# Low-dimensional visualization of experts' preferences in urgent group decision making under uncertainty[*]

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**Abstract**

Urgent or critical situations, such as natural disasters, often require that stakeholders make crucial decisions, by analyzing the information provided by a group of experts about the different elements of interest in these contexts, with the aid of decision support tools. This framework can be modeled as a Group Decision Making problem defined under uncertain environments, which are characterized by the imprecision and vagueness of information about the problem tackled. One of the main aspects to consider when solving Group Decision Making problems in urgent situations, is the fact that decisions should be taken under the highest level of possible agreement amongst all participating experts, in order to avoid serious mistakes or undesired responsibilities by some experts. Due to the complexity of urgent situations and the great amount of information about experts' preferences that must be managed by stakeholders, it would be difficult to reach consensus in the group within a reasonable time period, since it requires an adequate analysis of experts' preferences. Such an analysis might become a difficult task in these contexts, due to the inherent complexity, time pressure, etc. In order to help making consensual decisions in urgent situations and urgent computing environments, we propose a visual decision support tool for Group Decision Making problems defined under uncertainty. Such a tool provides stakeholders with a two-dimensional visual representation of experts' preferences based on the similarities between them, and it enables the analysis of easily interpretable information about the state of the decision problem, as well as the detection of agreement/disagreement positions between experts. The tool is based on Self-Organizing Maps, an unsupervised learning technique aimed at the visual projection of information related to preferences of experts into a low-dimensional space.

*Keywords:*

## 1 Introduction

Unusual emergency events, such as natural disasters or mass traffic accidents, often bring devastating effects when they occur. Emergency planning and management are essential tasks to be carried out,

in order to minimize such effects in this kind of situations [32]. These tasks usually consist in decision processes [6], in which a number of stakeholders must make crucial decisions, by analyzing some information provided by a group of experts regarding the possible actions to perform in response to the event occurred. The existing computer-based decision support systems and tools are often utilized in these contexts to support stakeholders in decision making processes [7, 31].

Many real-life decision problems - including those taking place in urgent situations - are characterized not only by the participation of multiple experts with different attitudes and level of knowledge who must make a common decision, but also by the existence of uncertainty due to the imprecision and vagueness of information about the problem tackled [16]. These frameworks can be modeled as Group Decision Making (GDM) problems defined under uncertain environments [13]. Several works have been proposed by different authors to deal with this type of GDM problems [4, 11, 22].

One of the main aspects to consider when solving GDM problems in urgent situations, is the fact that decisions should be taken under the highest level of possible agreement amongst all participating experts, in order to make higher-quality decisions, as well as avoiding serious mistakes or undesired responsibilities by some experts [5, 28]. To do this, different consensus-based models and approaches aimed at achieving a high level of agreement in GDM problems under uncertainty, have been proposed in the literature [13, 22, 24]. Based on such consensus approaches, a number of Group Decision Support Systems (GDSS) have been developed to assist groups in the resolution of GDM problems under consensus [14, 23, 35].

Despite the progress made by the existing models and GDSS to support groups in decision making contexts, urgent situations normally present an inherently higher complexity and some constraints that would not be present in other types of contexts, e.g. time constraints, highly uncertain information, etc [6]. Such constraints make sometimes necessary the use of decision support tools that provide stakeholders with an insight on the current state of the problem to make quick decisions. These decisions would be more time-consuming with the aid of current GDSS only, since most of them are based on providing textual or numerical information, whose analysis is often not immediate.

Moreover, when a considerable group of experts must take part in the decision process, the great amount of information about their individual preferences that must be managed by stakeholders, makes more difficult to reach consensus in the group within a reasonable time period [20]. In large groups, the existence of experts with drastically different opinions to the rest of the group or experts who do not cooperate to reach an agreement, is also more frequent [21]. Therefore, it is necessary to develop appropriate decision support tools that facilitate an adequate analysis of experts' preferences in a group, simplifying and enabling efficient decision making processes under the inherent complexity, time pressure and other features normally present in urgent contexts.

In order to help making group decisions in urgent situations and urgent computing environments with different nature, we propose a visual decision support tool for GDM problems defined under uncertainty. Such a tool provides stakeholders with a two-dimensional visual representation of experts' preferences based on the similarities between them, and it enables the analysis of easily interpretable information about the state of the decision problem and the overall level of agreement in the group, as well as the detection of agreement/disagreement positions between experts. The tool is proposed as a complementary resource for existing GDSS based on numerical or textual information, and it is based on Self-Organizing Maps [15], an unsupervised learning technique widely utilized for data visualization purposes, which facilitates the visual projection of information related to preferences of experts into a low-dimensional space.

The remaining of this contribution is set out as follows: Section 2 reviews some basic concepts about GDM problems under uncertainty and consensus reaching. Section 3 presents the visual decision support tool, illustrating its usefulness in different kinds of urgent situations, and shows an example of its application to a emergency context. Finally, Section 4 draws some concluding remarks.

# 2  Preliminaries

In this section, some basic concepts about GDM problems under uncertainty and Consensus Reaching Processes are revised, since they are the basis of our proposal to support decision processes in urgent situations.

## 2.1  GDM problems under Uncertainty

GDM problems, where a group of experts with different viewpoints must make a common decision together, are frequently utilized in many complex real-life situations [4, 16]. Formally, a GDM problem can be defined as a decision situation characterized by the participation of a group of experts, having each one their own knowledge and attitudes, in a decision problem consisting of a set of alternatives or possible solutions to the problem [13, 16]. Thus, the following elements are found in any GDM problem:

- A set $X = \{x_1, \ldots, x_n\}$, $(n \geq 2)$ of *alternatives*.

- A set $E = \{e_1, \ldots, e_m\}$, $(m \geq 2)$ of *experts*, who express their opinions or *preferences* on the alternatives in $X$.

Each expert $e_i \in E$, $i \in \{1, \ldots, m\}$, expresses his/her preferences over alternatives in $X$, by means of a preference structure. A commonly utilized preference structure in GDM is the preference relation [19]. Given $X$ finite, a preference relation $P_i$ associated to $e_i$ is given by a $n \times n$ matrix,

$$P_i = \begin{pmatrix} - & \cdots & p_i^{1n} \\ \vdots & \ddots & \vdots \\ p_i^{n1} & \cdots & - \end{pmatrix}$$

with each element $p_i^{lk}$ being an *assessment*, representing the degree to which $x_l$ is better than $x_k$, $l, k \in \{1, \ldots, n\}$, $l \neq k$, according to $e_i$. Other preference structures that have been considered in multiple GDM approaches are utility vectors [4] and preference orderings [29].

GDM problems are usually defined in environments of uncertainty, characterized by the existence of vague and imprecise information [1]. Fuzzy modeling and linguistic information have been widely utilized in such situations [33, 34], in which experts must express their preferences over alternatives by means of information domains that allow them to deal with such uncertainty. Some of these information domains are [11]:

- *Numerical* [19]: Assessments are represented as numerical values belonging to a specific scale, e.g. values in the [0,1] interval.

- *Interval-valued* [8]: Assessments are represented as intervals, $I([0, 1])$.

- *Linguistic* [25]: Assessments are represented as linguistic terms $s_u \in S$, $u \in \{0, \ldots, g\}$, belonging to a linguistic term set $S = \{s_0, \ldots, s_g\}$ with granularity $g$.

The solution to a GDM problem can be obtained by applying either a direct approach or an indirect approach. In a *direct approach*, the solution is directly obtained from experts' preferences, whereas in an *indirect approach*, a collective opinion is computed before determining the solution alternative/s [9]. Regardless of the approach considered, the selection process to solve GDM problems consists of two phases (see Figure 1) [26]: (i) an *aggregation phase*, where individual preferences are combined, and (ii) an *exploitation phase*, where an alternative or subset of alternatives are obtained as the solution to the problem.
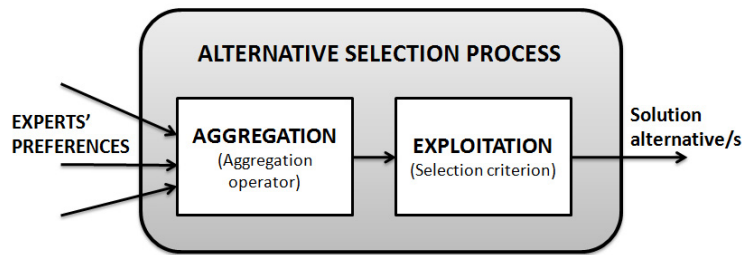
Figure 1: Classical selection process for the resolution of GDM problems

## 2.2 Consensus Reaching Processes

The classical alternatives selection process reviewed above does not regard the existing level of agreement amongst experts before making the decision [5]. As a result, it may occur that some individuals do not accept the solution chosen, because they might consider that their opinions have not been taken into account sufficiently [28]. A highly accepted solution by the whole group is an important aspect to consider in many GDM problem. For this reason, an additional phase called *Consensus Reaching Process* (CRP) was introduced in the resolution process for GDM problems (see Figure 2), in order to achieve a high degree of agreement amongst experts before making a decision. In a CRP, experts discuss and modify their preferences, bringing them closer to each other and towards a collective opinions which is viewed as satisfactory by all of them. Consensus has attained a great importance to reach more appreciated solutions in GDM problems, and it has become a major research topic in the last decades [3, 10, 14, 24].
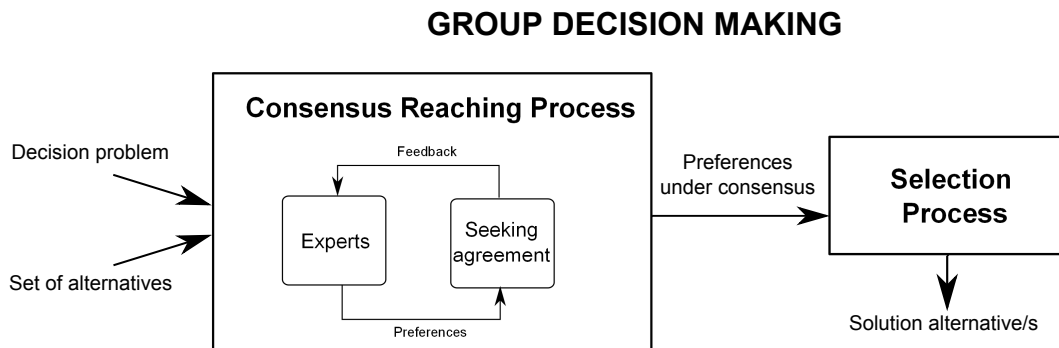
## GROUP DECISION MAKING



Figure 2: Resolution scheme of consensus-based GDM problems

The process to reach consensus is a dynamic and iterative discussion process, frequently coordinated by a human figure known as moderator, who is responsible for supervising and guiding experts over the course of the CRP [5, 17, 28]. A general scheme of CRPs is shown Figure 3. Its phases are briefly described below:

(1) *Consensus measurement*: Preferences of all experts over $X$, $P_i$, $i \in \{1, \ldots, m\}$, are gathered to compute the current degree of consensus in the group by means of a consensus measure, which determines how close the opinions of experts are from unanimous agreement.

(2) *Consensus control*: The consensus degree computed in the previous phase is checked to decide whether it is enough or not. If consensus is enough, the group moves on to the selection process.
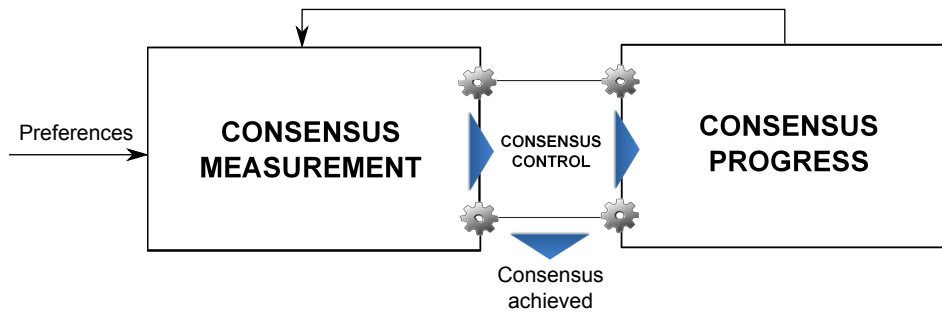
Figure 3: General consensus reaching scheme in GDM

Otherwise, it is necessary to carry out another round of discussion. Two parameters, whose values are fixed a priori by the group, could be utilized in this phase:

- A consensus threshold $\mu$, whose value indicates the minimum level of agreement required amongst members in the group. Many consensus models compute the consensus degree as a value in the unit interval [14, 18, 24], being a value of 1 interpreted as full and unanimous agreement, therefore $\mu \in [0, 1]$ in such cases.

- A maximum number of discussion rounds allowed, $Maxround \in \mathbb{N}$. If the number of rounds carried out exceeds this value, then the CRP ends without having reached consensus.

(3) *Consensus Progress*: If the current degree of consensus is not enough, a procedure is applied to increase the level of agreement in the following round of the CRP. Such a procedure has been traditionally based on providing experts with some feedback, which indicates them how to modify their preferences, but some approaches that conduct this process automatically have been also proposed:

(a) *Feedback Generation*: Many existing consensus models incorporate feedback mechanisms based on this process [12, 18], in which the moderator identifies the farthest experts' assessments from consensus in the current round. He/she then provides experts with some advices to modify the value of assessments previously identified, in order to bring them closer to the rest of the group and increase the consensus degree in the following round.

(b) *Automatic Updates*: Some consensus models do not incorporate a feedback mechanism, and instead they implement approaches that update information (e.g. assessments of experts) to increase consensus in the group automatically [2, 23]. Therefore, once experts provide their initial preferences at the beginning of the CRP, they do not need to supervise them at each round. The main advantage of implementing consensus models based on this approach is their low time cost due to the full automation of the CRP, thus making them more adequate in GDM problems defined in time-constrained urgent situations. On the other hand, their main weakness is the total loss of experts' sovereignty to decide whether they accept changes suggested on their assessments or not.

# 3 Visual Decision Support Tool for GDM in Urgent Situations

In this section, we present a visual decision support tool to facilitate the resolution of GDM problems under uncertainty that take place in urgent situations. Such a tool aims at providing stakeholders with easily interpretable knowledge problem state, thus giving them additional support under certain circumstances present in emergency contexts:

- Highly time-constrained events that require making group decisions as quickly as possible, without investing a substantial amount of time in analyzing the non-visual information managed by existing GDSS.

- The presence of experts or subgroups of them with conflicting opinions within a large group, and the difficulty to reach consensus in such a case.

- The possible existence of experts who do not cooperate with the rest of the group to reach an agreement [21], and the necessity of detecting them visually to avoid delays in reaching consensus.

The proposed tool attempts to monitor these features both in classical GDM problems and in CRPs, in order to analyze the positions of experts' preferences with respect to the group based on their preferences, and it is based on Self-Organizing Maps (SOMs) [15], one of the most utilized techniques to create low-dimensional visual projections of high-dimensional data sets - e.g. preferences of a group of experts over a set of alternatives - based on the closeness degree between data objects.

The proposal description is divided into three parts:

- An overview of the decision support tool architecture and technologies utilized.

- A description of the main features of the tool regarding its use in GDM problems taking place in urgent contexts.

- An example of its application to a specific urgent scenario.

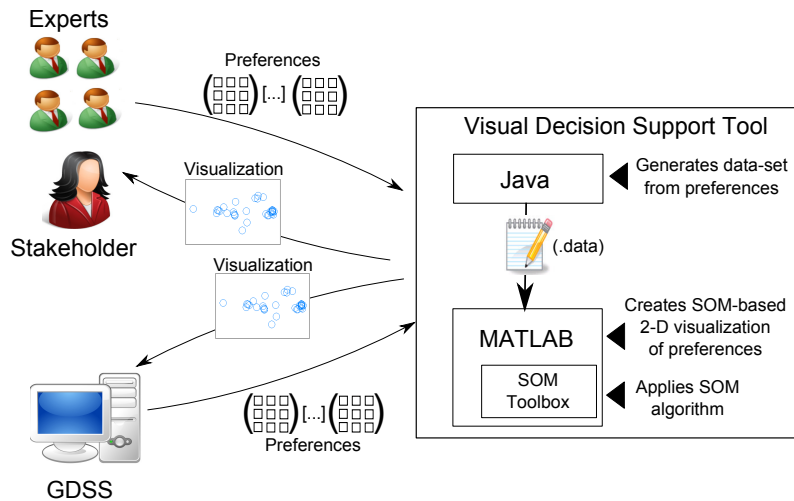## 3.1  Architecture of the Decision Support Tool



Figure 4: Architecture of the tool

Figure 4 shows the architecture of the proposed tool. It has been developed as a local application that receives a set of experts' preferences associated to a GDM problem and outputs a two-dimensional graphical representation of such preferences. As illustrated in the figure, the tool can be either used directly used by decision groups who provide their preferences, or by integrating it with existing GDSS to provide a more interpretable representation of the information provided by them. The following technologies have been considered for its development:

- *Java*: Utilized to read input data, i.e. experts' preferences and generate a preference data-set upon them.

- *MATLAB*: Utlized to create a SOM-based two-dimensional projection from a preference data-set, by applying a SOM algorithm. Further detail on SOM techniques and algorithms can be found in [15].
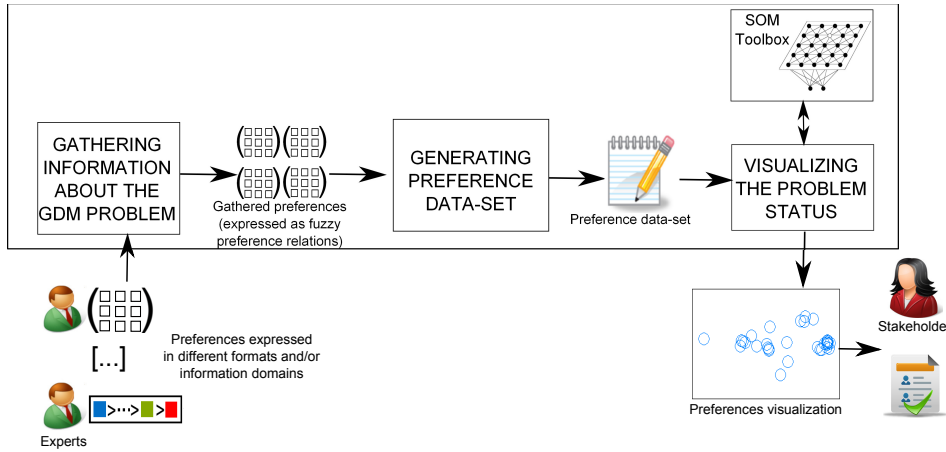


Figure 5: Scheme followed by the tool to visualize group preferences

A scheme of the tool operation is shown in Figure 5. The procedure followed to generate a graphical representation about the status of the GDM problem consists of three phases:

(1) *Gathering information about the GDM problem*: Preferences of all experts in the group, $P_i, i \in \{1, \ldots, m\}$, are gathered in this phase. In many GDM problems, especially those requiring a high level of consensus, the collective preference of the group $P_c$ should be computed (by aggregating individual preferences) as gathered as well, with the aim of visualizing how close experts opinions are from such a collective preference.

The proposed tool deals with preferences expressed as fuzzy preference relations (i.e. preference relations in which assessments $p_i^{lk} \in [0, 1]$), to generate a data-set from them. Nevertheless, different preference structures can be also managed by transforming them into fuzzy preference relations [12]. Preferences expressed under different information domains (e.g. intervals or linguistic values), can be also transformed into numerical values in [0,1] by applying the adequate transformation functions [11]. These functionalities are implemented in the Java module of the tool.

(2) *Generating preference data-set*: Once information to be visualized has been gathered and expressed as fuzzy preference relations, in this phase a preference data-set file with extension *.data* is generated upon them. As occurred with the previous phase, this process is also carried out by the Java module. In a preference data-set, each row contains the assessments $p_i^{lk}$, $l \neq k$, corresponding to a single fuzzy preference relation $P_i$. Therefore, for a GDM problem with $n$ alternatives, the dimension of each data object obtained from preferences must be equal to $n(n-1)$.

A particularly interesting aspect of the visualization tool is the fact that each data sample can be optionally tagged by placing an alphanumerical tag at the end of the corresponding row in the data-set, with informative purposes. Tagging provides additional information about a specific preference relation (for example, the name, identifier or role of the corresponding expert who provided it). Tags are visualized together with their corresponding preference in the two-dimensional representation of the GDM problem state. Some examples that illustrate

the usefulness of tagging preferences in urgent GDM problems will be presented in the following subsection.

(3) *Visualizing the problem status*: The preference data-set is used as an input by the MATLAB module of the proposed tool, to apply a SOM-based technique that generates a two-dimensional graphical projection of data contained in it. Such a projection may be utilized by one or several stakeholders for analyzing aspects of interest about the GDM problem and receiving additional support in emergency situations (see Section 3.2).

A SOM algorithm [15] must be invoked to create the two-dimensional map on which data will be visualized. To do so, a third-party MATLAB library so-called SOM Toolbox[1] [30], has been utilized to create and manage SOMs. Once constructed the map, each preference in the data-set is projected into it, so that similar data objects are projected into closer positions to each other in the map.

As previously stated, tagging data might be useful for several visualization purposes, e.g. viewing the collective preference of the group, by including and tagging it in the preference data-set.

## 3.2   Use of the Tool in Different Urgent Scenarios

Once the architecture and operation scheme of the visual decision support tool have been presented, here we show some guidelines regarding its use to facilitate the resolution of GDM problems in several types of urgent scenarios with different degrees of time-constraint.

- *Problems that require highly accepted decisions*: Response to emergency events is usually the result of a collaborative effort amongst a number of experts with different roles and belonging to diverse entities, hence it is essential to determine the existing level of agreement between experts' preferences, which should be as high as possible. Consensus models implemented by existing GDSS [14, 23, 22, 35] can be used to aid groups reaching consensus, by applying a CRP. The choice between an automatic or a feedback-based consensus model (see Section 2.2), might be subject to the level of urgency present at each particular problem, e.g. in emergencies in which decision must be made as soon possible, it would be more appropriate to use automatic models. In either case, the proposed visualization tool can be a valuable aid for stakeholders to analyze some aspects of interest throughout a CRP:

  (a) It can be iteratively used to visualize the state of the GDM problem at each consensus round: visualizing the evolution of experts' preferences across the time may provide a better insight on the overall performance of the CRP and the foresight of the problem state in upcoming discussion rounds.

  (b) In large groups, when consensus is difficult to reach due to highly conflicting opinions between experts, analyzing numerical or textual information their preferences to identify conflicting opinions can be a complex task. A two-dimensional visual representation of preferences can provide a visual insight on conflicting opinions in these cases. Stakeholders can identify the experts associated with such opinions by tagging preferences, e.g. by means of a numerical identifiers (see Figure 6).

  (3) Experts may adopt different behaviors in a CRP, regarding their willingness to modify their initial opinions and bring them closer to the collective opinion. Coalitions of experts with similar interests might sometimes try to move their preferences strategically to deviate the collective opinion in their favor [27]. If the necessary mechanisms to identify such behaviors [21] are implemented in a GDSS, then it is possible to tag experts involved in such behaviors to facilitate their visual detection.
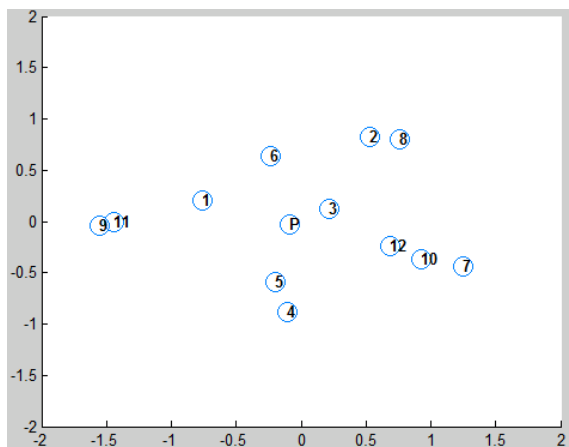
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[1]http://www.cis.hut.fi/somtoolbox/

Figure 6: Visualization of tagged preferences: $P$ indicates the position of the collective preference and $1-12$ represent experts' preferences.

In the cases that consensus is not reached after a number of rounds, the decision group would prefer to adopt a different strategy to solve the GDM problem (as described below), in order not to invest a substantial amount of time in making a decision.

- *Highly time-constrained problems*: Some critical situations, e.g. those in which lives may be at stake [6], require making highly time-constrained decisions, such that CRPs can not be applied. In such cases, the proposed tool would be used to gather experts' preferences and provide their visual representation to stakeholders immediately, so that they decide the action to be taken, for instance making the decision based on the preferences of an expert who represents a majority opinion in the group (see $e_9$ in Figure 7).
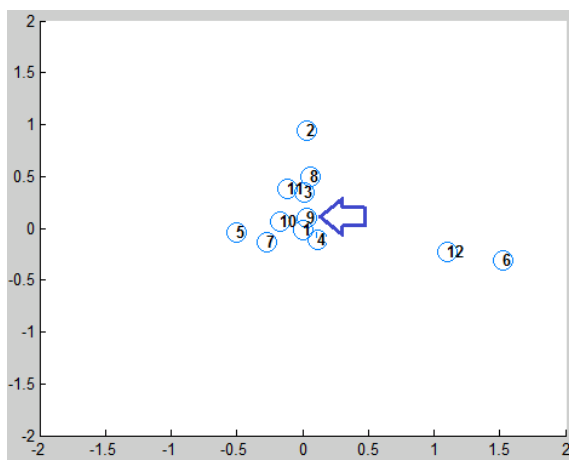


Figure 7: Selecting an expert preference as the solution to the GDM problem in highly-constrained scenarios.

## 3.3   Application Example

Finally, an example of application of the tool is presented to show its usefulness in a real-life urgent GDM problem.

The GDM problem is formulated as follows: due to an overnight snowfall, four towns situated similarly close to the city of Jaén, $X = \{x_1, x_2, x_3, x_4\}$ ended up isolated. Such phenomena are extremely uncommon in this area, therefore only one snowplow is available and a county panel formed by 45 members $E = \{e_1, \ldots, e_{45}\}$, must jointly decide in which order should each town be helped. Since there were no human lives at stake, time-constrains were low enough to consider making a decision under consensus. Therefore the panel members used a GDSS that implements a feedback-based consensus model [22], to provide their preferences and modify them at each consensus round. This consensus model allows expert to provide their preferences by means of either numerical, interval-valued or linguistic preference relations [11], depending on the level of knowledge and uncertainty they present about the problem. The visualization tool has been integrated with such a GDSS to gather experts preferences and the collective preference obtained as each consensus round and show the existing agreement/disagreement positions graphically.
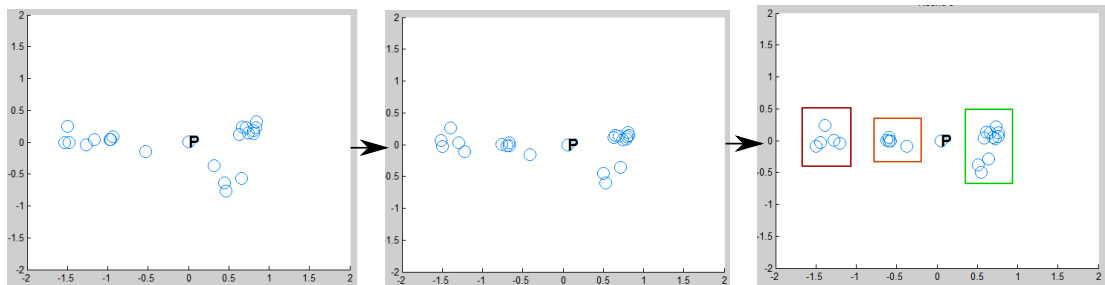


Figure 8: Overall state of the GDM problem at three consecutive consensus rounds.

Figure 8 shows experts' preferences and their closeness to the collective preference (obtained by applying an arithmetic mean operator) at three consecutive discussion rounds. Based on the visual representation of preferences, the stakeholder could easily notice the existence of several subgroups with very similar opinions within each one. Additionally, the efforts made by the group to reach consensus were not enough after these rounds, since their opinions did not move significantly towards $P$. Depending on the level or urgency and the context of the problem, the stakeholder could perform the following possible actions based on visual information received:

- Notify group members about the need for cooperating to achieve an agreement and encourage them to apply more substantial changes based on the feedback received by means of the consensus model utilized.

- Consider a majority group only (see green rectangle at the right side of the third plot) to reach consensus more easily, excluding the opinions of the rest of expert in the group.

- In the case of having injured citizens in the four towns, consider making a quick decision, based on a single expert's opinion from a majority position (as previously shown in Figure 7).

# 4   Concluding Remarks

In this contribution, we have presented a visual decision support tool, based on Self-Organizing Maps, for Group Decision Making problems defined under uncertainty. Such a tool provides a two-dimensional visual representation of experts' preferences thus facilitating the analysis of information and obtaining useful knowledge about the state of the decision problem or the level of consensus in the group. An

application example has been presented to show the usefulness of the tool as a visual complement for existing Group Decision Support System, in real-life group decision problems that take place in urgent situations.

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