Eliciting Comparative Linguistic Expressions in Group Decision Making

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***Abstract*—The complexity and relevance of the real world decision making problems have made necessary to use multiple points of view to achieve a common solution by using the knowledge provided for a group of experts. Usually, this knowledge is vague and imprecise. In such cases, the use of linguistic information has provided successful results, although sometimes they are limited because of the linguistic models restrict the elicitation of the linguistic information to single linguistic terms. In qualitative settings there is a high degree of uncertainty, in which experts can hesitate among several linguistic terms to provide their preferences. Therefore, richer expressions than single linguistic terms might support experts in such hesitant situations and improve the preferences elicitation and decision results. In this contribution, it is proposed a new group decision making model able to manage complex linguistic expressions based on hesitant fuzzy linguistic term sets and context-free grammars. The proposed model defines the necessary operations and tools to deal with such linguistic expressions.**

I. Introduction

Decision making is a common process in human beings life, which can be the final outcome of some mental and reasoning processes that lead to the selection of the best alternative or set of alternatives among a group of them. Real world decision making problems with a high degree of uncertainty usually need multiple points of view to reach common solutions from the preferences provided by different experts. In the literature there are many approaches for group decision making (GDM) that model and manage the uncertainty by differ types of information [2], [6], [17], [20]. The use of linguistic modelling has provided successful and reliable results in such problems. Nevertheless, different researchers [9], [16] have pointed out that the linguistic approaches are limited because most of them deal with linguistic terms defined a priori and experts are constrained to express their preferences by using single linguistic terms that sometimes are not enough to reflect their knowledge because they need more elaborated expressions.

In [9], [16], [18] different linguistic approaches to elicit more elaborated expressions than single linguistic terms have been presented. Such approaches improved the preferences elicitation when experts hesitate among different linguistic terms, but the expressions generated are far from common language used by experts in real world. Additionally these approaches have been applied in multicriteria decision making but there is not any application either model for GDM problems, because of the complexity to deal with the expressions generated in linguistic preference relations.

Recently, it has been introduced the hesitant fuzzy linguistic terms set (HFLTS) [15] that not only provides a way to generate linguistic expressions richer than single linguistic terms, but also such expressions are quite flexible and close to experts' language because they are based on context-free grammars. Besides, a computational model to carry out processes of computing with words (CWW) [11], [12] with HFLTS was presented in [15].

Because of all previous linguistic approaches [15], [16], [18] dealing with complex linguistic expressions have been applied to multicriteria decision making, but not to GDM. In this contribution it is introduced a novel GDM model dealing with comparative linguistic expressions close to the common language used by experts involved in such problems. To develop this model a context-free grammar to generate comparative linguistic expressions suitable for expressing preferences in preference relations is defined together its representation by means of HFLTS. Finally to accomplish the processes of CWW, the 2-tuple computational model is applied.

This contribution is organized as follows. Section 2 reviews some basic concepts about GDM problems, linguistic information and the 2-tuple linguistic representation model. Section 3 presents the elicitation of comparative linguistic expressions. Section 4 introduces a linguistic GDM model that deals such linguistic expressions. Section 5 shows an illustrative example, and Section 6 concludes the paper.

II. Preliminaries

To better understand our proposal, this section reviews briefly some basic and necessary concepts about GDM, the fuzzy linguistic approach and the 2-tuple linguistic representation model.

A. Group Decision Making

Usually, a GDM is defined as a decision situation in which two or more experts that have their own knowledge and attitudes provide their preferences to achieve a collective decision [8]. The complexity and importance of decisions in real world lead to the necessity of taking into account different points of view in decision problems.

A GDM solving process applies a *selection process* to reach a common solution that obtains the best alternative or subset of alternatives according to experts' preferences. However, sometimes the aim of GDM is not to achieve the best solution but a satisfactory solution for all experts involved in the problem. In such a case, it is necessary to apply consensus reaching processes [14]. In this contribution, we focus on the *selection process* for GDM, because they are always necessary even for obtaining satisfactory solutions after consensus reaching processes.

A GDM problem is defined as a decision situation where a finite set of experts $E = \{e_1, \ldots, e_m\}$ $(m \geq 2)$, provides their preferences over a finite set of alternatives, $X = \{x_1, \ldots, x_n\}$, $(n \geq 2)$ to obtain a solution set of alternatives for the decision problem [7]. Each expert e_k, expresses her/his preferences over alternatives on X by means of a preference relation P^k, μ_{P^k} $X \times X \longrightarrow D$,

$$P^k = \begin{pmatrix} p_{11}^k & \cdots & p_{1n}^k \\ \vdots & \ddots & \vdots \\ p_{n1}^k & \cdots & p_{nn}^k \end{pmatrix}$$

where each assessment, $\mu_{P^k}(x_i, x_j) = p_{ij}^k$, represents the degree of preference of the alternative x_i over x_j according to expert e_k.

Decision problems are usually defined under uncertain and imprecise situations. In such cases, the use of linguistic information is suitable for modelling experts' preferences. This fact has led to the use of fuzzy linguistic approach [23] to model and manage this type of uncertainty.

B. Fuzzy Linguistic Approach

To model linguistically the information in the fuzzy linguistic approach is necessary to choose appropriate linguistic descriptors for the linguistic term set and their semantics. There are different approaches for such selections [4], [10]. One of the most common ways to choose the linguistic descriptors consists of supplying directly the term set by considering all the terms distributed on a scale in which a total order has been defined [21]. The semantics of the linguistic descriptors is represented by fuzzy numbers defined in the interval [0,1] described by membership functions [1].

A solving scheme of a GDM problem that manages linguistic information was proposed by Herrera and Herrera-Viedma [4]. It is compound in three phases:

- *Definition of syntax and semantics*: It establishes the domain of linguistic expression in which experts will provide their preferences about alternatives according to their knowledge and experience.
- *Selection of an aggregation operator for linguistic information*: A linguistic aggregation operator is chosen to aggregate the preferences provided by experts.
- *Selection of the best alternative*: The best alternative/s are chosen according to the preferences provided by experts. It is carried out by two phases.
  i) *Aggregation*: it aggregates the preference relations provided by experts by using the selected aggregation operator to obtain a collective preference relation.
  ii) *Exploitation*: it selects the best alternative as solution of the problem by applying a choice function to the collective preference relation.

Looking at this solving scheme for linguistic decision making, it is clear the need of using linguistic computational models that carry out processes of CWW to obtain accurate results. In our proposal, we will use the 2-tuple linguistic representation model [5], because it provides precise results and easy to understand.

C. 2-Tuple Linguistic Representation Model

The 2-tuple linguistic representation model was proposed to avoid the loss of information and improve the precision in processes of CWW [5].

This model represents the linguistic information by means of a pair of values called *2-tuple* (s,α), where s is a linguistic term and α is a numerical value representing the symbolic translation.

Definition 1: [5] The symbolic translation is a numerical value assessed in $[-0.5, 0.5)$ that supports the "difference of information" between a counting of information β assessed in the interval of granularity $[0, g]$ of the term set S and the closest value in $\{0, \ldots, g\}$ which indicates the index of the closest linguistic term in S.

This representation model defines a set of functions to facilitate the computational process with 2-tuples [5].

Definition 2: [5] Let $S = \{s_0, \ldots, s_g\}$ be a set of linguistic terms. The 2-tuple set associated with S is defined as $\langle S \rangle = S \times [-0.5, 0.5)$. The function $\Delta : [0, g] \longrightarrow \langle S \rangle$ is defined as follows,

$$\Delta(\beta) = (s_i, \alpha), \quad \text{with} \quad \begin{cases} i = \text{ round } (\beta), \\ \alpha = \beta - i, \end{cases} \tag{1}$$

where *round* assigns to β the integer number $i \in \{0, \ldots, g\}$ closest to β.

We note that Δ is bijective [5] and $\Delta^{-1} : \langle S \rangle \longrightarrow [0, g]$ is defined by $\Delta^{-1}(s_i, \alpha) = i + \alpha$.

Remark 1: The transformation between a linguistic term into a linguistic 2-tuple value consists of adding a value 0 as symbolic translation, $s_i \in S \Rightarrow (s_i, 0)$.

A linguistic computational model based on Δ and Δ^{-1} functions was also defined in [5] with the following operations:

1) *Comparison of 2-tuples*: Let (s_k, α_1) and (s_l, α_2) be two 2-tuples, with each one representing a counting of information:
   - if $k < l$ then $(s_k, \alpha_1) < (s_l, \alpha_2)$
   - if $k = l$ then
     a) if $\alpha_1 = \alpha_2$ then (s_k, α_1), (s_l, α_2) represents the same information
     b) if $\alpha_1 < \alpha_2$ then $(s_k, \alpha_1) < (s_l, \alpha_2)$
     c) if $\alpha_1 > \alpha_2$ then $(s_k, \alpha_1) > (s_l, \alpha_2)$

2) *Negation of a 2-tuple*: The negation operator over 2-tuples was defined as:

$$Neg((s_i, \alpha)) = \Delta(g - (\Delta^{-1}(s_i, \alpha)))$$

where $g+1$ is the cardinality of S, $S = \{s_0, ..., s_g\}$.
3) *Aggregation of 2-tuples*: The aggregation of information consists of obtaining a value that summarizes a set of values, therefore, the result of the aggregation of a set of 2-tuples must be a 2-tuple. There exists several 2-tuple aggregation operators [5].

III. Elicitation of Comparative Linguistic Expressions

Our interest is focused on decision situations where there is a high degree of uncertainty and experts hesitate among different linguistic terms to provide their preferences. In such situations, classical linguistic approaches are limited because they use single valued and predefined terms that restrict the richness of eliciting freely preferences to the experts. To avoid this limitation, different proposals have been introduced in the literature [9], [16], [18] to improve the flexibility of the linguistic expressions in hesitant situations.

- Tang and Zheng presented in [16] a linguistic approach that builds linguistic expressions by using linguistic terms S and logical connectives $\vee, \wedge, \neg, \rightarrow$, whose semantics are represented by fuzzy relations that describe the degree of similarity between linguistic terms.
- Wang and Hao introduced in [18] a linguistic model based on the proportions of two consecutive linguistic terms represented by 2-tuples. A proportional 2-tuple value is compound by a pair of 2-tuple in which the linguistic term represents the linguistic information and the numerical value establishes its proportion in the linguistic expression.
- Ma et al proposed in [9] a linguistic model to improve the flexibility of the linguistic expressions by using multiple linguistic terms that are integrated in "synthesized comments". But it does not provide a formalization to fix the syntax of the synthesized comments.

In spite of the previous proposals improve the flexibility to express the linguistic expressions in hesitant situations, they are far from the human beings cognitive model and/or do not have defined any formalization to generate the linguistic expressions.

Therefore, in this contribution it is considered another approach [15] based on HFLTS and context-free grammars. In this section it is presented a context-free grammar that generates typical expressions elicited by experts in GDM problems, and a computational model to deal with this type of information.

A. Context-free Grammar

Definition 3: Let G_H be a context-free grammar and $S = \{s_0, \ldots, s_g\}$ a linguistic term set. The elements of $G_H = (V_N, V_T, I, P)$ are defined as follows:

$V_N = \{\langle primary\ term\rangle, \langle composite\ term\rangle, \langle unary\ relation\rangle, \langle binary\ relation\rangle, \langle conjunction\rangle\}$

$V_T = \{lower\ than, greater\ than, at\ least, at\ most, between, and, s_0, s_1, \ldots, s_g\}$

$I \in V_N$

The production rules are defined in an extended Backus Naur Form so that the brackets enclose optional elements and the symbol | indicates alternative elements [1]. For the context-free grammar, G_H, the production rules are the following:

$P = \{I ::= \langle primary\ term\rangle | \langle composite\ term\rangle$

$\langle composite\ term\rangle ::= \langle unary\ relation\rangle\langle primary\ term\rangle | \langle binary\ relation\rangle\langle primary\ term\rangle\langle conjunction\rangle \langle primary\ term\rangle$

$\langle primary\ term\rangle ::= s_0|s_1|\ldots|s_g$

$\langle unary\ relation\rangle ::= lower\ than|greater\ than|at\ least| at\ most$

$\langle binary\ relation\rangle ::= between$

$\langle conjunction\rangle ::= and\}$

The set of expressions generated by the context-free grammar G_H defines the expression domain S_{ll}, that can be used to provide the preferences in a GDM problem.

B. Hesitant Fuzzy Linguistic Term Set and Computing with Words

These comparative linguistic expressions generated by G_H cannot be directly used for CWW, therefore, they are transformed into HFLTS by means of a transformation function.

Definition 4: [15] An HFLTS, H_S, is an ordered finite subset of consecutive linguistic terms of S, where $S = \{s_0, \ldots, s_g\}$ is a linguistic term set.

In [15] a transformation function E_{G_H}, was defined to obtain HFLTS from the comparative linguistic expressions.

Definition 5: [15] Let E_{G_H} be a function that transforms comparative linguistic expressions, ll, obtained by G_H, into HFLTS, H_S, where S is the linguistic term set used by G_H and S_{ll} is the set of linguistic expressions generated by G_H,

$$E_{G_H} : S_{ll} \longrightarrow H_S$$

The comparative linguistic expressions built by the context-free grammar G_H, are transformed into HFLTS by using the following transformations:

- $E_{G_H}(s_i) = \{s_i | s_i \in S\}$
- $E_{G_H}(\text{at most } s_i) = \{s_j | s_j \in S\ and\ s_j \leq s_i\}$
- $E_{G_H}(\text{lower than } s_i) = \{s_j | s_j \in S\ and\ s_j < s_i\}$
- $E_{G_H}(\text{at least } s_i) = \{s_j | s_j \in S\ and\ s_j \geq s_i\}$
- $E_{G_H}(\text{greater than } s_i) = \{s_j | s_j \in S\ and\ s_j > s_i\}$
- $E_{G_H}(\text{between } s_i \text{ and } s_j) = \{s_k | s_k \in S\ and\ s_i \leq s_k \leq s_j\}$

To facilitate the computations with HFLTS was defined the concept of envelope $env(H_S)$ of an HFLTS.

Definition 6: [15] The envelope of a HFLTS, $env(H_S)$, is a linguistic interval whose limits are obtained by means of its upper bound (*max*) and its lower bound (*min*):

$$env(H_S) = [H_{S^-}, H_{S^+}], \quad H_{S^-} \leq H_{S^+}$$

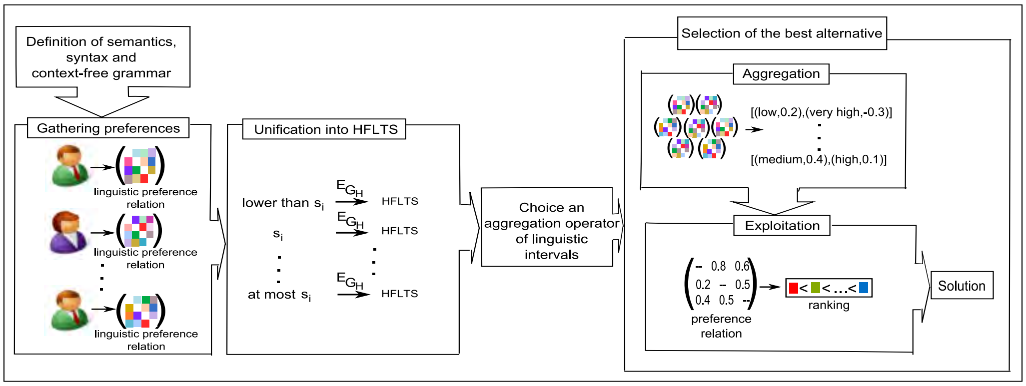

Fig. 1. Scheme of the proposed group decision making model

IV. A Linguistic Group Decision Making Model Based on Comparative Linguistic Expressions

In this section is proposed a linguistic GDM model that copes with hesitate situations in qualitative settings in which experts provide their preferences by using single linguistic terms or comparative linguistic expressions based on context-free grammar and HFLTS. The use of comparative linguistic expressions extends and modifies the linguistic solving scheme as follows (see Fig. 1).

1) *Definition of semantics, syntax and context-free grammar*
The use of linguistic information implies to define the syntax of a linguistic term set $S = \{s_0, \dots, s_g\}$ and its semantics. Nevertheless, the use of comparative linguistic expressions and HFLTS makes necessary to extend this phase to define a context-free grammar G_H as the one presented in Def. 3, that generates comparative linguistic expressions.
2) *Gathering of preferences*
Experts provide their preference relations P^k by using single linguistic terms or comparative linguistic expressions $\mu_{P^k} : X \times X \longrightarrow S_{ll}$,

$$P^k = \begin{pmatrix} p^k_{12} & \dots & p^k_{1n} \\ \vdots & \ddots & \vdots \\ p^k_{n1} & \dots & p^k_{nn} \end{pmatrix}$$

where each assessment $p^k_{ij} \in S_{ll}$, represents the preference degree of the alternative x_i over x_j according to expert e_k, expressed in the information domain S_{ll}..
3) *Unification of linguistic expressions into HFLTS*
To carry out the processes of CWW in the phase *Selection of the best alternative*, the linguistic expressions provided by experts are unified into a same domain. To do so, it is used the transformation function E_{G_H}, introduced in Def. 5 that transforms both single linguistic terms and comparative linguistic expressions into HFLTS.
Afterwards, it is computed an envelope for each HFLTS that obtains a linguistic interval that will be used to aggregate the preferences provided by experts, $env(H_S(p^k_{ij}) = [p^{k-}_{ij}, p^{k+}_{ij}]$. Therefore, each preference relation P^k, will be represented as follows,

$$P^k = \begin{pmatrix} [p^{k-}_{11}, p^{k+}_{11}] & \dots & [p^{k-}_{1n}, p^{k+}_{1n}] \\ \vdots & \ddots & \vdots \\ [p^{k-}_{n1}, p^{k+}_{n1}] & \dots & [p^{k-}_{nn}, p^{k+}_{nn}] \end{pmatrix}$$

4) *Selection of an aggregation operator for linguistic intervals*
To aggregate the linguistic intervals obtained in the previous phase, it is necessary to choice an appropriate aggregation operator $\varphi : [s_0, s_g]^n \rightarrow [s_0, s_g]$. Usually, it is necessary one or two aggregation operators to obtain a solution of the GDM problem depending on the resolution scheme.
5) *Selection of the best alternative/s*
It selects the best alternative or set of alternatives as solution of the GDM problem. It consists of two steps:
   i) *Aggregation of the preference relations represented by linguistic intervals*
   In this step the preference relations are aggregated to obtain a collective preference for each alternative. Taking into account that the preference relations are represented by linguistic intervals, such preferences are considered from two points of view, being the lower bound of the interval the pessimistic perception and the greater bound the optimistic one. To do so, a double aggregation process is carried out.
   - The right and left limits of the linguistic intervals are aggregated separately by using the aggregation operator φ chosen in the previos phase. The result will be two collective preference relations P^-_C and P^+_C represented by 2-tuple linguistic values that represent the collective pessimistic and optimistic perceptions of the aggregated preferences.

$$(s_r, \alpha)^+_{ij} = \Delta\left(\varphi(\Delta^{-1}(p^{k+}_{ij}))\right) \quad \forall k \in \{1, \dots, m\} \quad (2)$$

$$(s_r, \alpha)^-_{ij} = \Delta\left(\varphi(\Delta^{-1}(p^{k-}_{ij}))\right) \quad \forall k \in \{1, \dots, m\} \quad (3)$$

- The collective preference relations P_C^- and P_C^+ are aggregated by using an aggregation operator ϕ that may be the same as φ or not. The results are the pessimistic p_i^- and optimistic p_i^+ collective preferences for each alternative x_i.

$$p_i^+ = \Delta\,(\phi(\Delta^{-1}(s_r,\alpha)_{ij}^+)) \quad \forall j \in \{1,\ldots,n\} \tag{4}$$
$$p_i^- = \Delta\,(\phi(\Delta^{-1}(s_r,\alpha)_{ij}^-)) \quad \forall j \in \{1,\ldots,n\} \tag{5}$$

Afterwards, a collective linguistic interval is built for each alternative.

$$V^R = (p_1^R, \ldots, p_n^R) \tag{6}$$

where $p_i^R = [p_i^-, p_i^+]$ and $i \in \{1,\ldots,n\}$.

ii) *Exploitation*
In this step the vector of collective linguistic intervals for the alternatives is used to obtain a ranking of alternatives and select the best one/s. There are different approaches to order the alternatives [6], [19]. We will use the approach introduced by Wang et al. in [19] because it is suitable to order alternatives by using intervals.
Usually, a choice function is used to select the best alternative as solution of the GDM problem. In the literature has been proposed different approaches [3]. We will use a non dominance choice degree NDD, that indicates the degree to which an alternative is not dominated by the remaining ones [13].

V. Illustrative Example of the Proposed Model

Let us suppose a small company that wants to renew the computers of the sales employees $E = \{e_1, e_2, e_3\}$, and the manager of the company ask them their preferences to know which computer is better according to their requirements. The alternatives are the following ones $X = \{x_1 : PC, x_2 : Laptop, x_3 : Netbook, x_4 : Imac\}$. Due to the fact the employees are not experts in computer science, they might hesitate among different linguistic terms to provide their preferences. To facilitate the elicitation of their preferences, they can use comparative linguistic expressions.

To solve the defined GDM problem, we follow the phases of the proposed model.

1) *Definition of semantics, syntax and context-free grammar*
The linguistic term set defined is:
$S = \{neither(n), very\ low(vl), low(l), medium(m), high(h),$
$very\ high(vh), absolute(a)\}$
The context-free grammar used to generate the comparative linguistic expressions is the one presented in Def. 3.

2) *Gathering of preferences*
The preferences relations provided by the employees are the following ones:

$$P^1 = \begin{pmatrix} - & lower\ than\ l & vh & at\ least\ h \\ greater\ than\ h & - & between\ h\ and\ vh & h \\ l & at\ most\ vl & - & a \\ lower\ than\ l & l & vl & - \end{pmatrix}$$

$$P^2 = \begin{pmatrix} - & lower\ than\ m & h & between\ vl\ and\ l \\ between\ h\ and\ vh & - & h & vl \\ vl & l & - & h \\ at\ least\ h & vh & vl & - \end{pmatrix}$$

$$P^3 = \begin{pmatrix} - & h & between\ vl\ and\ l & h \\ vl & - & greater\ than\ m & at\ least\ vh \\ at\ least\ h & lower\ than\ l & - & vh \\ l & at\ most\ l & vl & - \end{pmatrix}$$

3) *Unification of linguistic expressions into HFLTS*
The linguistic preference relations provided by the employees are unified into HFLTS by means of the transformation function E_{G_H}:

$$P^1 = \begin{pmatrix} - & \{n,vl\} & \{vh\} & \{h,vh,a\} \\ \{vh,a\} & - & \{h,vh\} & \{h\} \\ \{l\} & \{n,vl\} & - & \{a\} \\ \{n,vl\} & \{l\} & \{vl\} & - \end{pmatrix}$$

$$P^2 = \begin{pmatrix} - & \{n,vl,l\} & \{h\} & \{vl,l\} \\ \{h,vh\} & - & \{h\} & \{vl\} \\ \{vl\} & \{l\} & - & \{h\} \\ \{h,vh,a\} & \{vh\} & \{vl\} & - \end{pmatrix}$$

$$P^3 = \begin{pmatrix} - & \{h\} & \{vl,l\} & \{h\} \\ \{vl\} & - & \{h,vh,a\} & \{vh,a\} \\ \{h,vh,a\} & \{n,vl\} & - & \{vh\} \\ \{l\} & \{n,vl,l\} & \{vl\} & - \end{pmatrix}$$

The envelopes for each HFTLS are the following ones:

$$P^1 = \begin{pmatrix} - & [n,vl] & [vh,vh] & [h,a] \\ [vh,a] & - & [h,vh] & [h,h] \\ [l,l] & [n,vl] & - & [h,h] \\ [n,vl] & [l,l] & [vl,vl] & - \end{pmatrix}$$

$$P^2 = \begin{pmatrix} - & [n,l] & [h,h] & [vl,l] \\ [h,vh] & - & [h,h] & [vl,vl] \\ [vl,vl] & [l,l] & - & [h,h] \\ [h,a] & [vh,vh] & [vl,vl] & - \end{pmatrix}$$

$$P^3 = \begin{pmatrix} - & [h,h] & [vl,l] & [h,h] \\ [vl,vl] & - & [h,a] & [vh,a] \\ [h,a] & [n,vl] & - & [vh,vh] \\ [l,l] & [n,l] & [vl,vl] & - \end{pmatrix}$$

4) *Selection of an aggregation operator for linguistic intervals*
For the sake of simplicity and without loss of generality, the aggregation operators used in the aggregation phase are the OWA and arithmetic mean based on 2-tuple [5].

5) *Selection of the best alternative/s*

i) *Aggregation of the preference relations represented by linguistic intervals*
Obtain the pessimistic and optimistic collective preference relations by using the OWA operator and the linguistic quantifier "most" [22].

$$P_C^- = \begin{pmatrix} - & (n,.24) & (m,.22) & (m,.22) \\ (m,.22) & - & (h,0) & (vl,-.33) \\ (l,-.16) & (n,.12) & - & (h,.06) \\ (l,-.44) & (l,-.38) & (vl,0) & - \end{pmatrix}$$

$$P_C^+ = \begin{pmatrix} - & (l,-.16) & (vh,-.22) & (h,-.44) \\ (h,-.06) & - & (a,-.33) & (h,-.06) \\ (l,-.04) & (l,.06) & - & (h,.06) \\ (l,-.04) & (l,.18) & (vl,0) & - \end{pmatrix}$$

Compute a pessimistic and optimistic collective preference for each alternative by using the arithmetic mean based on 2-tuple (see Table I).

TABLE I
PESSIMISTIC AND OPTIMISTIC COLLECTIVE PREFERENCE FOR EACH ALTERNATIVE

	PC	$Laptop$	$Netbook$	$Imac$
pessimistic	l,.21	m,.48	l,.01	vl,.39
optimistic	m,-.03	h,.22	l,.36	vl,.39

Build the interval vector for the alternatives (see Table II).

TABLE II
LINGUISTIC INTERVALS FOR EACH ALTERNATIVE

PC	$Laptop$	$Netbook$	$Imac$
[(l,.21),(m,-.03)]	[(m,.48),(h,.22)]	[(l,.01),(l,.36)]	[(vl,.39),(vl,.39)]

ii) *Exploitation*

By using the approach introduced by Wang et al is built a preference relation P_D.

$$P_D = \begin{pmatrix} - & 0 & 0.86 & 1 \\ 1 & - & 1 & 1 \\ 0.14 & 0 & - & 1 \\ 0 & 0 & 0 & - \end{pmatrix}$$

A non dominance choice degree is applied.

$$P_D^S = \begin{pmatrix} - & 0 & 0.72 & 1 \\ 1 & - & 1 & 1 \\ 0 & 0 & - & 1 \\ 0 & 0 & 0 & - \end{pmatrix}$$

$NDD_1 = 0$ $NDD_2 = 1$, $NDD_3 = 0$, $NDD_4 = 0$
The set of alternatives is ordered according to their non dominance degree.

$$x_2 > x_1 = x_3 = x_4$$

And the best alternative is

$$x_2 = [(m,.48),(h,.22)]$$

VI. CONCLUSIONS

This contribution focuses on linguistic GDM problems. Sometimes, the linguistic approaches restrict the elicitation of the linguistic information by using just one linguistic term. However, in hesitant situations, experts might hesitate among different linguistic terms to express their preferences and demand more elaborated linguistic expressions than single linguistic terms. In this contribution has been presented a GDM model that deals with comparative linguistic expressions based on context-free grammars and HFLTS. It carries out the processes of CWW has been used the 2-tuple computational linguistic model. Finally, an illustrative example has been solved to show its performance.

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