A Linguistic 2-Tuple Multicriteria Decision Making Model dealing with Hesitant Linguistic Information

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Abstract—**Decision making has become a core research area in different fields such as evaluation, engineering, medicine, etc. Usually, decision making problems are defined in contexts with vague and imprecise information. The use of linguistic modeling has provided successful results in decision making problems. However, most of the linguistic approaches are limited, because they restrict the elicitation of linguistic information to single linguistic terms and sometimes due to the lack of information, time or knowledge, decision makers hesitate among several linguistic terms to elicit their assessments and the use of only one linguistic term cannot reflect their assessments in a proper way. Therefore, more elaborated expressions than single linguistic terms might support decision makers in such hesitant situations and improve the elicitation of hesitant linguistic information. The use of hesitant fuzzy linguistic term sets (HFLTS), allows modeling this hesitation and facilitates the generation of comparative linguistic expressions similar to the expressions used by human beings in real world decision making problems using context-free grammars. There are different decision making models that deal with HFLTS, however they do not provide linguistic results as the computing with words scheme proposed to facilitate their comprehension. Therefore, the aim of this contribution is to present a multicriteria decision making model that not only improves the elicitation of hesitant linguistic information, but also obtains linguistic results easy to understand by decision makers. To achieve this latter goal the proposed model will make use of the linguistic 2-tuple model.**

I. Introduction

Decision making is a daily task in the human beings' life which consists of selecting the best alternative(s) among a set of possible alternatives. Real world decision making problems are usually defined in contexts where the information is vague, uncertain and imprecise. In such situations, decision makers may feel more comfortable expressing their knowledge by using linguistic terms that are closer to human beings' cognitive model. Fuzzy logic and fuzzy linguistic approach have provided tools to model linguistically this type of uncertainty by means of linguistic variables [25]. The use of linguistic information in decision making implies to carry out computations with words. Computing with words (CWW) [10], [26] is defined as a methodology able to reason, compute and make decisions using linguistic information similar to the natural language used by human beings. It follows a computational scheme [16], [18], [21] (see Fig. 1) in which input and output information should be linguistically expressed. Yager [22], [23] points out the importance in CWW of the *retranslation process* to obtain results in a understandable way for decision makers.

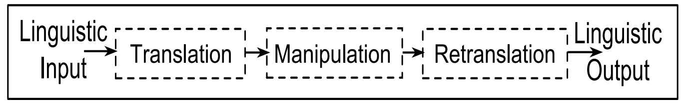

Fig. 1. Computing with words scheme

In spite of the use of linguistic information has provided successful results in decision making [2], [3], [6], [11], most of the linguistic approaches limit decision makers to express their assessments using only one linguistic term that sometimes it is not enough, because decision makers do not have sufficient knowledge or information about the problem and they hesitate among different linguistic terms to elicit their assessments. Therefore, more elaborated linguistic expressions than single linguistic terms are necessary to assess the alternatives and criteria in decision making problems. In the literature can be found different approaches [8], [17], [19], [20] that try to improve the elicitation of hesitant linguistic information, but the expressions generated by such approaches are not close to the expressions used by human beings in real world decision making problems or they do not formalize the generation of such expressions in a proper way.

Recently, Rodríguez et al. have introduced the concept of Hesitant Fuzzy Linguistic Term Sets (HFLTS) [13] which allows modeling the uncertainty provoked by hesitation and provides a way to generate comparative linguistic expressions richer than single linguistic terms and close to the human beings' cognitive model. In addition, such expressions are formally built by using context-free grammars based on HFLTS. This novel concept has been spread quickly [15] and many researchers have already proposed decision making models that use hesitant linguistic information [1], [7], [14], [19], [24]. In spite of such models improve the elicitation of hesitant linguistic information, they do not consider the retranslation process to obtain linguistic results easy to understand by human beings as it was indicated in Fig. 1.

Therefore, the aim of this contribution is to propose a multicriteria decision making (MCDM) model which deals with comparative linguistic expressions close to the common language used by decision makers involved in such problems and obtain linguistic intermediate and final results comprehensible by them. To do so, we will use the linguistic 2-tuple model [5] which has been applied in different fields and diverse applications [9], because it is simple and provides linguistic, precise and easy to understand results.

This contribution is organized as follows: Section 2 makes a brief introduction about decision making and CWW; the linguistic 2-tuple model which will be used to accomplish the CWW processes in the proposed decision making model and obtain comprehensible results; and introduces the elicitation of hesitant linguistic information by means of context-free grammars and HFLTS. Section 3 presents a novel MCDM model able to deal with hesitant linguistic information and obtain linguistic results easy to understand. Section 4 shows an illustrative example that is solved applying the proposed model, and finally some conclusions are pointed out in Section 5.

II. Preliminaries

This section revises some basic and necessary concepts about linguistic decision making, CWW, the linguistic 2-tuple model and the elicitation of hesitant linguistic information. All of them are necessary to understand our proposal of a linguistic 2-tuple MCDM model which deals with comparative linguistic expressions.

A. Linguistic Decision Making and Computing with Words

Usually, in real world decision making problems the information is vague and imprecise. Fuzzy logic and fuzzy linguistic approach [25] have improved the reliability and flexibility of the results by using linguistic information to model this type of uncertainty. A linguistic decision making problem consists of a finite set of alternatives $X = \{x_1, \dots, x_n\}$, in which a finite set of decision makers $E = \{e_1, \dots, e_m\}$, express their assessments using a linguistic term set $s_i \in S = \{s_0, \dots, s_g\}$.

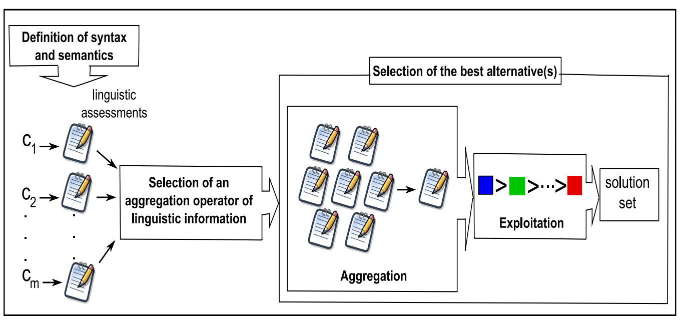

Fig. 2. Solving scheme for linguistic decision making

A solving scheme for linguistic decision making problems was proposed by Herrera and Herrera-Viedma [4]. It consists of three main phases as shows Fig. 2.

1) *Definition of syntax and semantics:* it is defined the linguistic expression domain in which decision makers provide their assessments about alternatives and criteria.

2) *Selection of an aggregation operator of linguistic information:* a linguistic aggregation operator suitable to aggregate the assessments provided by decision makers is chosen.

3) *Selection of the best alternative:* it consists of selecting the best alternative or subset of alternatives and it is divided into two steps:

i) *Aggregation:* it aggregates the assessments by using the selected aggregation operator to obtain a collective value for each alternative.

ii) *Exploitation:* it establishes a ranking of alternatives to select the best one as solution of the problem.

This scheme shows the necessity of carrying out CWW processes and obtaining linguistic results easy to understand according to the CWW scheme (see Fig. 1). Consequently, different linguistic computing models have been proposed [5], [20] with the aim of improving the accuracy of the decision solving processes and facilitating the comprehension of their results. One of the linguistic computing models more widely used in linguistic decision making is the linguistic 2-tuple model [5], because it keeps the fuzzy linguistic approach [12] and follows the CWW scheme shown in Fig. 1, in which the input and output information is linguistically expressed making it easy to understand by human beings. Following a brief review of the linguistic 2-tuple model is done to facilitate the comprehension of our proposal.

B. Linguistic 2-tuple Model

The linguistic 2-tuple model was proposed by Herrera and Martínez to improve the accuracy of the linguistic computations and avoid the loss of information [5], [9] keeping the CWW scheme [12] shown in Fig. 1. This model represents the linguistic information by means of a pair of values called *2-tuple* $(s_i, \alpha) \in \bar{S} = S \times [-0.5, 0.5)$, where $s_i \in S$ is a linguistic term and $\alpha \in [-0.5, 0.5)$ is a numerical value that represents the symbolic translation.

Definition 1: [5] The symbolic translation is a numerical value assessed in $[-0.5, 0.5)$ that supports the “difference of information” between a counting of information β assessed in the interval of granularity $[0, g]$ of the linguistic term set S, and the closest value in $\{0, \dots, g\}$ which indicates the index of the closest linguistic term in S.

This representation model defines the functions Δ and Δ^{-1} to facilitate the CWW processes [5].

Definition 2: [9] Let $S = \{s_0, \dots, s_g\}$ be a set of linguistic terms and $\beta \in [0, g]$ a value supporting the result of a symbolic aggregation operation. A linguistic 2-tuple value that expresses the equivalent information to β is obtained as follows.

$$\Delta : [0, g] \longrightarrow \bar{S}$$

$$\Delta(\beta) = (s_i, \alpha), \quad \text{with} \quad \begin{cases} i = \text{ round } (\beta), \\ \alpha = \beta - i, \end{cases} \tag{1}$$

being *round* the round operation, i the index of the closest label s_i, to β and α the symbolic translation.

We note that Δ is a bijective function [5] and $\Delta^{-1} : \bar{S} \longrightarrow [0, g]$ is defined by $\Delta^{-1}(s_i, \alpha) = i + \alpha$.

Remark 1: The transformation between a linguistic term into a linguistic 2-tuple value consists of adding a value 0 as symbolic translation, $s_i \in S \Rightarrow (s_i, 0) \in \bar{S}$.

Let us suppose that $\beta = 4.25$ is a value that represents the result of a symbolic aggregation operation on the linguistic term set $S = \{nothing, very\ bad, bad, medium, good, very\ good, perfect\}$. The linguistic 2-tuple value that represents the equivalent information to β is $(good, 0.25)$ (see Fig. 3).

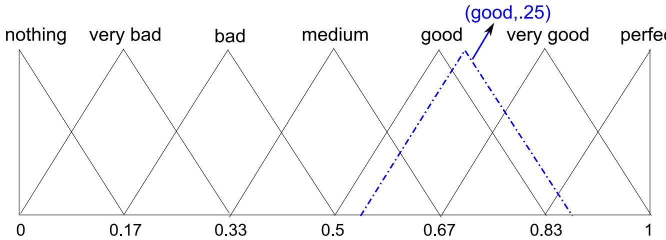

Fig. 3. Representation of a linguistic 2-tuple value

The linguistic 2-tuple model defined a computational model based on the functions Δ and Δ^{-1} and introduced a negation operator, several aggregation operators and the comparison between two linguistic 2-tuple values [5].

This model has been widely used in different applications [9], because it provides linguistic, precise and easy to understand results.

C. Elicitation of Comparative Linguistic Expressions: Hesitant Linguistic Information

In many decision making problems the information is vague and imprecise. In these situations, the use of linguistic information has provided successful results modeling this type of uncertainty. Nevertheless, most of the linguistic approaches limit decision makers to express their assessments by using only one linguistic term and sometimes it is not enough, because decision makers might hesitate among different linguistic terms due to the lack of information or knowledge about the problem. Therefore, it is necessary to generate more complex linguistic expressions than single linguistic terms that allow to reflect such hesitation. Different approaches [8], [17], [19], [20] have been introduced in the literature to improve the elicitation of hesitant linguistic information. Nevertheless, the linguistic expressions generated by such approaches are far from the natural language used by human beings to provide their opinions and/or they do not define any formalization to generate such expressions.

Therefore, in this contribution, it is considered another recent approach based on the concept of HFLTS [13] that models this type of hesitation and facilitates the generation of comparative linguistic expressions close to the human beings' cognitive model by using context-free grammars.

The following context-free grammar G_H, builds comparative linguistic expressions suitable to elicit assessments in decision making problems.

Definition 3: [7] Let G_H be a context-free grammar and $S = \{s_0, \ldots, s_g\}$ be a linguistic term set. The elements of $G_H = (V_N, V_T, I, P)$ are defined as follows:

$V_N = \{\langle primary\ term\rangle\ , \langle composite\ term\rangle\ \langle unary\ relation\rangle\ \langle binary\ relation\rangle\ , \langle conjunction\rangle\}$,

$V_T = \{at\ most,\ at\ least,\ between, and,\ s_0, \ldots, s_g\}$,

$I \in V_N$,

$P = \{I ::= \langle primary\ term\rangle | \langle composite\ term\rangle$

$\langle composite\ term\rangle\ ::=\ \langle unary\ relation\rangle\langle primary\ term\rangle | \langle binary\ relation\rangle\langle primary\ term\rangle\langle conjunction\rangle\langle primary\ term\rangle$

$\langle primary\ term\rangle ::= s_0|s_1|\ldots|s_g$

$\langle unary\ relation\rangle ::= at\ most|at\ least$

$\langle binary\ relation\rangle ::= between$

$\langle conjunction\rangle ::= and\}$.

The set of expressions ll, generated by the context-free grammar G_H, defines the expression domain $ll \in S_{ll}$.

Let us suppose the previous context-free grammar G_H, and the linguistic term set $S = \{nothing, very\ bad, bad, medium, good, very\ good, perfect\}$, some linguistic expressions might be the following ones:

$ll_1 = at\ most\ bad$

$ll_2 = between\ medium\ and\ very\ good$

In order to facilitate the CWW processes with these expressions, they are transformed into HFLTS by means of a transformation function.

Definition 4: [13] Let $S = \{s_0, \ldots, s_g\}$ be a linguistic term set, a HFLTS H_S, is defined as an ordered finite subset of consecutive linguistic terms of S:

$$H_S = \{s_i, s_{i+1}, \ldots, s_j\} \quad such\ that \quad s_k \in S, \quad k \in \{i, \ldots, j\} \tag{2}$$

For example, let ϑ be a linguistic variable and $S = \{nothing, very\ bad, bad, medium, good, very\ good, perfect\}$ be a linguistic term set, two different HFLTS might be,

$H_S(\vartheta) = \{very\ bad, bad\}$

$H_S(\vartheta) = \{good, very\ good, perfect\}$

A transformation function was defined to obtain HFLTS from comparative linguistic expressions.

Definition 5: [13] Let E_{G_H} be a function that transforms comparative linguistic expressions $ll \in S_{ll}$, obtained from a context-free grammar G_H, into HFLTS H_S, where S is the linguistic term set used by G_H, and S_{ll} is the set of linguistic expressions generated by G_H.

$$E_{G_H} : S_{ll} \longrightarrow H_S \tag{3}$$

E_{G_H} performance depends on the comparative linguistic expressions generated by the context-free grammar G_H. The transformations for the context-free grammar G_H, introduced in Def. 3 are as follows:

- $E_{G_H}(s_i) = \{s_i | s_i \in S\}$
- $E_{G_H}(\text{at most } s_i) = \{s_j | s_j \in S\ and\ s_j \leq s_i\}$
- $E_{G_H}(\text{at least } s_i) = \{s_j | s_j \in S\ and\ s_j \geq s_i\}$
- $E_{G_H}(\text{between } s_i \text{ and } s_j) = \{s_k | s_k \in S\ and\ s_i \leq s_k \leq s_j\}$

In order to facilitate the computations with HFLTS, a fuzzy envelope for HFLTS was proposed in [7] which represents the linguistic expressions by means of a fuzzy membership function obtained by the aggregation of the linguistic terms that compound the HFLTS.

Definition 6: [7] Let $H_S = \{s_i, s_{i+1}, \ldots, s_j\}$ be a HFLTS, so that $s_k \in S = \{s_0, \ldots, s_g\}$, $k \in \{i, \ldots, j\}$.

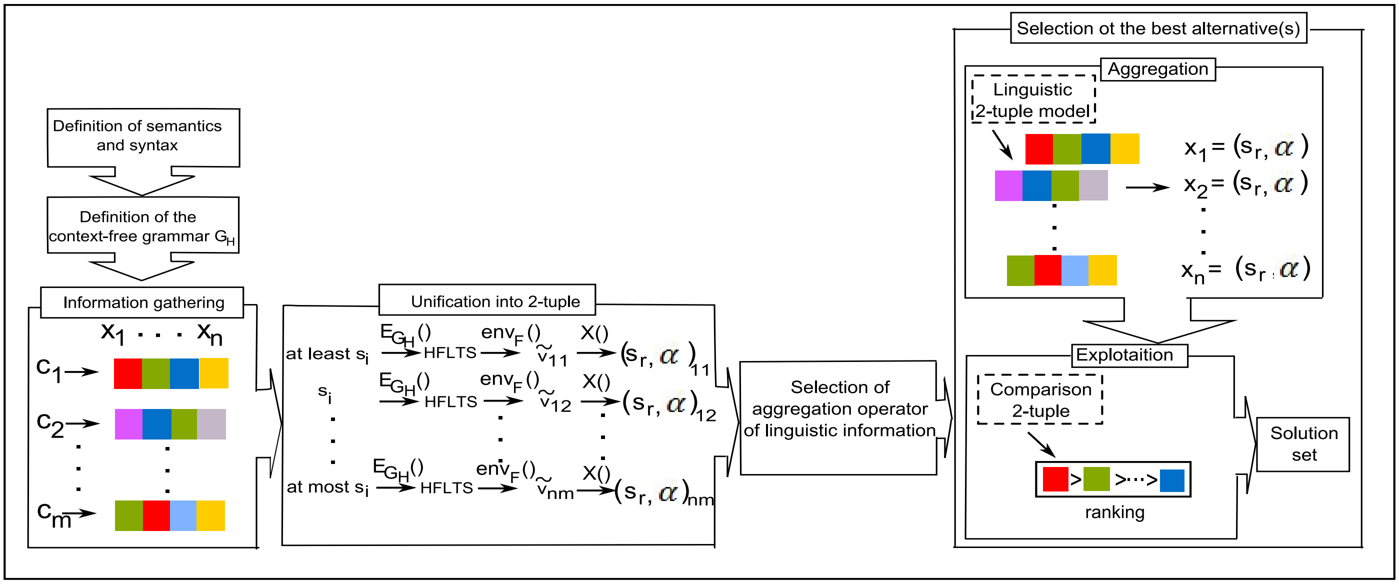

Fig. 4. Solving scheme for the proposed linguistic decision making model

$$env_F\left(H_S\right) = T(a,b,c,d), \tag{4}$$

being $T(\cdot)$ a trapezoidal or triangular fuzzy membership function (see [7] for further details).

III. A Novel Linguistic 2-Tuple Multicriteria Decision Making Model dealing with Comparative Linguistic Expressions

Despite the concept of HFLTS is quite new, it has been already used to propose different decision making models [1], [7], [14], [24]. However, these models do not follow the CWW scheme shown in Fig. 1 in which a retranslation process is necessary to obtain linguistic results easy to understand by decision makers involved in the decision problem. Therefore, this section presents a new MCDM model which copes with hesitant situations in qualitative contexts where decision makers can elicit their assessments by using single linguistic terms or comparative linguistic expressions based on a context-free grammar and HFLTS. This model does include a *retranslation process* by using the linguistic 2-tuple model to obtain linguistic intermediate and final results easy to understand.

Therefore, we extend the linguistic decision solving scheme revised in Fig. 2 by adding two new phases to deal with HFLTS and obtain linguistic results. Such phases are described below.

- *Definition of the context-free grammar G_H*: It defines the context-free grammar G_H utilized to generate the comparative linguistic expressions that will be used by decision makers to express their assessments in the decision making problem.
- *Unification into 2-tuple*: All the assessments are conducted into linguistic 2-tuple values in a linguistic term set S, to carry out the CWW processes through the linguistic 2-tuple model which follows the CWW scheme by means of the linguistic computational model introduced in Section II-B.

Before describing in further detail the proposed MCDM model and its phases, it is necessary to establish the notation of the elements of a MCDM problem dealing with HFLTS. Let $X = \{x_1, \ldots, x_n\}$, be a finite set of alternatives where each alternative is defined by means of a finite set of criteria $C = \{c_1, \ldots, c_m\}$ which will be assessed by using comparative linguistic expressions $ll_{ij} \in S_{ll}$ such that $i \in \{1, \ldots, n\}$, $j \in \{1, \ldots, m\}$ or single linguistic terms $s_k \in S = \{s_0, \ldots, s_g\}$. Hence, the six main phases of the linguistic 2-tuple MCDM model dealing with hesitant linguistic information are graphically shown in Fig. 4.

1) *Definition of semantics and syntax*

In this phase is chosen a appropriate linguistic term set $S = \{s_0, \ldots, s_g\}$ which will be used by decision makers to assess the criteria of the linguistic decision making problem. To do this, we have to study the granularity of the uncertainty, fix the syntax of the linguistic term set S, and define its semantics.

2) *Definition of context-free grammar*

Once, the syntax and semantics has been established, it is defined a context-free grammar G_H, to generate comparative linguistic expressions $ll_{ij} \in S_{ll}$. The definition of the context-free grammar will depend on the specific problem, therefore it is crucial to define in a proper way the elements of the context-free grammar $G_H = (V_N, V_T, P, I)$. The context-free grammar introduced in Def. 3 builds comparative linguistic expressions suitable for decision makers provide their assessments in a linguistic decision making problem.

3) *Information gathering process*

The decision makers provide their assessments over a set of criteria $C = \{c_1, \ldots, c_m\}$ for each alternative $X = \{x_1, \ldots, x_n\}$ using single linguistic terms or comparative linguistic expressions (see Table I).

TABLE I. ASSESSMENTS PROVIDED FOR THE MCDM PROBLEM

		criteria		
		c_1	$\dots$	c_m
alternatives	x_1	v_{11}	$\dots$	v_{1m}
	$\vdots$	$\vdots$	$\ddots$	$\vdots$
	x_n	v_{n1}	$\dots$	v_{nm}

where each assessment v_{ij}, represents the assessment for the alternative x_i and the criterion c_j expressed by means of single linguistic terms or comparative linguistic expressions.

4) *Unification into 2-tuple*

The assessments can be single linguistic terms or comparative linguistic expressions, therefore in order to carry out the CWW processes, it is necessary to conduct all the assessments into a unique domain. In this model all the assessments are unified into linguistic 2-tuple values to keep the CWW scheme obtaining linguistic intermediate and final results easy to understand. This phase is divided into three steps.

i) *Transformation into HFLTS*
The assessments are transformed into HFLTS by means of the transformation function $E_{G_H}(\cdot)$ introduced in Def. 5.

$$E_{G_H}(v_{ij}) = H_S(v_{ij})$$

where H_S is the HFLTS obtained according to the function $E_{G_H}(\cdot)$.

ii) *Computation of fuzzy envelope*
For each HFLTS is computed its fuzzy representation by $env_F(\cdot)$.

$$env_F(H_S(v_{ij})) = \tilde{v}_{ij}$$

being $\tilde{v}_{ij}$ a trapezoidal fuzzy membership function.

iii) *Obtaining of linguistic 2-tuple values*

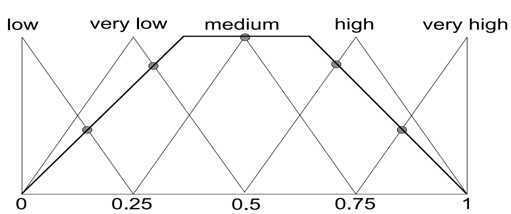

Fig. 5. Transformation of a fuzzy envelope into a linguistic 2-tuple value

The trapezoidal fuzzy membership function is transformed into a linguistic 2-tuple value by means of the function $\chi(\cdot)$ introduced in [9] that initially computes a fuzzy set of S in the fuzzy envelope and then computes its central value to obtain a linguistic 2-tuple value in S (see Fig. 5).

Definition 7: [9] Let $F(S)$ be a fuzzy set in S, the function $\chi : F(S) \to \bar{S}$ is defined as:

$$\chi(F(S)) = \Delta\left(\frac{\sum_{j=0}^{g} j\gamma_j}{\sum_{j=0}^{g} \gamma_j}\right) = \Delta(\beta) = (s_i, \alpha) \quad (5)$$

5) *Selection of an aggregation operator for linguistic information*

Taking into account that the assessments are represented by linguistic 2-tuple values and the computations are carried out by means of the linguistic 2-tuple model to obtain linguistic intermediate and final results easy to understand, in this phase an aggregation operator φ, based on 2-tuple is chosen to aggregate the set of criteria for each alternative. In the literature can be found different aggregation operators for 2-tuple [5].

6) *Selection of the best alternative*

The selection process finds the best alternative or set of alternatives as solution of the MCDM problem. It consists of two steps.

i) *Aggregation of linguistic information*
In this step the assessments represented by linguistic 2-tuple values are aggregated, by using the aggregation operator chosen previously, to obtain a collective value for each alternative.

$$(s_r, \alpha)_i = \Delta(\varphi(\Delta^{-1}(s_r, \alpha)_{ij})) \quad \forall j \in \{1, \dots, m\} \quad (6)$$

ii) *Exploitation*
Due to the collective values obtained for the alternatives are represented by linguistic 2-tuple values, in order to obtain a ranking of alternatives, it is used the comparison operator for linguistic 2-tuple values. Finally, it is selected the solution set of alternative(s) for the decision making problem as the alternative that maximizes the collective value.

$$X_{sol} = \{x_i \in X | i = \max_j\{(s_r, \alpha)_j\}\} \quad (7)$$

Once, the 2-tuple MCDM model that deals with hesitant linguistic information has been explained, an algorithm to solve MCDM problems defined in qualitative contexts is introduced.

a) Defining the semantics and syntax of the linguistic term set S.
b) Defining the context-free grammar G_H.
c) Gathering the assessments v_{ij}, $i = \{1, \dots, n\}$ and $j = \{1, \dots, m\}$.
d) FOR each row $i = \{1, \dots, n\}$ DO
  d.1) FOR each column $j = \{1, \dots, m\}$ DO

$$E_{G_H}(v_{ij}) = H_S(v_{ij})$$

  END FOR
END FOR
e) FOR each row $i = \{1, \dots, n\}$ DO
  e.1) FOR each column $j = \{1, \dots, m\}$ DO

$$env_F(H_S(v_{ij})) = \tilde{v}_{ij}$$

  END FOR
END FOR
f) FOR each row $i = \{1, \dots, n\}$ DO
  f.1) FOR each column $j = \{1, \dots, m\}$ DO

$$\chi(\tilde{v}_{ij}) = (s_r, \alpha)_{ij}$$

END FOR

END FOR

g) Selecting an aggregation operator of linguistic information φ.

h) FOR each row $i = \{1, \ldots, n\}$ DO

$$(s_r, \alpha)_i = \Delta(\varphi(\Delta^{-1}(s_r, \alpha)_{ij})) \quad \forall j \in \{1, \ldots, m\}$$

END FOR

i) Selecting the best alternative

$$X_{sol} = \{x_i \in X | i = \max_j\{(s_r, \alpha)_j\}\}$$

IV. Illustrative Example

Let suppose the manager of a company wants to develop a new variety of handbags and needs to select a material supplier. After preliminary screening, the manager has considered three possible alternatives $\{x_1, x_2, x_3\}$ to be assessed according to four criteria $C = \{c_1 = quality, c_2 = price, c_3 = business\ reputation, c_4 = reliability\}$. Due to the lack of information and knowledge about the decision making problem, the manager of the company might hesitate among several linguistic terms to elicit their assessments. In order to facilitate the elicitation of assessments, he/she can use comparative linguistic expressions based on a context-free grammar and HFLTS or single linguistic terms.

In order to solve this decision making problem, we follow the phases of the MCDM model proposed in Section III.

1) *Definition of semantics and syntax*

A linguistic term set S, appropriate for this problem is shown in Fig. 6.

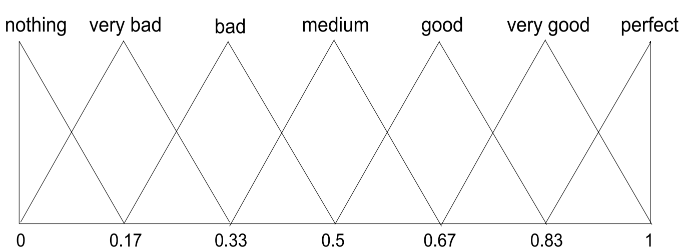

Fig. 6. Semantics and syntax of the linguistic term set S

2) *Defining the context-free grammar*

The context-free grammar used to generate the comparative linguistic expressions is the one introduced in Def. 3.

3) *Information gathering process*

The company's manager provides his/her assessments taking into account the available information about the three material suppliers. Table II shows the assessments provided over the criteria defined for each alternative.

TABLE II. Assessments provided by the manager

	c_1	c_2	c_3	c_4
x_1	*bt g and vg*	*vg*	*g*	*at least vg*
x_2	*m*	*bt m and vg*	*vg*	*at most m*
x_3	*vg*	*g*	*at least vg*	*at least g*

where bt stands for between.

4) *Unification into 2-tuple*

i) *Transformation into HFLTS*
The assessments elicited by the company's manager are transformed into HFLTS by means of the function $E_{G_H}(\cdot)$. Table III shows the comparative linguistic expressions and single linguistic terms unified into HFLTS.

TABLE III. Assessments unified into HFLTS

	c_1	c_2	c_3	c_4
x_1	$\{g, vg\}$	$\{vg\}$	$\{g\}$	$\{vg, p\}$
x_2	$\{m\}$	$\{m, g, vg\}$	$\{vg\}$	$\{n, vb, b, m\}$
x_3	$\{vg\}$	$\{g\}$	$\{vg, p\}$	$\{g, vg, p\}$

ii) *Computation of fuzzy envelope*
$\tilde{v}_{11} = T(0.5, 0.67, 0.83, 1)$
$\tilde{v}_{14} = T(0.67, 0.97, 1, 1)$
$\tilde{v}_{22} = T(0.33, 0.64, 0.7, 1)$
$\tilde{v}_{24} = T(0, 0, 0.35, 0.67)$
$\tilde{v}_{33} = T(0.67, 0.97, 1, 1)$
$\tilde{v}_{34} = T(0.5, 0.85, 1, 1)$

As the remaining assessments are represented by single linguistic terms, they are directly transformed into linguistic 2-tuple values by adding the value 0 as symbolic translation.

iii) *Obtaining of linguistic 2-tuple values*
The linguistic 2-tuple values are obtained by means of the function $\chi(\cdot)$. Table IV shows the assessments represented by linguistic 2-tuple values.

TABLE IV. Assessments represented by linguistic 2-tuple values

	c_1	c_2	c_3	c_4
x_1	$(vg, -0.5)$	$(vg, 0)$	$(g, 0)$	$(p, -0.16)$
x_2	$(m, 0)$	$(g, 0)$	$(vg, 0)$	$(b, 0.12)$
x_3	$(vg, 0)$	$(g, 0)$	$(p. -0.16)$	$(vg, 0.11)$

5) *Selection of an aggregation operator for linguistic information*

For the sake of simplicity and without loss of generality, the aggregation operator used in the aggregation phase will be the weighted mean based on 2-tuple [5]. The weighting vector is $W = (0.2, 0.2, 0.25, 0.35)$.

6) *Selection of the best alternative*

i) *Aggregation of linguistic information*
The assessments represented by linguistic 2-tuple values are aggregated to obtain a collective value for each alternative (see Table V).

TABLE V. Collective values for the alternatives

x_1	$(vg, -0.06)$
x_2	$(m, 0.4)$
x_3	$(vg, 0.05)$

ii) *Exploitation*

Finally, the solution set of alternatives is obtained.

$$X_{sol} = \{x_i \in X | i = \max_{j\in\{1,2,3\}} \{(vg, -.06)_1, (m, .4)_2, (vg, .05)_3\}\} =$$
$$= \{x_3\}$$

Therefore, the material supplier is,

$$\boldsymbol{x_3 = (vg, 0.05)}.$$

We can see that the final result represented by a linguistic 2-tuple value is a linguistic value close to the human beings' cognitive model and therefore it is comprehensible by the company's manager.

V. Conclusions

Due to the complexity of the decision making problems, decision makers might hesitate to express their assessments because they do not have enough knowledge about the problem. In such situations the use of just one single linguistic term is not appropriate to reflect their hesitation being necessary to use linguistic expressions more elaborated and flexible than single linguistic terms. Recently, it has been introduced an approach to facilitate the elicitation of hesitant linguistic information by means of context-free grammars and hesitant fuzzy linguistic term sets. Different decision making models have been already presented by using such a novel approach. However, they do not follow the computing with words (CWW) scheme introduced to obtain linguistic results easy to understand by decision makers. Since, in this contribution is presented a multicriteria decision making model able to deal with comparative linguistic expressions close to the natural language used by human beings in decision situations. This model uses the linguistic 2-tuple model to accomplish the CWW processes keeping the CWW scheme, therefore it obtains linguistic results comprehensible.

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