Attitude-Based Consensus Model for Heterogeneous Group Decision Making

R. M. Rodríguez, I. Palomares and L. Martínez

Abstract Usually, human beings make decisions in their daily life providing their preferences according to their knowledge area and background. Therefore, when a high number of decision makers take part in a group decision-making problem, it is usual that they use different information domains to express their preferences. Besides, it might occur that several subgroups of decision makers have different interests, which may lead to situations of disagreement amongst them. Therefore, the integration of the group's attitude toward consensus might help optimizing the consensus reaching process according to the needs of decision makers. In this contribution, we propose an attitude-based consensus model for heterogeneous group decision-making problems with large groups of decision makers.

Keywords Group decision making · Heterogeneous information · Attitude · Consensus reaching

1 Introduction

Decision making is a usual process for human beings in their daily life. In group decision-making (GDM) problems, a group of decision makers try to reach a solution to a problem that consists of a set of possible alternatives, providing their preferences [3]. An important aspect in GDM problems is to achieve a common solution which is accepted by all decision makers involved in the problem.

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Usually, GDM problems have been solved by applying approaches that do not guarantee to reach such a collectively accepted solution. Hence, Consensus Reaching Processes (CRPs) become necessary to obtain accepted solutions by all decision makers participating in the GDM problem [8].

Classically, consensus models proposed in the literature to deal with CRPs focused on the resolution of GDM problems where a low number of decision makers take part. However, nowadays new trends like social networks [9] and e-democracy [4] imply the participation of larger groups of decision makers in discussion processes. When large groups of decision makers are involved in a GDM problem, it is usual that each one expresses her/his preferences in different information domains, such as, numerical, interval valued or linguistic values, according to their profile, the area of knowledge they belong to, and the nature of alternatives. When alternatives are quantitative in nature, they are normally assessed by means of numerical or interval-valued values; however, when their nature is qualitative, the use of linguistic information might be more suitable [1]. In such cases, the GDM problem is defined in an heterogeneous framework. Different approaches to deal with heterogeneous information have been presented [2, 5].

Another issue in GDM problems with a large number of decision makers is that there might exist several subgroups of decision makers with conflicting interests, which may lead to situations of disagreement amongst such subgroups, thus making it hard to achieve an agreed solution and delaying the decision process. Therefore, the integration of the group's attitude toward consensus, i.e., the capacity of decision makers to modify their own preferences during the CRP, becomes an important aspect to consider in any CRP involving large groups [6].

The aim of this paper is to propose a new consensus model for GDM problems defined in heterogeneous contexts, which is able of integrating the attitude of decision makers toward consensus in CRPs involving large groups.

This paper is structured as follows: Sect. 2 revises some preliminary concepts about management of heterogeneous information and attitude integration in CRPs. Section 3 presents the heterogeneous consensus model that integrates the group's attitude toward consensus. Section 4 shows an illustrative example of the proposed model, and Sect. 5 points out some conclusions.

2 Preliminaries

In this section, some concepts about managing heterogeneous information in GDM and integrating attitudes in CRPs are briefly reviewed.

2.1 Heterogeneous Information in GDM

In GDM problems, where a large number of decision makers are involved, it is frequent that they have different background or they have different degrees of knowledge about the problem. For these reasons, the use of different information domains might allow decision makers to express their preferences in a more suitable way, thus leading to better results than those obtained if they had to express such preferences in a single domain imposed for the whole group. In this paper, we consider an heterogeneous framework compound by the following domains:

- *Numerical*: $N = \{v | v \in [0,1]\}$.
- *Interval-valued*: $I = P([0,1]) = \{[l,u] | l,u \in [0,1] \wedge l \leq u\}$.
- *Linguistic*: $S = \{s_0, \ldots, s_g\}$, where S is a linguistic term set defined in the unit interval. It is assumed that each linguistic term $s_j \in S, j \in \{0, \ldots, g\}$, has associated a fuzzy membership function, denoted as $\mu_{s_j}(y), y \in [0,1]$ [12].

In spite of the different approaches to deal with heterogeneous information in the literature [2, 5], each one is based on different features. Here, we use the method proposed by Herrera et al. [2] to unify assessments expressed in different domains into fuzzy sets $F(S_T)$, in a common linguistic term set $S_T = \{s_0, \ldots, s_g\}$ chosen according to the rules introduced in [2], by means of the transformation functions defined below for each type of information.

Definition 1 [2] Let $v \in [0,1]$ be a numerical value, the function $\tau_{NS_T} : [0,1] \rightarrow F(S_T)$ transforms a numerical value into a fuzzy set in S_T.

$$\tau_{NS_T}(v) = \{(s_0, \gamma_0), \ldots, (s_g, \gamma_g)\} \quad s_k \in S_T, \gamma_k \in [0,1]$$

$$\gamma_k = \mu_{s_k}(v) = \begin{cases} 0 & if \quad v \notin support\ (\mu_{s_k}(x)), \\ \frac{v-a_k}{b_k-a_k} & if \quad a_k \leq v \leq b_k, \\ 1 & if \quad b_k \leq v \leq d_k, \\ \frac{c_k-v}{c_k-d_k} & if \quad d_k \leq v \leq c_k. \end{cases}$$

being $\mu_{s_k}(\cdot)$ a membership function for linguistic term $s_k \in S_T$, represented by a parametric function (a_k, b_k, d_k, c_k).

Definition 2 [2] Let $I = [l,u]$ be an interval valued in [0, 1], the function $\tau_{IS_T} : I \rightarrow F(S_T)$ transforms an interval valued into a fuzzy set in S_T.

$$\tau_{IS_T}(I) = \{(s_k, \gamma_k)/k \in \{0, \ldots, g\}\}$$
$$\gamma_k = max_y min\{\mu_I(y), \mu_{s_k}(y)\}$$

where $\mu_I(\cdot)$ and $\mu_{s_k}(\cdot)$ are the membership functions of the fuzzy sets associated with the interval valued I and linguistic term s_k, respectively.

Definition 3 [2] Let $S = \{l_0, \ldots, l_p\}$ and $S_T = \{s_0, \ldots, s_g\}$ be two linguistic term sets, such that $g \geq p$, a linguistic transformation function $\tau_{SS_T} : S \rightarrow F(S_T)$ transforms a linguistic value l_i into a fuzzy set in S_T.

$$\tau_{SS_T}(l_i) = \{(s_k, \gamma_k^i)/k \in \{0, \ldots, g\}\} \qquad \forall l_i \in S$$
$$\gamma_k^i = max_y min\{\mu_{l_i}(y), \mu_{s_k}(y)\}$$

where $i \in \{0, \ldots, p\}$. $\mu_{l_i}(\cdot)$ and $\mu_{s_k}(\cdot)$ are the membership functions of the fuzzy sets associated with the terms l_i and s_k, respectively.

2.2 Attitude Integration in CRPs: Attitude-OWA

CRPs in GDM problems attempt to find a collective agreement amongst decision makers before making a decision, so that a more accepted solution by the whole group is achieved [8]. Although a large number of consensus models have been proposed to support groups in CRPs, most of them do not consider the integration of the group's attitude toward consensus in situations where several subgroups of decision makers, with different (and often conflicting) interests and attitudes, take part in the GDM problem. Since this aspect might help optimizing the CRP according to the needs of decision makers and each particular problem, a consensus model that integrates such an attitude was recently proposed in [6], where the different types of group's attitudes to be considered were also introduced:

- *Optimistic attitude*: Achieving an agreement is more important for decision makers than their own preferences. Therefore, more importance is given to positions in the group with higher agreement.
- *Pessimistic attitude*: Decision makers consider more important to preserve their own preferences. Therefore, positions in the group with lower agreement are given more importance.

In the following, we briefly review the definition of an aggregation operator, so-called Attitude-OWA, that will be used to integrate the attitude of decision makers in CRPs in the proposed consensus model. Such an operator extends OWA aggregation operators [10], and it is specially suitable for dealing with large groups of decision makers [6].

Definition 4 [10] An OWA operator on a set $A = \{a_1, \ldots, a_h\}$, $a_i \in R$ is a mapping $F : R^h \rightarrow R$, with an associated weighting vector $W = [w_1 \ldots w_h]^\top$:

$$F(a_1, \ldots, a_h) = \sum_{j=1}^{h} w_j b_j \tag{1}$$

with $w_i \in [0, 1]$, $\sum_i w_i = 1$. b_j is the jth largest a_i value.

The Attitude-OWA operator extends the OWA operators by introducing two *attitudinal parameters* that must be provided by the decision group, $\vartheta, \varphi \in [0, 1]$:

- ϑ represents the group's attitude, which can be optimistic ($\vartheta > 0.5$), pessimistic ($\vartheta < 0.5$), or indifferent ($\vartheta = 0.5$). It is equivalent to the measure of optimism (*orness*) that characterizes OWA operators [10].
- φ indicates the amount of agreement positions that are given nonnull weight in the aggregation process. The higher φ, the more values are considered.

Attitude-OWA operator is then defined as follows:

Definition 5 [6] An Attitude-OWA operator of dimension h on a set $A = \{a_1, \ldots, a_h\}$, is an OWA operator based on attitudinal parameters ϑ, φ given by a group of decision makers to indicate their attitude toward consensus,

$$\textit{Attitude-OWA}_W(A, \vartheta, \varphi) = \sum_{j=1}^{h} w_j b_j \tag{2}$$

where b_j is the j-th largest of a_i values and A is the set of values to aggregate.

Attitude-OWA is characterized by a weighting vector W, computed according to attitudinal parameters, so that weights reflect an specific attitude adopted by decision makers. The following scheme was proposed in [6] to compute Attitude-OWA weights.

(i) The group provides values for ϑ, φ, based on their interests and/or the nature of the GDM problem.
(ii) A RIM (Regular Increasing Monotone) linguistic quantifier with membership funcion $Q(r)$, $r \in [0, 1]$:

$$Q(r) = \begin{cases} 0 & \text{if } r \leq \alpha, \\ \frac{r-\alpha}{\beta-\alpha} & \text{if } \alpha < r \leq \beta, \\ 1 & \text{if } r > \beta. \end{cases} \tag{3}$$

is defined upon ϑ, φ, by computing $\alpha = 1 - \vartheta - \frac{\varphi}{2}$ and $\beta = \alpha + \varphi$.
(iii) The following method proposed by Yager in [11] is applied to compute weights w_i:

$$w_i = Q\left(\frac{i}{h}\right) - Q\left(\frac{i-1}{h}\right), i = 1, \ldots, h \tag{4}$$

3 Attitude-Based Consensus Model for GDM in Heterogeneous Contexts

This section presents a consensus model for GDM problems that deals with heterogeneous information and integrates the attitude in CRPs when there are large groups of decision makers.

Classically, consensus models proposed in the literature consider the existence of a human figure so-called moderator, who is responsible for coordinating the overall CRP [8]. Nevertheless, this approach facilitates the automation of his/her tasks, by implementing such a model into a *Consensus Support System* based on intelligent techniques [7].

GDM problems considered in this model are formed by a set $E = \{e_1, \ldots, e_m\}$, $(m \geq 2)$, of decision makers who express their preferences over a set of alternatives $X = \{x_1, \ldots, x_n\}$, $(n \geq 2)$ by using a preference relation P_i:

$$P_i = \begin{pmatrix} - & \ldots & p_i^{1n} \\ \vdots & \ddots & \vdots \\ p_i^{n1} & \ldots & - \end{pmatrix}$$

Each assessment $p_i^{lk} \in D_i$ represents the degree of preference of the alternative x_l over x_k, $(l \neq k)$ for the decision maker e_i, expressed in an information domain, $D_i \in \{N, I, S\}$ (see Sect. 2.1).

A scheme of the consensus model is depicted in Fig. 1, and its phases are described in detail below:

1. *Determining Group's Attitude*: The first phase consists of determining the group's attitude towards the measurement of consensus, gathered by means of the attitudinal parameters ϑ, φ.
2. *Gathering Preferences*: Given that the GDM problem is defined in an heterogeneous framework, each e_i provides his/her preferences on X by means of a preference relation P_i, consisting of a $n \times n$ matrix of assessments $p_i^{lk} \in D_i = \{N, I, S\}$.
3. *Making Heterogeneous Information Uniform*: Preferences expressed by decision makers in different information domains are unified by applying the approach proposed in [2], that unifies heterogeneous information into fuzzy sets

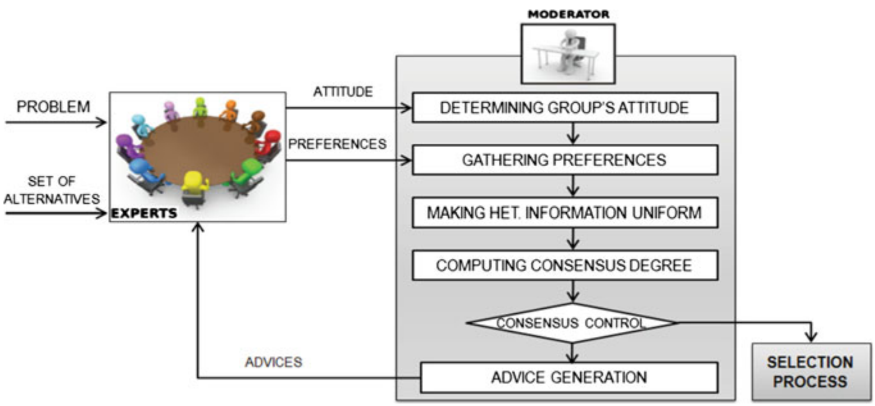

Fig. 1 Consensus model scheme

in a common linguistic term set (see Def. 1, 2, 3). Assuming that each unified assessment is represented by $p_i^{lk} = (\gamma_{i0}^{lk}, \ldots, \gamma_{ig}^{lk})$, each decision maker's preference relation is represented as follows:

$$P_i = \begin{pmatrix} - & \ldots & (\gamma_{i0}^{1n}, \ldots, \gamma_{ig}^{1n}) \\ \vdots & \ddots & \vdots \\ (\gamma_{i0}^{n1}, \ldots, \gamma_{ig}^{n1}) & \ldots & - \end{pmatrix}$$

4. *Computing Consensus Degree*: It computes the degree of agreement amongst decision makers [3], measured as a value in [0, 1]. The group's attitude toward consensus is integrated in step c) during this phase.

   (a) For each unified assessment (fuzzy set) $p_i^{lk} = (\gamma_{i0}^{lk}, \ldots, \gamma_{ig}^{lk})$, a *central value* $cv_i^{lk} \in [0, g]$ is obtained to facilitate further computations, as follows:

$$cv_i^{lk} = \frac{\sum_{j=0}^{g} index(s_j) \cdot \gamma_{ij}^{lk}}{\sum_{j=0}^{g} \cdot \gamma_{ij}^{lk}}, \quad s_j \in S_T \tag{5}$$

where $index(s_j) = j \in \{0, \ldots, g\}$.

   (b) For each pair of decision makers e_i, e_t, $(i<t)$, a *similarity matrix* $SM_{it} = (sm_{it}^{lk})_{n\times n}$ is computed. Each similarity value $sm_{it}^{lk} \in [0, 1]$ represents the agreement level between e_i and e_t in their opinion on (x_l, x_k), computed as:

$$sm_{it}^{lk} = 1 - \left| \frac{cv_i^{lk} - cv_t^{lk}}{g} \right| \tag{6}$$

   (c) A *consensus matrix* $CM = (cm^{lk})_{n\times n}$ is obtained by aggregating similarity values, by means of an Attitude-OWA operator defined upon ϑ, φ (see Sect. 2.2) to reflect the group's attitude [6]:

$$cm^{lk} = Attitude - OWA_W(SIM^{lk}, \vartheta, \varphi) \tag{7}$$

$SIM^{lk} = \{sm_{12}^{lk}, \ldots, sm_{1m}^{lk}, \ldots, sm_{(m-1)m}^{lk}\}$ is the set of all pairs of decision makers' similarities in their opinion on (x_l, x_k), with $|SIM^{lk}| = \binom{m}{2}$, being cm^{lk} the degree of consensus achieved by the group in their opinion on (x_l, x_k).

   (d) Consensus degrees ca^l on each alternative x_l, are computed as

$$ca^l = \frac{\sum_{k=1, k\neq l}^{n} cm^{lk}}{n-1} \tag{8}$$

   (e) Finally, an overall consensus degree, cr, is obtained as follows:

$$cr = \frac{\sum_{l=1}^{n} ca^l}{n} \tag{9}$$

5. *Consensus Control*: Consensus degree cr is compared with a consensus threshold $\mu \in [0, 1]$, established a priori by the group. If $cr \geq \mu$, the CRP ends and the group moves on the selection process; otherwise, the process requires further discussion. A parameter $Maxrounds \in \mathbb{N}$ can be used to control the maximum number of discussion rounds.

6. *Advice Generation*: When consensus required is not achieved, $cr < \mu$, decision makers are advised to modify their preferences to make them closer to each other and increase the consensus degree in the following CRP round. As stated above, despite a human moderator has been traditionally responsible for advising and guiding decision makers during CRPs, the proposed model allows an automation of his/her tasks [7], many of which are found in this phase of the CRP. The following steps are conducted in this phase (based on central values cv_i^{lk}):

   (a) Compute a collective preference and proximity matrices: A collective preference $P_c = (p_c^{lk})_{n \times n}$, $p_c^{lk} \in [0, g]$, is computed for each pair of alternatives by aggregating preference relations:

$$p_c^{lk} = v(cv_1^{lk}, \ldots, cv_m^{lk}) \tag{10}$$

Afterwards, a proximity matrix PP_i between each e_i's preference relation and P_c is obtained:

$$PP_i = \begin{pmatrix} - & \ldots & pp_i^{1n} \\ \vdots & \ddots & \vdots \\ pp_i^{n1} & \ldots & - \end{pmatrix}$$

Proximity values $pp_i^{lk} \in [0, 1]$ are obtained for each pair (x_l, x_k) as follows:

$$pp_i^{lk} = 1 - \left| \frac{cv_i^{lk} - p_c^{lk}}{g} \right| \tag{11}$$

Proximity values are used to identify the furthest preferences from the collective opinion, which should be modified by some decision makers.

   (b) Identify preferences to be changed (CC): Pairs of alternatives (x_l, x_k) whose consensus degrees ca^l and cp^{lk} are not enough, are identified:

$$CC = \{(x_l, x_k) | ca^l < cr \wedge cp^{lk} < cr\} \tag{12}$$

Afterwards, the model identifies decision makers who should change their opinions on each of these pairs, i.e. those e_is whose assessment p_i^{lk} on $(x_l, x_k) \in CC$ is such that cv_i^{lk} is furthest to p_c^{lk}. To do so, an average proximity $\overline{pp}^{lk}$ is calculated, by using an aggregation operator λ:

$$\overline{pp}^{lk} = \lambda(pp_1^{lk}, \ldots, pp_m^{lk}) \tag{13}$$

As a result, decision makers e_i whose $pp_i^{lk} < \overline{pp}^{lk}$ are advised to modify their assessments p_{ij}^{lk} on (x_l, x_k).

(c) Establish change directions: Several direction rules are applied to suggest the direction of changes proposed to decision makers, in order to increase the level of agreement in the following rounds. An acceptability threshold $\varepsilon \geq 0$, which should take a positive value close to zero is used to allow a margin of acceptability when cv_i^{lk} and p_c^{lk} are close to each other.

- DIR.1: If $(cv_i^{lk} - p_c^{lk}) < -\varepsilon$, then e_i should *increase* his/her assessment p_i^{lk} on (x_l, x_k).
- DIR.2: If $(cv_i^{lk} - p_c^{lk}) > \varepsilon$, then e_i should *decrease* his/her assessment p_i^{lk} on (x_l, x_k).
- DIR.3: If $-\varepsilon \leq (cv_i^{lk} - p_c^{lk}) \leq \varepsilon$, then e_i should not modify his/her assessment p_i^{lk} on (x_l, x_k).

4 Application Example

In this section, a real-life GDM problem is solved by using a Web-based Consensus Support System that facilitates the implementation of the previous consensus model [7].

Let us suppose a city council compound by 40 politicians with different background $E = \{e_1, \ldots, e_{40}\}$, must make an agreed decision about defining a budget allocation related to a recent income from the national government. The investment options proposed are, X = {x_1:Introduction of a new tram line in the city center,x_2:Construction of an indoor shopping center,x_3:Expand green areas and parks,x_4:Improve leisure centers and sports facilities}: The information domains defined are:

- Numerical: $[0, 1]$
- Interval-valued: $I([0, 1])$
- Linguistic: $S = \{s_0 : null\ (N), s_1 : very\_low\ (VL), s_2 : low\ (L), s_3 : medium\ (M), s_4 : high\ (H), s_5 : very\_high\ (VH), s_6 : perfect\ (P)\}$.

Without loss of generality, the term set S is chosen as the common linguistic term set used to unify heterogeneous information, i.e. $S_T = S$. The group's attitude given by ϑ, φ and other CRP parameters are summarized in Table 1.

Once the problem is defined, the CRP begins, following the phases shown in Sect. 3 and Fig. 1:

Table 1 Parameters defined at the beginning of the CRP

Attitudinal parameters	Consensus threshold	Maximum #rounds	Accept. threshold
$\vartheta = 0.35$, $\varphi = 0.6$	$\mu = 0.85$	$Maxrounds = 10$	$\varepsilon = 0.1$

(1) *Determining group's attitude*: The group's attitude toward consensus is gathered by means of $\vartheta, \varphi \in [0,1]$ (see Table 1). Based on them, a RIM quantifier's parameters are computed as: $\alpha = 1 - \vartheta - \frac{\varphi}{2} = 0.35$ and $\beta = \alpha + \varphi = 0.95$, respectively, thus obtaining the following quantifier $Q(r)$ (see Fig. 2a):

$$Q(r) = \begin{cases} 0 & \text{if} \quad r \le 0.35, \\ \frac{r-0.35}{0.6} & \text{if} \quad 0.35 < r \le 0.95, \\ 1 & \text{if} \quad r > 0.95. \end{cases} \tag{14}$$

Eq. (4) is then used to compute a weighting vector W of dimension $\binom{40}{2} = 780$, thus defining an $Attitude - OWA_W(SIM^{lk}, 0.35, 0.6)$ operator.

(2) *Gathering preferences*: Decision makers provide their preferences, expressed in their domains. An example of preference relations expressed by three experts, e_s, e_r, e_t in different information domains is given below:

$$P_s = \begin{pmatrix} - & [.6,.8] & [.5,.7] & [.1,.4] \\ [.2,.4] & - & [.7,.9] & [0,.2] \\ [.3,.5] & [.1,.3] & - & [0,0] \\ [.6,.9] & [.8,1] & [1,1] & - \end{pmatrix}, \; P_r = \begin{pmatrix} - & .9 & .8, & .5 \\ .1 & - & .4 & 0 \\ .2 & .6 & - & .2 \\ .5 & 1 & .8 & - \end{pmatrix},$$

$$P_t = \begin{pmatrix} - & P & P & H \\ N & - & M & L \\ N & M & - & VL \\ L & H & VH & - \end{pmatrix}$$

(3) *Making Heterogeneous Information Uniform*: The unification scheme described in Sect. 2.1 is applied to decision makers' preferences obtaining fuzzy sets

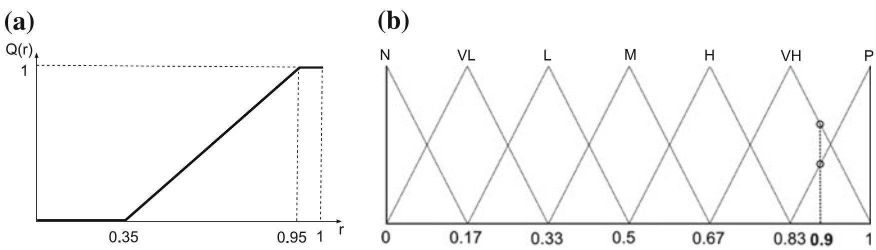

Fig. 2 **a** RIM quantifier defined upon attitudinal parameters $\vartheta = 0.35$, $\varphi = 0.6$. **b** Unification of heterogeneous information into fuzzy sets in S_T

Table 2 Global consensus degree for each round

Round 1	Round 2	Round 3	Round 4	Round 5
0.616	0.693	0.762	0.821	0.870

in S_T. For example, assessment $p_r^{12} = 0.9$ on (x_1, x_2) in the numerical preference shown above is unified into $(\gamma_{r0}^{12}, \ldots, \gamma_{r6}^{12})$=$(0, 0, 0, 0, 0, .59, .41)$ (see Fig. 2b).

(4) *Computing consensus degree*: The level of agreement is computed taking into account the Attitude-OWA operator defined above. A central value must be previously computed upon each unified assessment (see Eq. (5)).
Pairwise similarities are computed and aggregated, taking into account the Attitude-OWA operator defined above. Afterwards, the overall consensus degree is obtained as $cr = 0.616$.

(5) *Consensus control*: The global consensus degree, $cr = 0.616 < 0.85 = \mu$, therefore consensus achieved is not enough and the CRP must continue.

(6) *Advice generation*: Some recommendations are generated for each politician to modify his/her preferences and increase the level of collective agreement. Afterwards, the second CRP round begins.

In this problem, due to the moderately pessimistic attitude provided by the group, it was necessary to carry out a total of five rounds of discussion (see Table 2) to reach the consensus threshold $\mu = 0.85$.

The proposed consensus model allowed us to solve the GDM problem taking into account the attitude of decision makers toward consensus and giving them the possibility to use different information domains to express their preferences, which lead to make an agreed and highly accepted solution by the whole group.

5 Conclusions

Nowadays, new trends like e-democracy and social networks imply the participation in discussion processes of large groups of decision makers, who might have different backgrounds. Therefore, the use of heterogeneous information is common in GDM problems, where a high number of decision makers take part. In this contribution, we have presented a consensus model that deals with heterogeneous information and integrates the attitude of decision makers to achieve the consensus.

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