Computing with Words Based on a Hybrid Semantic-Symbolic Model

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The use of linguistic information based on the fuzzy linguistic approach to deal with uncertain and vague information it has been successfully used in many problems. It implies processes of computing with words (CW). The use of fuzzy numbers to accomplish the processes of CW provides accuracy and flexibility in the operations, but the results are fuzzy numbers that usually cannot be expressed linguistically. Hence symbolic approaches have been proposed to accomplish the processes of CW and improve the understanding of the results, but this has implied a lack of precision and a limitation of the operations in those processes. In this contribution, we present a hybrid linguistic computational model that carries out the operations by using fuzzy numbers, but the results are expressed linguistically in order to overcome the operational limitations and the lack of precision of symbolic approaches.

Keywords: computing with words, linguistic 2-tuple, semantic model

1. Introduction

There exist many situations in which problems deal with vague and imprecise information. In such cases, the linguistic modelling [11] have been used with successful results. The use of linguistic information implies the need of carrying out processes that operate with words, so-called Computing with Words (CW). In the literature can be found two classical linguistic computational models that provide linguistic operators for CW:

i) Semantic model based on the extension principle [1,2].
ii) Symbolic model [3,5,10].

Analyzing both models, the semantic one is accurate and easy to use but unfortunately its results are difficult to understand by people who are not experts in fuzzy logic. Due to this fact, recently it has paid too much attention to the symbolic model that was initially easy to understand but inaccurate and with limitations in its operations. In the literature we can find different proposals that have improved its lack of precision [3,5], however, they have paid much less attention in the limitations about the operations. Xu [9] presented a symbolic model that overcomes all the limitations, but with this model the linguistic information loses its syntax and semantics, therefore it is not a linguistic model in the sense of the fuzzy linguistic approach [11].

The aim of this contribution is to introduce a linguistic computational model which is able to accomplish different operations, such that the computations are carried out semantically by means of fuzzy numbers and the results are expressed linguistically. Due to the long limitation of the paper we just focus on the addition operation, although it can be easy to extend to other operations. Given that the current symbolic models cannot accomplish these operations, it is necessary to introduce a new linguistic representation model that is based on the 2-tuple and that allows to express linguistically the computing results.

The paper is structured as follows: In Section 2 we shall review briefly the model based on the Extension Principle and the linguistic 2-tuple representation. In Section 3, we shall present the hybrid computational model, and finally we shall point out some concluding remarks.

2. Preliminares

Due to the fact that our aim is to introduce a hybrid linguistic computational model which is able to represent linguistically the results computed semantically, in this section we will review the necessary concepts related to the semantic model and linguistic modelling.

2.1. *Model based on the Extension Principle*

This model carries out operations with linguistic terms by means of operations associated to their membership functions based on the Extension Principle. The Extension Principle is a basic concept in the fuzzy sets theory [4] which is used to generalize crisp mathematical concepts to fuzzy sets. The use of extended arithmetic based on the Extension Principle [4] increases the vagueness of the results. Therefore, the results obtained by the fuzzy linguistic operators based on the Extension Principle are fuzzy numbers that usually do not match with any linguistic term in the initial term set. For this reason, it is necessary to carry out a linguistic approximation process in order to express the results in the original expression domain.

Without loss of generality we shall use triangular fuzzy numbers as semantics of linguistic labels. Different operations with this type of fuzzy number are shown in the following. Let $S = \{s_0, \ldots, s_g\}$ be a linguistic term set and let $s_1 = (a_1, b_1, c_1)$ and $s_2 = (a_2, b_2, c_2)$ be two linguistic terms in [0,1], where $s_1, s_2 \in S$ and $g + 1$ is the granularity of S:

- Addition: $s_r = s_1 + s_2 = [a_1 + a_2, b_1 + b_2, c_1 + c_2]$
- Subtraction: $s_r = s_1 - s_2 = [a_1 - c_2, b_1 - b_2, c_1 - a_2]$
- Multiplication: $s_r = s_1 * s_2 = [a_1 * a_2, b_1 * b_2, c_1 * c_2]$
- Division: $s_r = s_1/s_2 = s_1 * \frac{1}{s_2} = [a_1 * \frac{1}{c_2}, b_1 * \frac{1}{b_2}, c_1 * \frac{1}{a_2}]$

2.2. *Linguistic 2-tuple model*

This symbolic model was presented in [6] to avoid the loss of information and express symbolically any counting of information in the universe of discourse. The representation of this model is based on the concept of *symbolic translation* and uses it for representing the linguistic information by means of a pair of values, so-called *2-tuples*, (s_i, α), where s is a linguistic term and α is a numerical value representing the symbolic translation.

From this concept, is developed a linguistic representation model which represents the linguistic information by means of 2-tuples (s_i, α), $s_i \in S$ and $\alpha_i \in [-0.5, 0.5)$. Besides, this representation model defines a set of functions, such as Δ and Δ^{-1} to facilitate computational processes with 2-tuples, see [6] for further details.

3. Hybrid computational model

In this section we are going to introduce how to accomplish processes of CW by using the Extension Principle but representing the results linguistically in a similar term set to the initial one. As it was aforementioned, we focus on the addition operation, although in the future could be easily extended to other operations. First we shall show the operational idea and then the representation model.

3.1. *Adding linguistic labels*

Let $s_1 = (a_1, b_1, c_1)$ and $s_2 = (a_2, b_2, c_2)$ be two linguistic terms in [0,1] and $s_1, s_2 \in S$, its addition is computed as we showed in Section 2.1

The main problem of this computation is that might not be represented linguistically, because the computed fuzzy number does not match with any semantics of the initial linguistic terms set and additionally the fuzzy number could be out of the initial universe of discourse (see figure 1).

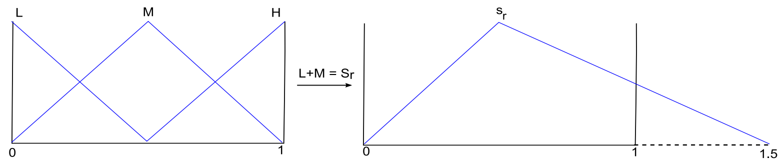

Fig. 1. Representation of the result out of the initial universe of discourse

Our idea consists of two steps: (i) to extend the initial term set to a similar one in the new universe of discourse according to the maximum value that can be achieved by the operation, (ii) to transform the fuzzy number obtained into a linguistic term in the new universe of discourse.

(1) Extended linguistic term set, S': the idea is to represent the initial term set in the new universe of discourse. So, the new universe of discourse in the addition is computed as the maximum value of the initial universe, multiplied by the number of terms added. To represent the new universe of discourse, we will use the parameter δ.

Let's suppose the example of the figure 1, where $s_i \in S$ and $U = [0, 1]$:

$$max_x \mu_{s_i}(x) = 1 \qquad L + M(2labels) \qquad \Rightarrow \qquad \delta = 1 * 2 = 2$$

Therefore, the new linguistic term set, (S'), would be the following one (see figure 2):

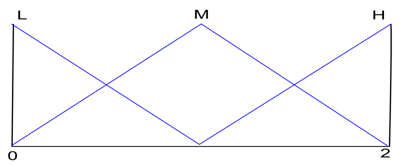

Fig. 2. Representation of the new linguistic term set, S'

Once we have the term set, S', in which will be expressed the result, we should represent the addition result linguistically (see figure 3).

(2) Transforming the result into linguistic value: in order to represent the fuzzy number obtained in the addition in a linguistic term, we have revised different proposals in the literature [6–9], but none of them is completely suitable for our aims. Therefore, we have chosen a matching process [7] and the linguistic 2-tuple model, although it should be extended with a new parameter to transform the fuzzy number into a linguistic term. First we follow the process presented in [6] in order

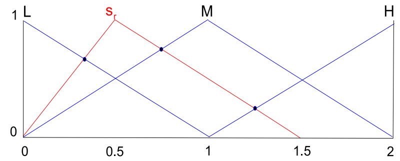

Fig. 3. Result of adding L + M

to transform the fuzzy number in a linguistic 2-tuple by means of the following function:

$$\tau_{s_r S'} : s_r \rightarrow F(S')$$
$$\tau_{s_r S'} = (s'_k, \gamma_k)/k \in \{0, \dots, g\}$$
$$\gamma_k = max_y min\{\mu_{s_r}(y), \mu_{s'_k}(y)\}$$

where $F(S')$ is the set of fuzzy sets defined in S', and $\mu_{s_r}(\cdot)$ and $\mu_{s'_k}(\cdot)$ are the membership functions of the fuzzy sets associated with the terms s_r and s'_k, respectively.

According to the previous example, see figure 3:

$$\gamma_0 = 0.63 \qquad \gamma_1 = 0.75 \qquad \gamma_2 = 0.25$$

After that, we calculate the *central value*:

$$cv(\tau_{s_r S'}) = \frac{\sum_{h=0}^{g} index(s_h) * \mu_h}{\sum_{h=0}^{g} \mu_h} \tag{1}$$

where $index(s_h)$ is the index of the linguistic term and μ_h is the membership function.

$$cv = \frac{(0 * 0.63) + (1 * 0.75) + (2 * 0.75)}{0.63 + 0.75 + 0.25} = \frac{1.25}{1.63} = 0.77$$

To transform the central value into 2-tuple, we use the function Δ of the 2-tuple linguistic representation model [6].

$$\Delta(0.77) = (M, -0.23)$$

But the previous 2-tuple is not enough because, what is the difference between $M \in S$ and $M \in S'$? It is the universe of discourse. In order to show such difference we use the parameter δ calculated in the previous step. Thus, the representation would be the following one:

$$(s, \alpha)^{\delta} = (M, -0.23)^2$$

4. Concluding Remarks

The use of the fuzzy linguistic approach implies processes of CW. In this contribution, we have presented a hybrid computational model that accomplishes the operations by means of the extension principle allowing the computation of any operation and then, the fuzzy results are transformed into linguistic values in order to facilitate its understanding.

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