{lk} = 1/2$ indicates indifference between x_l and x_k.

One of the main shortcomings found in classic GDM approaches, such as the majority rule, or minority rule, is the possible disagreement shown by one or more experts, since they might consider that their opinions have been taken into account enough. The need for a high agreement among all experts is becoming increasingly crucial in real GDM problems, hence consensus-based approaches to achieve a mutual agreement before making the decision have attained a great importance [14]. In order to achieve such agreement, it is usually necessary that experts modify their initial opinions in a discussion process, moving them towards a collective opinion which must be considered as satisfactory by all of them.

The notion of consensus may be interpreted in different ways, ranging from a strict view of consensus as a total agreement or *unanimity* [2], where consensus is only assumed when *all* experts have achieved a total mutual agreement in all their opinions, to a more flexible approach considering different degrees of partial agreement among experts (usually measured with a real number in the unit interval), to decide about the existence of consensus. One of the most widely accepted flexible approaches for measuring consensus is based on the concept of *soft consensus*, proposed by J. Kacprzyk in [6], where the concept of fuzzy linguistic majority is introduced, which establishes that there exists consensus if *most experts participating in a problem agree with the most important alternatives. Soft consensus*-based approaches have been used in different consensus models [3, 18], and consensus measures with OWA operators based on linear RIM quantifiers are appropriate to adopt this notion.

Overall, consensus approaches are based on a dynamic and iterative discussion process [14], frequently coordinated by a *moderator*, whose main responsibilities are evaluating the degree of agreement achieved in each round of discussion, identifying those alternatives that hamper reaching a consensus, and giving feedback to experts, regarding changes they should perform in their opinions, in order to increase the level of agreement in the next rounds. The experimental study conducted in this paper focuses on evaluating the collective degree of agreement achieved, by using different instances of OWA operators. Further detail on the consensus model and CSS used to perform this study can be found in [11, 12].

2.2 OWA OPERATORS BASED ON LINGUISTIC QUANTIFIERS

OWA (Ordered Weighted Averaging) operators were introduced by R. Yager in [15]. These operators fulfill some desirable properties in aggregation operators [4], including idempotence, continuity, monotonicity, neutrality and compensativeness.

Definition. Let $A = \{a_1, \ldots, a_n\}$ be a set of values to aggregate in [0,1]. An OWA operator of dimension n is defined as a mapping $OWA_W : [0,1]^n \to [0,1]$, with an associated weighting vector $W = [w_1 w_2 \ldots w_n]^\top$, where $w_i \in [0,1]$, $\sum_i w_i = 1$ and,

$$OWA_W(a_1, \ldots, a_n) = \sum_{j=1}^{n} w_j b_j \quad (1)$$

where b_j is the jth largest of the a_i values.

A key issue in OWA operators is the previous reordering of values a_i, so that a particular weight w_i is not associated with a value a_i, but rather with a particular ordered position in the set of values to aggregate. OWA operators lie between maximum ("or") and minimum ("and"), and thus allow an easy adjusting of the degree of optimism/pessimism by the appropriate choice of weights w_i. A measure of the optimism or *orness* associated with W was introduced as

$$orness(W) = \frac{1}{n-1} \sum_{i=1}^{n} (n-i) w_i \quad (2)$$

The closer $orness(W)$ is to one, the closer OWA_W is to "or" and the more optimistic the decision maker's attitude is, since higher values are given more importance in the aggregation. Analogously, the closer this value is to zero, the more similar to "and" the operator is, thus leading to the adoption of a pessimistic attitude. Therefore, a decision group represented by a moderator can express different attitudes regarding optimism/pessimism by choosing an appropriate set of weights W.

Another measure, the *dispersion* or *entropy*, is used as an indicator of the degree to which information contained in values $a_1, \ldots, a_n$ is considered when aggregating. Since OWA weights can be seen as a discrete probability distribution, dispersion is also an indicator of how uniformly weights are distributed. This measure is defined as

$$Disp(W) = -\sum_{i=1}^{n} w_i \ln w_i \quad (3)$$

where for any $n \geq 2$, $0 \leq Disp(W) \leq \ln(n)$. The higher dispersion is, the more weights are non-null. Some widely known special cases of OWA operators [16] are the "or" operator, where $w_1 = 1$ and $w_i = 0, \forall i \neq 1$, $orness(W) = 1$ and $Disp(W) = 0$; the "and" operator, where $w_n = 1$ and $w_i = 0, \forall i \neq n$, $orness(W) = 1$ and $Disp(W) = 0$; and the arithmetic mean, where $orness(W) = 0.5$, $Disp(W) = \ln(n)$ and $w_i = 1/n, \forall i$.

The problem of determining weights for an OWA operator can be addressed in different ways, for example with the use of the so-called *linguistic quantifiers*, introduced by L. Zadeh in [17]. A relative linguistic quantifier Q, such as *most, few, many* and *all*, can be represented as a fuzzy subset of the unit interval,

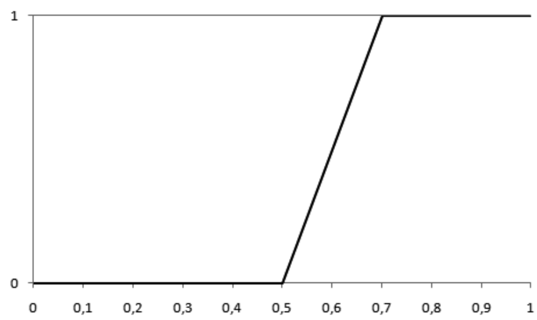

Figure 1: Example of linguistic quantifier $Q =$"*most*".

where for a given proportion $r \in [0,1]$ of the total of values to aggregate, $Q(r)$ indicates the extent to which this proportion satisfies the semantics defined in Q. For example, given $Q =$"*most*", if $Q(0.7) = 1$ then we would say that a proportion of 70% totally satisfies the idea conveyed by the quantifier *most*, whereas $Q(0.55) = 0.25$ indicates that the proportion 55% is barely compatible (only 0.25) with this concept.

Regular Increasing Monotone (RIM) quantifiers are especially interesting for their use in OWA operators. These quantifiers present the following properties:

1. $Q(0) = 0$
2. $Q(1) = 1$
3. If $r_1 > r_2$ then $Q(r_1) \geq Q(r_2)$.

In [15], Yager suggested the following way to compute weights w_i with the use of a RIM quantifier Q:

$$w_i = Q\left(\frac{i}{n}\right) - Q\left(\frac{i-1}{n}\right), i = 1, \ldots, n \quad (4)$$

where the membership function of a linear RIM quantifier $Q(r)$ is defined by two parameters $a, b \in [0,1]$ as

$$Q(r) = \begin{cases} 0 & \text{if } r < a, \\ \frac{r-a}{b-a} & \text{if } a \leq r \leq b, \\ 1 & \text{if } r > b. \end{cases} \quad (5)$$

An example of RIM quantifier $Q =$"*most*", with $a = 0.5$ and $b = 0.7$ (see Fig. 1) is

$$Q(r) = \begin{cases} 0 & \text{if } r < 0.5, \\ 5r - 2.5 & \text{if } 0.5 \leq r \leq 0.7, \\ 1 & \text{if } r > 0.7. \end{cases} \quad (6)$$

Since the use of OWA with RIM quantifiers captures Kacprzyk's notion of soft consensus properly, they can be adopted for our purpose of studying the effect of different aggregation operators on the resolution of a consensus problem with many experts, and expressing a desired group's attitude.

3 EXPERIMENTAL STUDY

In this section, we present the experimental study conducted on a CSS [11, 12], aimed to the repeated resolution of a consensus-based GDM problem under uncertainty, with a high number of experts involved. The proposed problem is always defined in a similar context and conditions attempting to simulate real-life situations, with the particularity of using flexible *consensus measures* in the different simulations performed, applying OWA operators based on different RIM quantifiers for aggregating in the consensus measurement phase.

The moderator determines consensus degrees as follows [10, 11]: for each pair of experts (e_i, e_j) $i < j$, a similarity matrix $SM_{ij} = (sm_{ij}^{lk})$ is obtained, where each element $sm_{ij}^{lk} \in [0, 1]$ represents the similarity between experts e_i and e_j in their opinion on the pair of alternatives (x_l, x_k), calculated using a similarity measure based on euclidean distance, $sm_{ij}^{lk} = 1 - |p_i^{lk} - p_j^{lk}|$. Experts' pairwise similarity matrices are then aggregated by means of an operator OWA_W to obtain a consensus matrix $CM = (cm^{lk})$, where each element $cm^{lk} \in [0, 1]$ indicates the collective degree of agreement achieved on pair (x_l, x_k).

$$cm^{lk} = OWA_W(sm_{12}^{lk}, sm_{13}^{lk}, \ldots, sm_{1m}^{lk}, sm_{23}^{lk}, \ldots, sm_{2m}^{lk}, \ldots, sm_{(m-1)m}^{lk}) \quad (7)$$

Elements in CM are successively aggregated with an arithmetic mean operator to obtain the degree of consensus on each alternative $x_l \in X$, as well as the overall consensus degree $cr \in [0, 1]$. Thereto, this study focuses exclusively in the use of different RIM quantifiers to aggregate similarity values into consensus values for each pair (x_l, x_k), as shown in Equation 7.

Regarding the decision group's attitude, they might adopt an optimistic attitude if they accept that only *a few* of the pairs of experts must have a high agreement in their opinion to consider the existente of consensus, i.e. higher similarity values are rather considered in aggregation. On the other hand, they shall be pessimistic if they accept the existence of consensus only when *all* or *a vast majority* of the pairs of experts agree themselves with such opinion, hence lower values are given more importance in aggregation. A neutral attitude is neither optimistic nor pessimistic, and implies no preference for higher or lower values in aggregation. Regarding the *dispersion* of the operator, it indicates the proportion of non-null weights given to elements to aggregate, i.e. the amount of information considered in aggregation.

The proposed problem is an example of frequent real situations usually solved through classic GDM criteria such as the majority rule. Since this way of making the decision may often lead to controversy as earlier discussed, we propose to simulate the problem's resolution through consensus by using a multi-agent based CSS. The problem's formulation is as follows:

A neighborhood community composed of fifty neighbors is meant to make changes related to their monthly fees. The possible alternatives $X = \{x_1, \ldots, x_4\}$ are: $x_1 =$ *Increase running costs by 10%*; $x_2 =$ *Increase maintenance costs by 20%*; $x_3 =$ *Reduce employees' fees*; and $x_4 =$ *Install a new lift and keep rest of services.*

We assume the following settings for *all* the simulations performed to solve the problem:

- Fifty experts $E = \{e_1, \ldots, e_{50}\}$ give their preferences on X using FPRs denoted as P_i.
- Before the process begins, each expert's initial preference relation is set by using random assessments p_i^{lk} in the unit interval, ensuring reciprocal assessments ($p_i^{kl} = 1 - p_i^{lk}$, $l \neq k$), in order to guarantee consistency in them.
- The moderator introduces the two initial parameters before the problem begins: (i) a consensus threshold $\mu = 0.85$, i.e. a value to compare the overall consensus degree *cr* achieved in each round and decide whether accepting or not the existence of consensus, and (ii) a maximum number of rounds allowed $\#MAX\_ROUNDS = 10$.
- When $cr < mu$, the moderator gives experts a series of advices to modify their opinions on some pairs of alternatives (x_l, x_k) [11]. We suppose for all simulations a real-like pattern of experts' behavior regarding obedience of these advices as follows: 70% of experts *always* follow advices, and the remaining ones may ignore each received advice with a probability of 0.5. For *all* experts, the extent of change (increase/decrease an assessment p_i^{lk}) is also variable and set randomly as either one of the values {0.05, 0.1, 0.15}.

Experiments consist in the repeated solution of the formulated problem using OWA operators based on different RIM quantifiers, where each quantifier will be denoted as $Q_{(a,b)}$, with parameters $a, b \in [0, 1]$, $a \leq b$ as stated in Equation 5. Table 1 shows the different quantifiers considered, including their approximate measures of orness and dispersion, computed taking into account that for m experts in the problem, Equation 7 requires the aggregation of $m(m-1)/2$ similarity values, therefore we have to make aggregations between 1.225 values.

Each quantifier is used in 20 simulations, and the results show the degree of convergence achieved, ex-

Table 1: RIM quantifiers considered in experiments.

Quantifier	Orness	Dispersion
$Q_{(0.7-1)}$	0.15	5.91
$Q_{(0.7-0.9)}$	0.2	5.5
$Q_{(0.7-0.8)}$	0.25	4.81
$Q_{(0.45-0.75)}$	0.4	5.91
$Q_{(0.45-0.65)}$	0.45	5.34
$Q_{(0.45-0.55)}$	0.5	4.81
$Q_{(0.2-0.5)}$	0.65	5.91
$Q_{(0.2-0.4)}$	0.7	5.5
$Q_{(0.2-0.3)}$	0.75	4.2

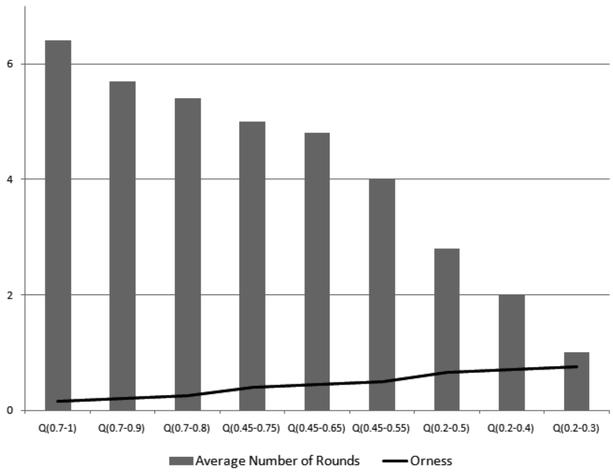

Figure 2: Average number of rounds required with different quantifiers.

pressed as the average number of rounds required to reach a consensus (exceeding threshold $\mu = 0.85$) with a particular quantifier. The reason to solve the problem with the same parameters repeatedly is the existence of some stochastic components, i.e. the random initial preferences of experts and their behavior pattern regarding the obedience of advice. These stochastic components have shown not to be significantly influential in the process performance, thus making our results with different quantifiers more trustworthy.

Figure 2 shows the average results obtained with each quantifier. RIM quantifiers based on an optimistic attitude ($orness > 0.5$) favor a greater convergence towards consensus, whereas RIM quantifiers based on a pessimistic attitude ($orness < 0.5$) favor a lower convergence and a further discussion process. However, no clear conclusions regarding dispersion in operators can be extracted from these results. Therefore, we decided to do some additional experiments considering the use of quantifiers with same orness and different dispersion degrees. Table 2 shows the characteristics of these quantifiers, and results are shown in Figure 3.

Table 2: Linguistic quantifiers with similar orness and different dispersion.

Quantifier	Orness	Dispersion
$Q_{(0.75-0.85)}$	0.2	4.8
$Q_{(0.7-0.9)}$	0.2	5.37
$Q_{(0.6-1)}$	0.2	6.19
$Q_{(0.35-0.45)}$	0.6	4.4
$Q_{(0.3-0.5)}$	0.6	5.5
$Q_{(0.2-0.6)}$	0.6	6.19

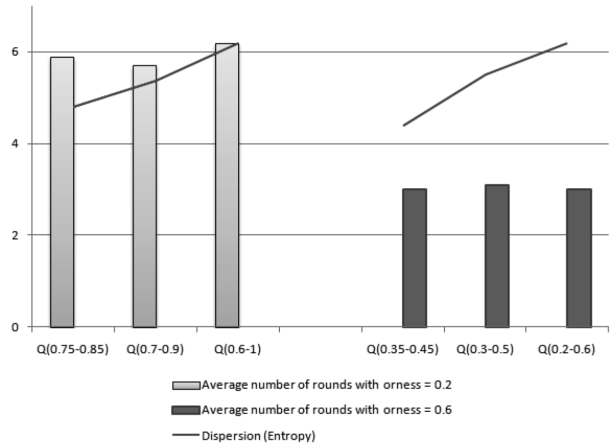

Figure 3: Average number of rounds with similar orness and different dispersion degrees.

Results show that dispersion does not exert a significant impact on the performance of consensus processes. Nevertheless, dispersion is another factor worth to consider in the reflection of a group's desired attitude, since depending on the context of the problem, decision makers may desire to consider different ranges of values to calculate consensus.

4 CONCLUSIONS AND FUTURE WORKS

This paper has applied OWA operators defined with different RIM quantifiers in a CSS to solve consensus problems with a high number of experts, in order to analyze the effects of varying some of the parameters inherent to these operators in the system's performance through quantifier's definition. Having conducted an experimental study, we conclude that the use of a specific OWA aggregation operator, based on a RIM quantifier $Q_{(a,b)}$ to solve a real-like problem with many experts, may depend on the needs of the decision team at a given moment. The use of optimistic operators is recommended when their priority is to achieve a quick agreement (greater convergence), giving more importance to higher similarity values, whereas the use of pessimistic operators is more appropriate when the problem to solve requires further discussion among experts (lower convergence).

In forthcoming stages of research, we are aiming to show in a more precise and quantitative way the relation between a group's attitude and the different parameters in OWA operators with RIM quantifiers, by adjusting them, thus defining an approach to determine an group's attitude, taking into account all experts' individual concerns, and translating it into an appropriate OWA operator. We also intend to apply different types of non-linear RIM quantifiers and analyze their effect in consensus processes.

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