UNIVERSIDAD DE JAÉN

Departamento de Informática

Sistema Multiagente para modelar Procesos de Consenso en Toma de Decisión en Grupo a Gran Escala usando técnicas de Soft Computing

Memoria de Tesis presentada por

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para optar al grado de Doctor en Informática

Director: Dr. Luis Martínez López

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La memoria titulada *Sistema Multiagente para modelar Procesos de Consenso en Toma de Decisión en Grupo a Gran Escala usando Técnicas de Soft Computing*, que presenta D. Iván Palomares Carrascosa para optar al grado de doctor, ha sido realizada dentro del Programa de Doctorado en Ingeniería y Arquitectura de la Universidad de Jaén bajo la dirección del doctor D. Luis Martínez López. Para su evaluación, esta memoria se presenta como conjunto de trabajos publicados, acogiéndose y ajustándose a lo establecido en el punto 3 del artículo 23 del *Reglamento de los Estudios de Doctorado de la Universidad de Jaén*, aprobado en Febrero de 2012.

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Capítulo 1

Introducción

El presente capítulo constituye una introducción a la memoria de tesis doctoral titulada: *Sistema Multiagente para modelar Procesos de Consenso en Toma de Decisión en Grupo a Gran Escala usando Técnicas de Soft Computing*. El capítulo comienza con una breve introducción al área de investigación en la que se centra esta memoria, y la motivación para la investigación realizada. A continuación, se exponen los objetivos fijados para llevar a cabo dicha investigación, seguidos de la estructura que se seguirá en el resto de la memoria.

1.1. Motivación

La Toma de Decisiones es un proceso habitual en las actividades cotidianas de los seres humanos [8, 72]. A menudo, nos enfrentamos a situaciones en las que existen varias alternativas, y debemos decidir cuál de ellas es la mejor o cuál llevar a cabo. Los problemas de Toma de Decisión en Grupo (TDG), caracterizados por la participación de múltiples individuos o expertos con diferentes puntos de vista, han adquirido especial importancia e interés investigador en el área de la Toma de Decisiones en las últimas décadas [40, 54].

Tradicionalmente, los problemas de TDG se han resuelto aplicando únicamente un proceso de selección de alternativas [30], en el que cada experto proporciona sus preferencias sobre las alternativas, y se escoge la mejor alternativa o subconjunto

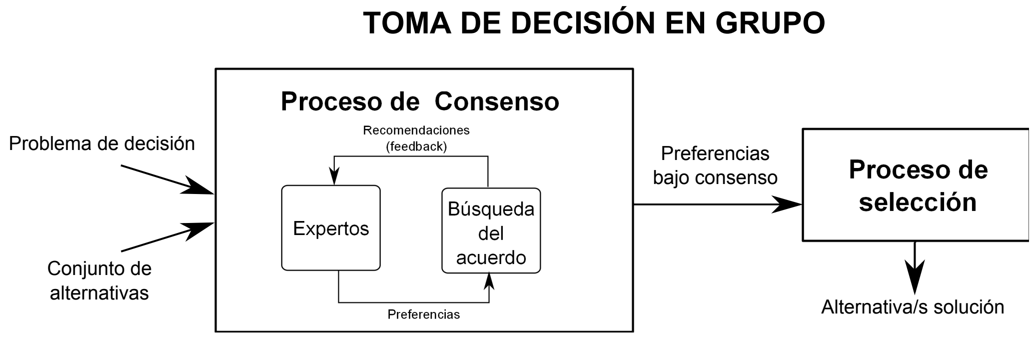

Figura 1.1: Proceso de resolución de problemas de TDG mediante consenso

de ellas. Este proceso de resolución no atiende al nivel de acuerdo existente entre los expertos según sus preferencias. Una consecuencia de ello es la posibilidad de tomar decisiones que no sean aceptadas como buenas por parte de algunos expertos, porque consideren que sus preferencias no han sido tenidas en cuenta. Por esta razón, el estudio de los *Procesos de Consenso* para alcanzar un acuerdo colectivo antes de tomar una decisión en grupo, se ha convertido en un importante tema de investigación dentro de la TDG. Los procesos de consenso se introducen como una nueva fase en el proceso de resolución de problemas de TDG (véase Figura 1.1). Se trata de procesos iterativos, compuestos por varias rondas en las que los expertos discuten y modifican sus preferencias, bajo la supervisión de una figura humana conocida como moderador, con el objetivo de acercar sus opiniones entre sí y alcanzar un alto nivel de acuerdo en el grupo [10, 57, 77].

Como resultado del estudio del consenso en TDG, en la literatura se han propuesto diferentes enfoques para dar soporte a procesos de consenso, tales como:

- Un gran número de modelos teóricos de consenso, que proporcionan las directrices necesarias para llevar a cabo procesos de consenso [9, 38, 60, 69, 77, 91].
- La definición de medidas de consenso, es decir, indicadores del nivel de acuerdo alcanzado entre los expertos [7, 32, 42]. Dichas medidas suelen estar basadas en el empleo de métricas de similitud y operadores de agregación [1, 2].
- El desarrollo de Sistemas de Apoyo al Consenso (SAC) basados en técnicas inteligentes [12,16,19,67], que implementan los modelos de consenso propuestos.

> Los SAC tienen como objetivo automatizar la labor del moderador humano de coordinar el proceso de discusión, eliminando así su posible parcialidad debida a factores subjetivos y, en ocasiones, permitiendo la celebración de reuniones no presenciales cuando los expertos están físicamente separados (por ejemplo, mediante el uso de tecnologías Web) [45, 46].

Clásicamente, los problemas de TDG que tienen lugar en la mayoría de organizaciones e instituciones, son realizados a un nivel estratégico, en el que únicamente un número reducido de personas se encarga de tomar la decisión (por ejemplo, los miembros directivos en un entorno empresarial) [10, 22, 23]. Sin embargo, la reciente evolución y creciente importancia de nuevos paradigmas y entornos tecnológicos en los últimos años, hace posible la participación de un mayor número de individuos en los procesos de toma de decisión. Algunos ejemplos de estos entornos y paradigmas son: las redes sociales [79, 81, 97], sistemas de democracia eléctronica o e-democracia [13, 49], sistemas de recomendación para grupos [59] y mercados electrónicos (*e-marketplaces*) para compras en grupo [11], entre otros. Como resultado, los llamados *problemas de TDG a gran escala*, en los que un gran número de expertos participa en el problema de decisión, se están convirtiendo en un importante tema de investigación a tener en cuenta, tanto en enfoques actuales de TDG y consenso, como en futuras propuestas en este ámbito [17, 85].

A pesar de los numerosos modelos y enfoques propuestos por diferentes autores para dar soporte a procesos de consenso en problemas de TDG, éstos se han centrado normalmente en problemas en los que participa un reducido número de expertos. Los resultados de investigación obtenidos en este ámbito hasta la fecha, no son suficientes al tratar con problemas de TDG a gran escala, ya que surgen nuevas dificultades y retos que requieren un estudio más profundo para la mejora de procesos de consenso en los que participa un elevado número de expertos. Algunos de estos retos y dificultades se describen a continuación:

- *Necesidad de Arquitecturas Escalables en Sistemas de Apoyo al Consenso*: La mayoría de SAC propuestos hasta la fecha se centran únicamente en tratar con un número de expertos bajo [12, 46, 103], por lo que las arquitecturas clásicas son suficientes para un desarrollo y puesta en práctica exitosa de dichos sistemas. Sin embargo, los problemas de TDG a gran escala requieren SAC basados en arquitecturas altamente escalables, que faciliten la gestión de grandes cantidades de información sobre las preferencias de los expertos. Por ello, es necesario proponer y desarrollar SAC basados en arquitecturas altamente escalables (por ejemplo, arquitecturas multiagente [88]), capaces de dar soporte a este tipo de problemas de TDG de manera efectiva.

- *Alto Coste en Supervisión de Preferencias*: En los procesos de consenso, a menudo los expertos deben revisar y modificar sus preferencias en cada ronda de discusión, con el fin de acercar sus opiniones a las del resto del grupo e incrementar el nivel de acuerdo [10, 77]. Cuando un grupo grande de expertos toma parte en el proceso de consenso, dicha revisión y modificación de preferencias podría suponer un mayor coste, en términos del tiempo invertido en alcanzar un acuerdo. Este aumento en el coste del proceso de discusión puede incluso provocar que algunos expertos terminen experimentando una pérdida de motivación e interés en el problema a abordar [64].

- *Individuos o Subgrupos con Comportamientos No Cooperativos*: Los procesos de consenso requieren que los expertos adopten una visión de cooperación mutua para alcanzar un acuerdo [77]. Sin embargo, la existencia de expertos o subgrupos de ellos que intentan manipular el proceso de discusión desviando la opinión colectiva a su favor, es frecuente en muchos procesos de consenso llevados a cabo durante la resolución de problemas de TDG reales [96]. Estos comportamientos no cooperativos suelen dificultar el alcance de consenso. Además, en problemas de TDG a gran escala, la existencia de subgrupos que presentan dichos comportamientos es más común, y la gestión de los mismos puede convertirse en una tarea compleja sin la ayuda de herramientas y enfoques adecuados para ello [65].

- *Entender el Estado Actual del Problema de TDG*: Disponer de una visión general sobre el estado del problema durante cada fase del proceso de consenso, basándose en las posiciones de los expertos según sus preferencias, podría ayudar a obtener conocimiento útil sobre el nivel de acuerdo entre expertos y el comportamiento de éstos. La información numérica y textual que proporcionan los modelos y SAC existentes, ha sido suficiente hasta ahora para obtener e interpretar el conocimiento sobre el estado del problema de TDG con facilidad [9, 103], debido al reducido número de expertos que normalmente participaba en dichos problemas. Sin embargo, la gran cantidad de información utilizada en problemas de TDG a gran escala, acentúa la necesidad de nuevas herramientas de apoyo basadas en la representación visual de la información, con el objetivo de hacerla más interpretable y permitir a los decisores monitorizar el estado actual del problema de manera sencilla [66].

- *Actitud de Grupo hacia el Consenso*: La actitud de los expertos hacia el consenso viene dada por la importancia que éstos dan a preservar sus preferencias individuales, en comparación con la importancia dada al objetivo de alcanzar un consenso. Conocer la visión o *actitud* de los expertos hacia el alcance de consenso en cada problema, es un aspecto importante a tener en cuenta para optimizar el proceso de consenso, adaptándolo a dicha actitud [63]. En el caso de problemas de TDG a gran escala, determinar y reflejar la actitud de grupo hacia el consenso en el proceso de discusión, a través de las medidas de consenso empleadas, puede ser una tarea compleja sin la ayuda de una medida adecuada para tener en cuenta dicha actitud.

La constante evolución y retos actuales encontrados en los problemas de TDG a gran escala, algunos de los cuales acabamos de describir, condujeron durante el comienzo de esta investigación a formular la siguiente hipótesis de partida:

Los modelos de consenso y SAC existentes no son capaces de satisfacer las necesidades actuales presentes en los problemas de TDG a gran escala: los procesos de consenso soportados por estos modelos y sistemas no son lo suficientemente flexibles para gestionar grandes grupos de expertos debido a múltiples factores, como se ha explicado anteriormente. Por esta razón, es necesario flexibilizar los procesos *de consenso mediante herramientas y enfoques adecuados para tal fin, facilitando así una efectiva gestión de las tareas y comportamiento de los expertos durante el proceso de discusión, y optimizando dichos procesos teniendo en cuenta la actitud de los expertos.*

1.2. Objetivos

Teniendo en cuenta los retos actuales de los procesos de consenso en TDG y la hipótesis expuesta en la sección anterior, el propósito inicial de esta investigación es el desarrollo de un SAC basado en el paradigma de sistemas multiagente, caracterizado por su alta escalabilidad y capacidad de computación distribuida [87, 88]. Dicho sistema permitirá implementar diferentes modelos de consenso, tanto nuevos como ya existentes, además de utilizar diversas técnicas de *soft computing* para la mejora y automatización de los procesos de consenso llevados a cabo en problemas de TDG a gran escala.

En base a este propósito inicial de SAC basado en una plataforma multiagente, nos planteamos los siguientes objetivos:

1. Desarrollo de un *modelo de autonomía semi-supervisada basado en agentes* [64], que permita un alto grado de automatización de las tareas realizadas por los expertos humanos, reduciendo así el coste temporal invertido en dichas tareas. El modelo debe permitir a los expertos humanos delegar en agentes inteligentes para que estos modifiquen sus preferencias de forma autónoma. Además, el modelo deberá solicitar supervisión humana en determinados casos en los que dicha supervisión sea conveniente, con el objeto de minimizar el coste global necesario para llevar a cabo el proceso de consenso, preservando al mismo tiempo la autoridad del experto humano en cierta medida.

2. Definición de mecanismos para *detectar y gestionar comportamientos no cooperativos en procesos de consenso* [65], que nos permitan el manejo de situaciones en las que un experto o subgrupo de expertos con intereses similares se nie-

guen a modificar sus opiniones iniciales para alcanzar un acuerdo de grupo, intentando desviar la opinión colectiva a su favor.

3. Desarrollo de una *herramienta gráfica de monitorización* [66], que facilite un análisis visual de las preferencias, y la evaluación de diversos aspectos, tales como posiciones de acuerdo o desacuerdo entre expertos y la presencia de individuos que no cooperan durante el proceso de consenso.

4. Integración de la *actitud de grupo hacia el consenso* [63] en el proceso de discusión. Para ello, tal actitud deberá ser reflejada en las medidas de consenso utilizadas para determinar el nivel de acuerdo alcanzado, con el fin de optimizar el proceso de consenso en función de la actitud adoptada por el grupo en cada problema en particular.

1.3. Estructura

Para alcanzar los objetivos que acabamos de plantear, y según lo establecido en el artículo 23, punto 3, de la normativa vigente para los Estudios de Doctorado en la Universidad de Jaén (Programa RD. 1393/2007), esta memoria de investigación será presentada como un conjunto de artículos publicados por el doctorando. Dichas publicaciones constituyen el núcleo de la tesis, y corresponden a cinco artículos científicos publicados en Revistas Internacionales indexadas por la base de datos JCR (*Journal Citation Reports*), producida por ISI (*Institute for Scientific Information*) además de otro artículo sometido a revisión en una Revista Internacional también indexada por JCR, en el momento de finalización de esta memoria.

Por tanto, la memoria se compone de un total de seis publicaciones, y se estructura en los siguientes capítulos:

- Capítulo 2: En él se presenta una revisión de TDG y consenso, haciendo un repaso de los conceptos básicos y antecedentes sobre problemas de TDG, modelado de preferencias, procesos de consenso, y una breve descripción de los enfoques existentes para dar soporte a dichos procesos: medidas de consenso, modelos de consenso y SAC. Para finalizar este capítulo, introduciremos una visión amplia sobre el consenso en TDG y una taxonomía que revisa y caracteriza un gran número de modelos de consenso existentes en la literatura, la cual será posteriormente presentada en la Sección 4.1, en el artículo titulado: *Consensus under a Fuzzy Context: Taxonomy, Analysis Framework and Experimental Case of Study.*

- Capítulo 3: Este capítulo presenta un resumen de la investigación realizada para alcanzar los objetivos planteados en esta memoria, y muestra una breve discusión de los resultados obtenidos en cada propuesta. Dichas propuestas son desarrolladas en los cinco artículos que se encuentran en las Secciones 4.2 a 4.6, y se organizan en torno a dos líneas de actuación principales (tal y como se explicará en el correspondiente capítulo): *Gestión Automatizada y Proactiva de Procesos de Consenso en TDG a Gran Escala* y *Gestión de la Actitud de Grupo hacia el Alcance de Consenso.*

- Capítulo 4: Este capítulo constituye el núcleo de la tesis doctoral, y contiene las seis publicaciones obtenidas como resultado de esta investigación.

- Capítulo 5: En este capítulo, se señalan las conclusiones y resultados más relevantes de la investigación realizada, así como las futuras líneas de investigación a seguir.

Por último, se añade un anexo que presenta un resumen en inglés de la memoria, para obtener la mención internacional de doctorado.

La memoria concluye con una recopilación bibliográfica de las contribuciones más destacadas en la materia estudiada.

Capítulo 2

Conceptos Teóricos y Antecedentes

En este capítulo, se revisa el contexto teórico y antecedentes necesarios para comprender la investigación que presentamos en esta memoria. Para ello, en primer lugar se introducen los conceptos y definiciones básicas sobre toma de decisión en grupo y consenso, seguidas de una descripción de los procesos de alcance de consenso. A continuación, se presentan de forma breve los principales tipos de enfoques existentes para dar soporte a grupos en procesos de consenso. Por último, se introducirá brevemente una visión general del consenso y una taxonomía de modelos de consenso existentes, cuyo artículo relacionado puede encontrarse en el Capítulo 4.

2.1. Toma de Decisión en Grupo

La Toma de Decisiones es una actividad inherente a los seres humanos en su vida cotidiana. Constantemente, debemos enfrentarnos a situaciones en las que existen varias alternativas y, en algunas ocasiones, hemos de decidir cuál de ellas es la mejor, o cuál debería llevarse a cabo. La Toma de Decisiones es un área que se ha aplicado en una amplia variedad de disciplinas, tales como: ciencias sociales, economía, ingeniería, planificación, medicina, psicología, etc. Como consecuencia de esta variedad de campos de aplicación, se han propuesto diferentes modelos de Toma de Decisiones, que han dado lugar a la llamada Teoría de la Decisión [8,54,71–73,76].

Los problemas clásicos de decisión presentan los siguientes elementos básicos:

1. Uno o varios objetivos por resolver.
2. Un conjunto de alternativas o decisiones posibles para alcanzar dichos objetivos.
3. Un conjunto de factores o estados de la naturaleza que definen el contexto en el que se plantea el problema de decisión.
4. Un conjunto de valores de utilidad asociados a los pares formados por cada alternativa y estado de la naturaleza.

Los procesos de Toma de Decisiones pueden tener lugar en diferentes situaciones, dependiendo del contexto en el que se define el problema de decisión:

1. *Ambiente de Certidumbre*: En esta situación, los valores de utilidad de las alternativas se conocen con exactitud.
2. *Ambiente de Riesgo*: Esta situación ocurre cuando el conocimiento que se tiene de cada alternativa se modela mediante una distribución de probabilidad.
3. *Ambiente de Incertidumbre*: En esta situación, no se dispone de conocimiento de naturaleza probabilística sobre las alternativas, por lo que los valores de utilidad de éstas deberán especificarse de forma aproximada.

La Teoría de la Decisión clásica proporciona un conjunto de métodos adecuados para tratar con problemas definidos en ambientes de certidumbre y riesgo. Sin embargo, dichos métodos no son adecuados para manejar problemas definidos bajo incertidumbre de naturaleza no probabilística, en los que la información sobre el problema es vaga e imprecisa [58]. Estas situaciones son también conocidas como problemas de toma de decisión en contextos difusos, o "Toma de Decisiones Difusa" [3]. La

Teoría de Conjuntos Difusos [52,99] y el Enfoque Lingüístico Difuso [100–102], propuestos por L.A. Zadeh, han demostrado ser un medio efectivo para el manejo de la incertidumbre en los problemas de decisión.

Los problemas de Toma de Decisiones pueden clasificarse según diferentes puntos de vista, siendo los dos siguientes algunos de ellos [54]:

- *Número de individuos o expertos*: Según el número de expertos que participan en el problema de decisión, tenemos problemas de Toma de Decisión individual y problemas de Toma de Decisión en Grupo [10,40].
- *Número de criterios*: Según el número de criterios o atributos que se han de valorar para cada alternativa, tenemos problemas de Toma de Decisión de un solo criterio y problemas de Toma de Decisión Multi-criterio [26,50,95].

En esta investigación nos centramos en problemas de decisión bajo incertidumbre donde participan varios expertos, más concretamente, en problemas de Toma de Decisión en Grupo (TDG). Tomar decisiones en grupo implica la participación de varios individuos, cada uno de ellos con sus propias motivaciones, ideas y actitudes, que han de tomar decisiones de forma colectiva, de cara a alcanzar una solución común a un problema. Un proceso de toma de decisión en el que participen varios individuos, donde cada uno de ellos aporta sus propios conocimientos y experiencia, dará como resultado, en ciertos ambientes, una decisión de mayor calidad que aquella aportada por un único experto. Formalmente, un problema de TDG se caracteriza por los siguientes elementos [40]:

- La existencia de un problema o cuestión común a resolver.
- Un conjunto X de *alternativas* o posibles soluciones al problema:

$$X = \{x_1, \ldots, x_n\}(n \geq 2) \tag{2.1}$$

- Un conjunto E de individuos o *expertos*, que expresan sus opiniones sobre el conjunto de alternativas X y que tienen la intención de alcanzar una solución en común al problema planteado.

$$E = \{e_1, \ldots, e_m\}(m \geq 2) \tag{2.2}$$

Cada experto debe utilizar una estructura de preferencia para representar su opinión sobre el conjunto de alternativas. Una de las estructuras más habituales en problemas de TDG bajo incertidumbre es la *relación de preferencia difusa* [62,68,84]. Dado un conjunto finito de alternativas X, una relación de preferencia difusa P_i asociada al experto e_i, viene dada por una matriz de dimensión $n \times n$ como sigue:

$$P_i = \begin{pmatrix} - & \ldots & p_i^{1n} \\ \vdots & \ddots & \vdots \\ p_i^{n1} & \ldots & - \end{pmatrix}$$

donde cada *valoración* $p_i^{lk} = \mu_{P_i}(x_l, x_k) \in [0,1]$ representa el grado de preferencia de la alternativa x_l sobre x_k, $l, k \in \{1, \ldots, n\}$, $l \neq k$, según el experto e_i, cumpliéndose:

- $p_i^{lk} > 0.5$ indica que e_i prefiere la alternativa x_l sobre x_k, y $p_i^{lk} = 1$ significa total preferencia de x_l sobre x_k.
- $p_i^{lk} < 0.5$ indica que e_i prefiere la alternativa x_k sobre x_l, y $p_i^{lk} = 0$ significa total preferencia de x_k sobre x_l.
- $p_i^{lk} = 0.5$ indica que x_l y x_k son indiferentes para e_i.

Otras estructuras de preferencia que han sido tenidas en cuenta por diversos investigadores en enfoques de TDG son: vectores de utilidad [9] y órdenes de preferencia [83], entre otros. Por otra parte, para el manejo de información incierta, los expertos pueden utilizar diferentes dominios de información para expresar sus preferencias sobre las alternativas, dependiendo de su área de conocimiento o experiencia [24]. Algunos dominios de información utilizados con frecuencia en problemas de TDG bajo incertidumbre son [36]:

- *Numérico* [9,62,104]: Las valoraciones vienen dadas por valores en [0,1] (como ocurre, por ejemplo, en las relaciones de preferencia difusas).

Figura 2.1: Proceso de selección para la resolución de problemas de TDG

- *Intervalar* [27,93]: Las valoraciones se representan mediante intervalos, $I([0, 1])$.
- *Lingüístico* [20, 29, 35, 55, 58]: Las valoraciones se expresan como términos lingüísticos $s_u \in S$, $u \in \{0, \ldots, g\}$, siendo $S = \{s_0, \ldots, s_g\}$ un conjunto de términos lingüísticos de granularidad g [100–102].

La solución a un problema de TDG se puede obtener aplicando un enfoque directo o bien un enfoque indirecto [30]. En un *enfoque directo*, la solución se obtiene a partir de las preferencias individuales de los expertos (sin obtener una opinión social o general antes de la resolución del problema), mientras que en un *enfoque indirecto* dicha solución se consigue determinando en primer lugar una opinión social o *preferencia colectiva*, y empleando dicha opinión para la obtención de la solución. Tal y como se observa en la Figura 2.1, independientemente del enfoque considerado, el proceso general para alcanzar una solución al problema de TDG se compone de dos fases [76]:

(i) *Fase de Agregación*: Se combinan las preferencias de los expertos.

(ii) *Fase de Explotación*: Consiste en obtener una alternativa o un subconjunto de alternativas que den solución al problema de decisión.

Además, dependiendo del problema al que nos enfrentemos, existirán diferentes situaciones cuando un grupo de individuos participa en un proceso de decisión, tales como: situaciones de colaboración entre expertos, de competitividad entre expertos, propuestas compatibles e incompatibles con uno o más expertos, e incluso propuestas que involucren a diferentes entornos (por ejemplo, entre compañías, gobiernos, etc.). Por esta razón, existen diferentes criterios clásicos que ayudan a resolver problemas de toma de decisión en grupo, basados en diferentes reglas para obtener la solución [10, 57]:

- *Regla de la Mayoría*: Se toma la decisión teniendo en cuenta la opinión de la mayoría de individuos que componen el grupo envuelto en el problema de decisión. Una vez adoptada la decisión de la mayoría, ésta debe ser respetada por las minorías del grupo, por lo que éstas no deben oponerse a la misma, ya que se asume que todos aceptan el uso de la regla. La noción de mayoría admite dos grandes modalidades de aplicación de la regla:

    1) *Mayoría absoluta*, cuando la opinión mayoritaria ha sido tenida en cuenta por más de la mitad del total de expertos.

    2) *Mayoría relativa o simple*, cuando solamente se requiere que la opinión mayoritaria haya sido la más numerosa en cuanto a expertos se refiere, aunque la suma del resto de expertos la supere.

- *Regla de la Minoría*: Se delega la toma de la decisión en un subgrupo de personas, ya que el problema requiere un nivel de experiencia que solamente presentan dichas personas. Es necesario que todos los expertos participantes acepten la regla y, por consiguiente, estén de acuerdo con delegar la toma de la decisión al subgrupo acordado.

- *Individual*: Esta situación se presenta cuando el grupo recurre a un experto para tomar la decisión o cuando existe un líder en el grupo.

- *Unanimidad*: Todos los miembros deben estar de acuerdo con la decisión tomada.

2.2. El Consenso en TDG: Procesos de Alcance de Consenso

En la mayoría de situaciones de TDG previamente descritas, puede ocurrir que al aplicar únicamente un proceso de selección de alternativas, algunos expertos no acepten la decisión tomada, porque consideren que sus opiniones no han sido tenidas en cuenta lo suficiente. Dado que en muchos problemas de TDG reales es necesario un alto nivel de acuerdo colectivo, surge la necesidad de aplicar un proceso de alcance de consenso o *proceso de consenso*, introduciendo una fase adicional en el proceso de resolución de problemas de TDG, con el objetivo de alcanzar un acuerdo global entre todos los expertos antes de tomar la decisión [10, 77].

La RAE[1] define el término consenso como el *acuerdo producido por consentimiento mutuo entre todos los miembros de un grupo o entre varios grupos*. Por su parte, en [77] Saint y Lawson definen el consenso como *un estado de acuerdo mutuo entre los miembros de un grupo, donde todas las opiniones e inquietudes de cada uno de los individuos han sido tenidas en cuenta para conseguir la satisfacción del grupo*. Estas definiciones asumen la idea de un proceso de TDG en el que ningún experto está en desacuerdo sobre las decisiones tomadas, aunque algunos expertos pueden seguir opinando que su solución preferida fuera mejor que la finalmente tomada. Para conseguir el acuerdo es necesario, pues, que *todos* los expertos cambien sus opiniones iniciales, tendiendo a aproximarlas entre sí, hacia una opinión colectiva considerada como satisfactoria por todo el grupo.

El concepto de consenso puede causar cierta controversia, ya que puede interpretarse de distintas formas, desde una visión clásica de consenso como acuerdo total (*unanimidad*) a otras interpretaciones más flexibles. El consenso como unanimidad [51] suele ser difícil o imposible de alcanzar en la práctica, o podría haber sido alcanzado mediante intimidación u otras circunstancias externas impuestas sobre el grupo, de forma que el acuerdo alcanzado no es verdadero (*consenso normati-*

[1]RAE (Real Academia de la Lengua Española): http://rae.es

vo) [86]. El consenso no debe entenderse como un acuerdo unánime, sino más bien como el resultado de un proceso de discusión tras el cual la decisión tomada podría no coincidir con las posiciones iniciales de los expertos. Esta visión de consenso es también conocida como *consenso cognitivo*, e implica que los expertos modifican sus opiniones iniciales tras varias rondas de discusión y negociación [57]. Basándose en esta idea, en la literatura se han propuesto algunos enfoques flexibles de consenso que consideran diferentes niveles de acuerdo parciales en el grupo [10,32,42]. Uno de los enfoques más aceptados para suavizar la visión clásica de consenso como unanimidad, es la noción de *soft consensus*, propuesta por Kacprzyk en [40]. Este enfoque está basado en el concepto de mayoría difusa, y establece que existe consenso en un grupo cuando *la mayoría de expertos importantes está de acuerdo en (sus testimonios concernientes a) casi todas las opciones relevantes* [41, 42]. Los conceptos de *soft consensus* y mayoría difusa están basados en la teoría de conjuntos difusos [99] y cuantificadores lingüísticos difusos [98]. Este enfoque ha proporcionado resultados satisfactorios en diferentes entornos de TDG [42,43,46].

El principal propósito de los procesos de consenso consiste en alcanzar un nivel de acuerdo mínimo antes de iniciar el proceso de selección de alternativas, mediante discusión de preferencias durante una o varias rondas [77]. Se trata, pues, de un proceso iterativo y dinámico, que suele estar coordinado por una figura humana: el *moderador*. El moderador es una figura clave en los procesos de consenso, y sus principales funciones son [57]:

- Evaluar el nivel de acuerdo alcanzado en cada ronda de consenso.
- Identificar las alternativas que impiden alcanzar el consenso deseado.
- Informar a los expertos sobre los cambios que éstos deben considerar sobre las preferencias en dichas alternativas.

Antes de iniciar el proceso de consenso, es fundamental que todos los expertos entiendan y acepten una serie de condiciones establecidas a priori [57]:

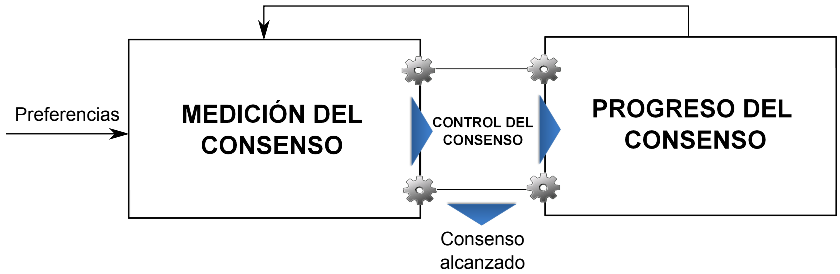

Figura 2.2: Esquema general de los procesos de consenso

- Todos los miembros del grupo *deben* entender el proceso llevado a cabo para alcanzar un acuerdo, clarificando cualquier posible duda o cuestión antes de comenzar el mismo.
- Aplicar un proceso de consenso implica que todos los expertos *aceptan* colaborar entre sí, con el objetivo de buscar una solución común mediante consenso.
- Cuando sea necesario, los expertos *deberán moverse* de sus posiciones iniciales, para acercar sus preferencias a las del resto del grupo.

La Figura 2.2 muestra un esquema general seguido por la mayoría de enfoques existentes en la literatura para realizar procesos de consenso. Sus principales fases son las siguientes:

(1) *Medición del consenso*: Se recopilan las preferencias de todos los expertos sobre X, P_i, $i \in \{1, \ldots, m\}$, para calcular el grado de consenso actual en el grupo mediante una medida de consenso, que determina cómo de próximas al acuerdo unánime están las opiniones de los expertos. Las medidas de consenso serán estudiadas con mayor detalle posteriormente en esta sección.

(2) *Control del Consenso*: Se comprueba el grado de consenso obtenido previamente para decidir si es suficiente o no. Si el nivel de acuerdo actual es suficiente, el grupo pasa al proceso de selección; de lo contrario, es necesario llevar a cabo otra ronda de discusión. En esta fase se pueden utilizar los dos parámetros siguientes, cuyos valores serán fijados por el grupo a priori:

- Un umbral de consenso μ, cuyo valor indica el mínimo nivel de acuerdo requerido en el grupo. Algunos modelos de consenso calculan el grado de consenso como un valor en el intervalo unitario [38, 43, 60, 68], de tal forma que un valor de 1 indica un acuerdo total y unánime entre todos los expertos, luego $\mu \in [0, 1]$ en estos casos.
- Un número máximo de rondas de discusión permitidas, $Maxround \in \mathbb{N}$. Si el número de rondas aplicadas excede este valor, el proceso de consenso finalizará sin haber alcanzado el nivel de acuerdo deseado.

(3) *Progreso del consenso*: Si el grado de consenso actual es insuficiente, se aplica un procedimiento para aumentar el nivel de acuerdo en la siguiente ronda del proceso. Tradicionalmente, dicho proceso ha consistido en proporcionar a los expertos una serie de recomendaciones o *feedback*, indicándoles cómo modificar sus preferencias. No obstante, también se han propuesto algunos enfoques para llevar a cabo este proceso de forma automática:

(a) *Generación de Recomendaciones (Feedback)*: Este es el proceso llevado a cabo normalmente en los procesos de consenso clásicos, en los que los expertos humanos discuten sobre sus preferencias, guiados por un moderador. El moderador identifica las valoraciones de expertos más alejadas del consenso en la ronda actual, y proporciona a dichos expertos una serie de recomendaciones para modificar el valor de estas valoraciones, con el fin de acercar sus opiniones a las del resto del grupo e incrementar el grado de consenso en la siguiente ronda. Numerosos modelos de consenso incorporan mecanismos de *feedback* basados en este proceso [9,12,38,60]. La Figura 2.3 ilustra un esquema general para procesos de consenso con mecanismo de *feedback*.

(b) *Actualizaciones Automáticas*: Algunos modelos de consenso propuestos no incorporan mecanismos de *feedback*, y en su lugar implementan enfoques que actualizan la información existente (normalmente valoraciones de los expertos) para aumentar el grado de consenso en el grupo automáticamente [4,28,89,90,92,104]. Así, una vez los expertos han proporcionado

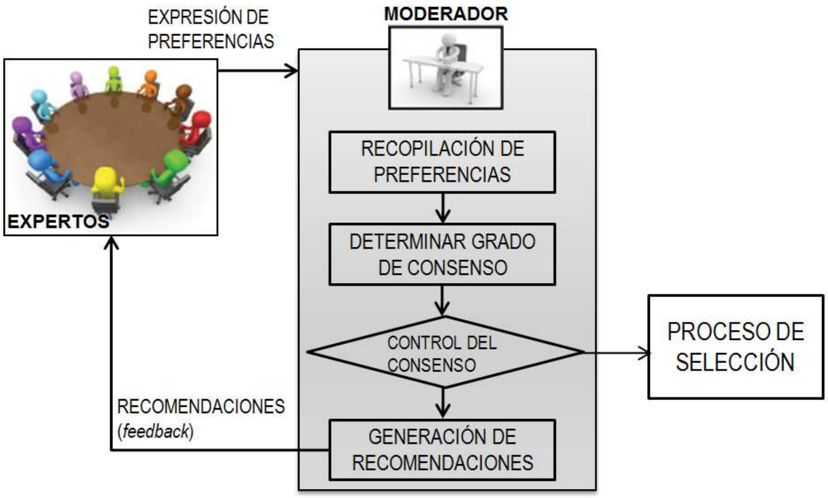

Figura 2.3: Esquema general de procesos de consenso basados en mecanismos de *feedback*

sus preferencias iniciales al comienzo del proceso de consenso, no será necesaria la supervisión de dichas preferencias tras cada ronda de discusión.

Como resultado de las investigaciones de los procesos de consenso realizadas dentro del área de la TDG en las últimas décadas, diferentes autores han propuesto un gran número de enfoques en la literatura, incluyendo:

i) Medidas de consenso [7, 15, 21, 32, 34, 42, 44, 48, 78, 82], es decir, medidas para calcular el nivel de acuerdo grupal a partir de las preferencias individuales de los expertos. Las medidas de consenso suelen estar basadas en el empleo de métricas de similitud o distancia para calcular el nivel de cercanía entre las preferencias de los expertos, así como en la utilización de operadores de agregación para obtener el nivel de acuerdo global en el grupo, mediante la agregación de los valores de similitud calculados previamente [1, 2].

ii) Modelos de consenso [9,33,37–39,47,60,68,69,77,91,92], que proporcionan a los grupos el soporte y directrices necesarias para realizar procesos de consenso, en problemas de TDG definidos en diferentes contextos. En la actualidad, existe una amplia variedad de modelos de consenso propuestos por múltiples investigadores para dar soporte a procesos de consenso en diferentes entornos de TDG, tales como: (i) entornos con un alto nivel de incertidumbre, que precisan de dominios basados en información lingüística apropiados para la expresión de preferencias [15,33,56,74,75], (ii) problemas de TDG en los que las alternativas deben ser evaluadas teniendo en cuenta múltiples criterios [69,92], y (iii) entornos en los que los expertos precisan del uso de diferentes estructuras de preferencia dependiendo de su grado de experiencia [38], entre otros.

iii) Sistemas de Apoyo al Consenso (SAC) [12,16,19,45,46,67,103], es decir, sistemas informáticos de apoyo a la decisión utilizados para llevar a cabo procesos de consenso, basándose en la implementación de diferentes modelos de consenso. Las principales ventajas que proporcionan los SAC son la automatización de las tareas asumidas por el moderador humano, y la posibilidad de celebrar reuniones no presenciales con la ayuda de los medios apropiados para ello, por ejemplo las tecnologías Web.

2.3. Taxonomía de Modelos de Consenso para TDG en Contextos Difusos

El objetivo de esta sección consiste en proporcionar una visión amplia de los antecedentes y conceptos básicos del consenso en TDG, necesarios para una mejor comprensión del resto de propuestas que se presentan en esta memoria de investigación. Dicha visión incluye una revisión de un gran número de modelos de consenso propuestos en la literatura, la cual puede encontrarse en uno de los artículos incluidos en la memoria (Capítulo 4).

El consenso se ha convertido en un importante tema de investigación en el área de la TDG: un gran número de modelos para dar soporte a procesos de consenso han sido propuestos por múltiples investigadores en las últimas décadas. Dada esta amplia variedad de modelos con diferentes características, y la necesidad de un marco de referencia para categorizar dichos modelos, proponemos una taxonomía de modelos de consenso para problemas de TDG en contextos difusos. Dicha taxonomía proporciona una visión general de un gran número de modelos de consenso, clasificándolos en diferentes categorías de acuerdo a sus principales características.

El artículo asociado a esta propuesta es (Sección 4.1):

- I. Palomares, F.J. Estrella, L. Martínez, F. Herrera, *Consensus under a Fuzzy Context: Taxonomy, Analysis Framework AFRYCA and Experimental Case of Study*. Information Fusion, sometido (2014).

Capítulo 3

Discusión de los Resultados

En este capítulo se presenta un resumen de las principales propuestas consideradas en esta memoria de investigación, poniendo de relieve los resultados obtenidos, así como las conclusiones extraídas de cada una de ellas. La discusión de resultados de esta investigación se organiza en torno a dos propuestas principales, las cuales a su vez se subdividen en varias partes:

1. *Gestión Automatizada y Proactiva de Procesos de Consenso en TDG a Gran Escala*. Esta propuesta se subdivide en dos partes:

    (a) *Modelo de Autonomía Semi-Supervisada de Agentes en un Sistema de Apoyo al Consenso basado en el Paradigma de Sistemas Multiagente.*

    (b) *Gestión de Comportamientos No Cooperativos en Procesos de Consenso con Grandes Grupos.*

2. *Gestión de la Actitud de Grupo hacia el Alcance de Consenso*. Esta propuesta se subdivide en dos partes:

    (a) *Medida de Consenso basada en un Operador que permite Reflejar la Actitud de Grupo.*

    (b) *Integración de la Actitud de Grupo hacia el Consenso en un Sistema de Apoyo al Consenso Web para TDG con Información Heterogénea.*

3.1. Gestión Automatizada y Proactiva de Procesos de Consenso en TDG a Gran Escala

En esta propuesta, se discuten las principales dificultades encontradas en los modelos y SAC actuales para el manejo de grandes grupos de expertos. Algunas de estas dificultades son: (i) la necesidad de supervisión constante por parte de los expertos humanos para revisar y modificar sus preferencias, lo que puede causar un excesivo coste temporal invertido en el proceso de consenso, y (ii) la presencia de expertos o subgrupos de ellos con intereses similares, cuyo comportamiento no contribuye a alcanzar un acuerdo en el grupo, ya que se muestran reacios a moverse de sus posiciones iniciales hacia el resto del grupo. Para vencer estas dificultades, proponemos los dos enfoques siguientes:

- Un modelo de autonomía semi-supervisada basado en agentes que minimice la cantidad de supervisión humana requerida en el proceso de consenso, y la integración de dicho modelo en un SAC con una arquitectura multiagente.
- Un modelo de consenso y una herramienta gráfica para la monitorización de preferencias de los expertos, que faciliten la detección y gestión de comportamientos no cooperativos en procesos de consenso con grandes grupos.

3.1.1. Modelo de Autonomía Semi-Supervisada de Agentes en un Sistema de Apoyo al Consenso basado en el Paradigma de Sistemas Multiagente

En este enfoque, se revisan brevemente las propuestas de modelos de consenso y SAC presentes en la literatura, haciendo hincapié en los logros actuales alcanzados en estos trabajos para dar soporte a procesos de consenso en problemas de TDG con pequeños grupos [5, 60, 69, 103]. En base a lo anterior, se destacan las limitaciones y debilidades que dichos trabajos presentan para manejar grandes grupos. Una de estas dificultades es la necesidad de SAC basados en arquitecturas altamente escalables y distribuidas, capaces de gestionar grandes cantidades de información sobre el problema de manera eficiente. Otra desventaja es la necesidad de supervisión constante de las preferencias por parte de los expertos humanos durante todo el proceso de discusión, lo cual a menudo conlleva un excesivo coste temporal invertido en revisar y modificar preferencias y, en ocasiones, la eventual pérdida de motivación e interés de los expertos en el problema a resolver [57].

Para afrontar estas dificultades, se propone un novedoso SAC semi-supervisado para problemas de TDG a gran escala, basado en una arquitectura multiagente. El paradigma de los sistemas multiagente se caracteriza por su alta escalabilidad y capacidades de computación distribuida [87, 88], facilitando computacionalmente el manejo de grandes cantidades de información asociadas a las preferencias de los expertos en dichos problemas [64], de ahí su elección como la tecnología utilizada en el sistema propuesto. El SAC incorpora un conjunto de agentes software con diferentes roles, responsables de llevar a cabo de manera autónoma las tareas clásicamente asumidas por el moderador humano en los procesos de consenso. Además, cada experto humano puede delegar en un agente software llamado agente experto, para la supervisión de sus preferencias. Dicho agente actúa en nombre del experto humano correspondiente, automatizando así sus tareas en buena medida. Los agentes se comunican entre sí mediante dos ontologías, intercambiando información sobre el problema a resolver, expresada bajo un lenguaje y semántica comunes [46, 80].

La principal novedad del sistema propuesto es un modelo de autonomía semi-supervisada de agentes, que minimiza la cantidad de supervisión requerida por los expertos para revisar y modificar sus preferencias. Dicha supervisión no es eliminada en su totalidad, ya que en ciertas circunstancias en las que el sistema propone cambios críticos en las valoraciones de los expertos, sería conveniente que el propio experto humano revise dichas propuestas de cambio y decida aceptarlos o no, en lugar de permitir al agente experto aplicar cambios directamente. De este modo, se preserva la autoridad del experto humano en cierta medida, a diferencia de como ocurre en algunas propuestas de modelos de consenso automáticos en la literatura, en las que dicha autoridad desaparece [90,92]. El enfoque semi-supervisado consta de dos componentes: (i) un conjunto de *perfiles de cambio* que implementan diferentes patrones adoptados por los agentes expertos para aplicar cambios en las valoraciones, inspirados en modelos de negociación de agentes como *Kasbah* [14]; y (ii) un conjunto de *reglas de supervisión*, que analizan las recomendaciones de cambio generadas para determinar en qué casos el agente software deberá solicitar supervisión humana. El modelo semi-supervisado ha sido integrado con un modelo de consenso para problemas de TDG basados en relaciones de preferencia difusas.

Se ha llevado a cabo un caso de estudio para mostrar los logros alcanzados usando el SAC semi-supervisado. Para ello, se ha resuelto un problema de TDG a gran escala dos veces, usando el sistema propuesto y otra versión del mismo que requiere una total supervisión por parte de los expertos humanos. Comparando los resultados obtenidos con ambos sistemas, se demuestra que tanto la cantidad de supervisión requerida como el número de expertos que necesitaron revisar alguna valoración en cada ronda del proceso de consenso, se ven significativamente reducidos con el sistema propuesto. Además, el SAC semi-supervisado contribuye a mejorar la convergencia hacia el consenso, que viene dada por el número de rondas de discusión necesarias para alcanzar el acuerdo deseado.

El artículo asociado a esta parte es (Sección 4.2):

- I. Palomares, L. Martínez, *A Semi-Supervised Multi-Agent System Model to support Consensus Reaching Processes.* IEEE Transactions on Fuzzy Systems, in press (2014). DOI:10.1109/TFUZZ.2013.2272588.

3.1.2. Gestión de Comportamientos No Cooperativos en Procesos de Consenso con Grandes Grupos

Como se ha dicho anteriormente, alcanzar un consenso implica que los expertos deben discutir y modificar sus preferencias iniciales, tendiendo a aproximarlas entre sí, hacia una solución colectiva que satisfaga a todo el grupo [77]. En esta parte, se estudia el problema de tratar con expertos que presentan un comportamiento no cooperativo en procesos de consenso, debido a que se nieguen a modificar sus opiniones para aproximarlas a las del resto del grupo. La presencia de individuos - o subgrupos de ellos - cuyo comportamiento no contribuye a alcanzar un acuerdo, es especialmente frecuente en problemas de TDG a gran escala: en grupos grandes, es habitual encontrar subgrupos o *coaliciones* de expertos con intereses similares. Algunas de estas coaliciones podrían decidir no modificar sus preferencias, o incluso modificarlas en contra del resto del grupo de forma coordinada, con el fin de desviar la opinión colectiva a su favor [65, 96]. Estos comportamientos pueden afectar negativamente al funcionamiento del proceso de consenso, ya que podrían dificultar significativamente alcanzar un acuerdo.

Dada la necesidad de detectar comportamientos no cooperativos y actuar en consecuencia para garantizar que el proceso de consenso no se vea afectado por los mismos, se propone una metodología para detectar y gestionar dichos comportamientos, y su integración en un modelo de consenso para TDG a gran escala. En dicho modelo, a cada experto se le asigna un peso de importancia, tal y como se ha propuesto en diversos enfoques existentes de TDG y consenso [54, 69, 92, 96].

La metodología propuesta para la gestión de comportamientos no cooperativos se aplica en cada ronda del proceso de consenso, utilizándose para ello una técnica de *clustering* difuso[1] [18, 70] basada en el algoritmo *Fuzzy C-Means* [6], para clasificar a los expertos en diferentes subgrupos, atendiendo a las similitudes entre sus preferencias. Se define además un conjunto de reglas basadas en análisis de clusters y lógica difusa [99], las cuales se aplican para detectar los posibles comportamientos no cooperativos de individuos y subgrupos. Una vez detectados los individuos no cooperativos, se aplica un proceso de actualización de los pesos asociados a estos en función de su comportamiento.

El modelo de consenso se ha implementado y utilizado para mostrar un ejemplo ilustrativo de su utilidad en la práctica. Los resultados del mismo muestran que la

[1] *Clustering* y *clustering difuso* son técnicas de aprendizaje no supervisado para la clasificación de datos en grupos, basándose en la similitud entre estos. La segunda de ellas se caracteriza por el empleo de la lógica difusa en el proceso de clasificación [18].

detección y gestión de comportamientos no cooperativos mediante el modelo propuesto, mejoran la convergencia del proceso de consenso, además de contribuir a obtener una solución que sea más ampliamente aceptada en el grupo.

Por otra parte, un análisis visual del proceso de consenso sería conveniente para facilitar la detección de expertos que no cooperan y visualizar su posición con respecto al resto del grupo. Los procesos de consenso clásicos llevados a cabo en grupos reducidos, han sido normalmente monitorizados mediante herramientas de apoyo basadas en información textual o numérica [9,103]. Sin embargo, la gran cantidad de información utilizada en problemas de TDG a gran escala hace necesario el uso de nuevas herramientas capaces de proporcionar información interpretable sobre el estado del proceso de consenso en cada ronda [66]. Por tanto, se propone también una herramienta gráfica de monitorización llamada MENTOR, para la visualización de preferencias en problemas de TDG a gran escala. MENTOR está basada en mapas auto-organizativos, una técnica de aprendizaje no supervisado ampliamente utilizada en visualización de datos, caracterizada por proyectar datos de alta dimensionalidad (tales como relaciones de preferencia difusas de los expertos) en un espacio de dimensión baja [53]. La herramienta de monitorización facilita la observación de diversos aspectos de interés en los procesos de consenso, tales como: posiciones de desacuerdo entre expertos, expertos que no cooperan, cardinalidad del acuerdo (dada por el número de expertos que presentan un alto nivel de acuerdo entre sí), etc. Por consiguiente, MENTOR constituye una útil herramienta complementaria para el modelo de consenso que acabamos de proponer. Se presenta un ejemplo de aplicación de la herramienta para ilustrar su utilidad, aplicando un proceso de consenso para la resolución de un problema de TDG a gran escala en el que participan varios subgrupos de expertos con diferentes comportamientos.

Los artículos asociados a esta parte son (Secciones 4.3 y 4.4):

- I. Palomares, L. Martínez, F. Herrera, *A consensus model to detect and manage non-cooperative behaviors in large scale group decision making.* IEEE Transactions on Fuzzy Systems, in press (2014). DOI:10.1109/TFUZZ.2013.2262769.

- I. Palomares, L. Martínez, F. Herrera, *MENTOR: A graphical monitoring tool of preferences evolution in large-scale group decision making.* Knowledge-based Systems, in press (2014). DOI:10.1016/j.knosys.2013.07.003.

3.2. Gestión de la Actitud de Grupo hacia el Alcance de Consenso

Además de la anterior propuesta para el manejo de comportamientos no cooperativos de expertos, también es necesario tener en cuenta la posible existencia de expertos sin una visión común sobre el problema de TDG considerado: no sólo la penalización de individuos no cooperativos nos permitirá mejorar la convergencia del proceso de consenso, sino que también integrar la actitud de grupo hacia el consenso en dicho proceso ayudará a optimizarlo. Por ello, en esta propuesta se estudia el problema de integrar la actitud de los expertos hacia el consenso. Esta propuesta se divide en dos partes:

- Medida de Consenso basada en un Operador que permite Reflejar la Actitud de Grupo.
- Integración de la Actitud de Grupo hacia el Consenso en un Sistema de Apoyo al Consenso Web para TDG con Información Heterogénea.

3.2.1. Medida de Consenso basada en un Operador que permite Reflejar la Actitud de Grupo

La investigación en esta parte se centra en el concepto de actitud de grupo hacia el consenso, es decir, la importancia que los expertos dan al alcance de consenso, con respecto a la importancia dada a preservar sus propias preferencias iniciales. A pesar de que resulta habitual que los expertos adopten diferentes actitudes en cada problema de TDG en el que participan (por ejemplo, optimista, pesimista o indiferente [63]), los modelos de consenso clásicos no han tenido en cuenta este aspecto de forma apropiada aún. En un problema de TDG a gran escala, en el que la existencia de subgrupos de expertos con diferentes visiones acerca del problema es más frecuente, conocer la actitud de los expertos hacia el alcance de consenso es una importante labor a tener en cuenta antes de iniciar el proceso de discusión.

Con el fin de optimizar el proceso de consenso adaptándolo a la actitud específica del grupo en cada problema a resolver, proponemos un método para integrar la actitud de grupo hacia el consenso en dicho proceso. Para ello, definimos un operador de agregación ponderado llamado *Attitude-OWA*, que extiende los operadores OWA (*Ordered Weighted Averaging*) [25, 94]. Dicho operador está basado en dos parámetros de actitud que indican la actitud de grupo, y en el empleo de un cuantificador lingüístico [98] para el cálculo de pesos a partir de dichos parámetros. A continuación, se define una medida flexible de consenso que utiliza el operador *Attitude-OWA* para agregar valores de similitud entre los expertos, obteniéndose los grados de consenso a partir de los mismos. Seguidamente, se extiende un modelo de consenso para TDG con relaciones de preferencias difusas que recoge ideas de [61,67], incorporando la medida de consenso definida e introduciendo una fase inicial en el proceso para determinar la actitud de grupo.

Se ha llevado a cabo una simulación experimental mediante la implementación del modelo propuesto en el prototipo de SAC basado en una plataforma multiagente propuesto en [64, 67]. El objetivo de dicha simulación es ilustrar los efectos de considerar diferentes actitudes de grupo en el funcionamiento del proceso de consenso. Los resultados de los experimentos realizados muestran que la convergencia hacia el consenso es mayor cuanto más optimista sea la actitud adoptada, debido al comportamiento flexible que presenta la medida de consenso utilizada, en función de dicha actitud. Finalmente, se proporcionan algunas directrices para que los grupos sean capaces de reflejar correctamente su actitud en las medidas de consenso, dependiendo de las necesidades específicas de éstos en cada problema de TDG.

El artículo asociado a esta parte es (Sección 4.5):

- I. Palomares, J. Liu, Y. Xu, L. Martínez, *Modelling experts' attitudes in group decision making.* Soft Computing, 16:10 (2012) pp. 1755-1766. DOI:10.1007/s00500-012-0859-8.

3.2.2. Integración de la Actitud de Grupo hacia el Consenso en un Sistema de Apoyo al Consenso Web para TDG con Información Heterogénea

Algunos aspectos adicionales que suelen requerir especial atención en TDG a gran escala son: (i) la presencia de expertos con diferentes perfiles, quienes pueden sentir predilección por expresar sus preferencias por medio de diferentes dominios de información, de acuerdo a su experiencia o área de conocimiento [36]; y (ii) la necesidad de SAC basados en teconologías Web, para hacer posible procesos de consenso ubicuos en aquellas situaciones en las cuales los expertos están físicamente separados y no pueden organizar reuniones presenciales. En esta parte nos centraremos en ambos aspectos, junto con el problema de integrar la actitud de grupo en el proceso de consenso (el cual ya se ha estudiado en la Sección 3.2.1).

Para abordar los aspectos descritos anteriormente, proponemos un modelo de consenso para TDG a gran escala en contextos heterogéneos. Sus principales características son, por un lado, un enfoque para manejar información heterogénea proporcionada por los expertos, y por otro, la integración de la actitud mediante una medida de consenso basada en el operador *Attitude-OWA* [63]. La metodología para tratar con información heterogénea [36] consiste en unificar las preferencias expresadas en diferentes dominios de información (numérico, intervalar y lingüístico), en un formato común utilizado para realizar las operaciones necesarias en el modelo de consenso. Una vez presentado dicho modelo, se ha desarrollado un SAC Web que implementa dicho modelo y automatiza totalmente las tareas del moderador humano, eliminando su posible subjetividad en el problema de TDG. Además, la interfaz Web del sistema facilita la celebración de reuniones no presenciales para llevar a cabo procesos de consenso ubicuos. Los expertos introducen sus preferencias a trávés de dicha interfaz, y reciben el *feedback* necesario para modificarlas a lo largo del proceso de consenso.

Una vez desarrollado el SAC Web, se ha utilizado para ilustrar su funcionamiento en la práctica. Para ello, se ha resuelto un problema de TDG a gran escala en el que cada experto escoge el dominio de información que desee para expresar sus preferencias. El problema ha sido resuelto varias veces con diferentes parámetros para la actitud de grupo, con el objetivo de remarcar los efectos de dicha actitud en la convergencia hacia el consenso.

El artículo asociado a esta parte es (Sección 4.6):

- I. Palomares, R.M. Rodríguez, L. Martínez, *An attitude-driven Web consensus support system for heterogeneous group decision making*. Expert Systems with Applications, 40:1 (2013) pp. 139-149. DOI:10.1016/j.eswa.2012.07.029.

Capítulo 4

Publicaciones

En virtud de lo establecido en el artículo 23, punto 3, de la normativa vigente para los Estudios de Doctorado de la Universidad de Jaén, correspondiente al programa RD. 1393/2007, en este capítulo se presentan las publicaciones que componen el núcleo de la presente tesis doctoral.

Dichas publicaciones se corresponden a cinco artículos científicos publicados en Revistas Internacionales indexadas por la base de datos JCR (*Journal Citation Reports*), producida por ISI (*Institute for Scientific Information*), además de un artículo sometido a revisión en una Revista Internacional también indexada por JCR en el momento de finalización de esta memoria.

4.1. Consensus under a Fuzzy Context: Taxonomy, Analysis Framework AFRYCA and Experimental Case of Study

- *Estado:* Sometido a revisión.
- *Título:* Consensus under a Fuzzy Context: Taxonomy, Analysis Framework AFRYCA and Experimental Case of Study.
- *Autores:* Iván Palomares, Francisco Javier Estrella, Luis Martínez y Francisco Herrera.
- *Revista:* Information Fusion.
- *ISSN:* 1566-2535.
- *Factor de Impacto (JCR 2012):* 2,262
    - *Cuartiles por Área de Conocimiento:*
        - **Cuartil 1** en *Computer Science, Artificial Intelligence*. Ranking 19/115.
        - **Cuartil 1** en *Computer Science, Theory & Methods*. Ranking 6/100.

Consensus under a Fuzzy Context: Taxonomy, Analysis Framework AFRYCA and Experimental Case of Study

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Abstract

Consensus reaching processes play an increasingly important role in the resolution of group decision making problems: a solution acceptable to all the experts participating in a problem is necessary in many real-life contexts. A large number of consensus approaches have been proposed to support groups in such processes, each one with its own characteristics, such as the methods utilized for the fusion of information regarding the preferences of experts. Given this variety of existing approaches in the literature to support consensus reaching processes, this paper considers two main objectives. Firstly, we propose a taxonomy that provides an overview and categorization of some existing consensus models for group decision making problems defined in a fuzzy context, taking into account the main features of each model. Secondly, the paper presents AFRYCA, a simulation-based analysis framework for the resolution of group decision making problems by means of different consensus models. The framework is aimed at facilitating a study of the performance of each consensus model, as well as determining the most suitable model/s for the resolution of a specific problem. An experimental study is carried out to show the usefulness of the framework.

Keywords: Group Decision Making, Consensus Reaching Process, Consensus Model, Consensus Support System, Consensus Measure.

1. Introduction

Decision making is a common process in daily life, characterized by the existence of several alternatives and the need to decide which one/s are the best or should be chosen as the solution to a problem. Group Decision Making (GDM) problems, in which several individuals or experts with different points of view take part in a decision problem with the aim of achieving a common solution, frequently occur in many organizations nowadays [1, 2]. Although decision problems may take place in different environments (certainty, risk or uncertainty), most real-life GDM problems are often defined in uncertain environments. Due to the difficulty of dealing with uncertainty of a non-probabilistic nature, which is mainly caused by the imprecision and vagueness

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of information, experts must express their preferences over alternatives by means of information domains that allow them to deal with such uncertainty. To do so, fuzzy modeling and linguistic information has been utilized in such situations [3, 4, 5].

Traditionally, GDM problems have been solved by applying an alternative selection process [6], in which the preferences of each expert over the alternatives are gathered and the best alternative or subset of alternatives is chosen [7]. This resolution scheme does not take into account the existing level of agreement between experts, therefore some experts may not accept the decision made because they might consider that their individual preferences have not been taken into account sufficiently [8, 9]. For this reason, Consensus Reaching Processes (CRPs) were introduced as an additional phase in the resolution of GDM problems [9]. In a CRP, experts discuss and modify their preferences, frequently coordinated by a human moderator, bringing their opinions closer to each other with the aim of increasing the level of agreement in the group.

Consensus has become a major research topic within the field of GDM. As a result, a large number of models and approaches to supporting CRPs have been proposed by several authors in the last few decades [10, 11, 12, 13, 14, 15, 16, 17]. The earliest proposals of consensus approaches were developed with the objective of reaching a full degree of agreement in the group, i.e. unanimity [18], which is normally difficult to achieve in practice. Therefore, more flexible notions of consensus in which different partial degrees of agreement can be obtained, have since been proposed [2, 19]. Consensus measures that are based on such flexible notions of agreement indicate how close experts' opinions are to unanimity. To do this, consensus degrees can be assessed in different ways, e.g. with numerical values in the unit interval [16, 20, 21], or linguistically [22, 23, 24, 25].

A large number of consensus models have been proposed for dealing with GDM problems in fuzzy contexts, therefore they may present a high variety of features, such as: (i) the type of consensus measures utilized to determine the level of agreement, based on the fusion of information about experts' preferences [19, 23, 26], (ii) the use of different mechanisms to guide the discussion process [27], or (i) the type of preference structures (e.g. preference relations, preference orderings, utility vectors, etc. [28]) or information domains (e.g. numerical or linguistic information [22, 29]) used by experts to express their preferences over alternatives, amongst others. Additionally, some models are focused on multiple criteria GDM problems (MCGDM) [29, 30], in which information fusion approaches are often utilized to combine preferences evaluated according to several criteria, whilst other models have been defined to deal with a particular type of real-life decision problems [10, 31].

Given this variety of existing consensus models, it would be desirable to have a clear characterization of them, with regard to the needs of each problem to be solved (type of preferences used by experts, necessity of giving the experts different importance weights, etc.), so that the most suitable models would be identified for solving such a problem. Moreover, some challenges are still present in the research topic of consensus, such as: (i) the large number of existing consensus models in the literature without a clear vision about which ones would be suitable for solving a specific type of GDM problem, and (ii) the lack of a frame of reference for the practical study of consensus models, which makes the analysis of their main features, their advantages and weaknesses, as well as comparisons amongst them, more difficult. Such a comparison would be particularly useful for evaluating new proposals of consensus models, in order to determine their main contributions with respect to other existing ones.

As a result of a thorough literature review on consensus approaches in a fuzzy context, in this paper we tackle two objectives: (i) proposing a taxonomy of existing works, and (ii) presenting an analytic framework called AFRYCA:

- We firstly present a taxonomy that provides an overview of a number of consensus models, with the main goal of providing a characterization of them, as well as pointing out the main characteristics of each proposal. The consensus models reviewed will be categorized into four groups, based on a double axis: (i) the use or not of feedback mechanisms to guide discussion, and (ii) the type of consensus measures applied (based on the method utilized for the fusion of information related to the preferences of the experts).
- Secondly, the paper introduces a prototype of simulation-based analysis framework called AFRYCA (A FRamework for the analYsis of Consensus Approaches). The framework has been developed to simulate the resolution of GDM problems by means of the different consensus models implemented in it. Therefore, its main purpose is to enable the analysis of the performance of each consensus model, as well as studying the results obtained by using different models for the resolution of a particular problem. AFRYCA has been implemented using Java and R technologies, and it incorporates several extendable modules and features, such as libraries that implement consensus models or patterns of expert behavior for its simulation, amongst others.

An experimental study is also presented to illustrate the usefulness of the analysis framework developed. For this, six consensus models of those reviewed in the taxonomy, have been implemented and used for the resolution of GDM problems.

The paper is organized as follows: in Section 2, some basic concepts regarding consensus in GDM are reviewed, together with some related works on consensus measures. Section 3 presents a taxonomy of consensus models. The analysis framework AFRYCA is presented in Section 4, followed by an experimental study that illustrates its usefulness in Section 5. Section 6 contains remarks on some of the lessons learnt and future directions in the use of AFRYCA. Finally, some conclusions are drawn in Section 7.

2. Background

In this section, we revise some basic concepts and approaches presented in the literature about GDM problems and consensus, in order to provide readers with a better understanding of the consensus models reviewed in the taxonomy presented in Section 3.

2.1. Group Decision Making Problems

A GDM problem can be formally defined as a decision situation where [1]:

(i) There exists a group of m individuals or *experts*, $E = \{e_1, \dots, e_m\}$, who each have their own knowledge and attitudes.
(ii) There is a decision problem consisting of n *alternatives* or possible solutions to the problem, $X = \{x_1, \dots, x_n\}$.
(iii) The experts try to achieve a common solution.

In a GDM problem, each expert $e_i \in E$, $i \in \{1, \dots, m\}$, expresses his/her preferences over alternatives in X, by means of a preference structure. One of the most common preference structures in GDM is the so-called preference relation [29]. A preference relation P_i associated to expert e_i can be represented, for X finite, as an $n \times n$ matrix as follows:

$$P_i = \begin{pmatrix} - & \dots & p_i^{1n} \\ \vdots & \ddots & \vdots \\ p_i^{n1} & \dots & - \end{pmatrix}$$

where each *assessment*, p_i^{lk}, represents the degree to which the alternative x_l is better than x_k, $l, k \in \{1, \dots, n\}$, $l \neq k$, according to e_i. Other preference structures that have been considered in some GDM approaches are utility vectors [32] and preference orderings [33, 34], amongst others.

Some problems are characterized by the existence of several attributes or criteria, $C = \{c_1, \dots, c_q\}$ (e.g. location, neighborhood and size, in a problem about buying a new house). In such situations, experts must assess alternatives according to each of these criteria, $c_y \in C$, i.e. a Multi-Criteria Group Decision Making (MCGDM) problem is defined [1].

GDM problems are often defined in environments of uncertainty, characterized by the existence of vague and imprecise information. Such situations are also known as GDM problems in fuzzy contexts or *fuzzy GDM problems* in the literature [3]. In order to deal with such uncertainty, experts may utilize different information domains to provide their preferences out of the existing alternatives, depending on their knowledge area or level of expertise in the problem. Some information domains frequently utilized in GDM problems under uncertainty are [35]:

- *Numerical* [36]: Assessments are represented as values in [0,1].
- *Interval-valued* [37]: Assessments are represented as intervals, $I([0, 1])$.
- *Linguistic* [38]: Assessments are represented as linguistic terms $s_u \in S$, $u \in \{0, \dots, g\}$, being $S = \{s_0, \dots, s_g\}$ a set of linguistic terms with granularity g.

The solution for a GDM problem can be derived by applying either a direct approach or an indirect approach [6]. In a *direct approach*, the solution is directly obtained from the individual preferences of experts, without constructing a social opinion first. In an *indirect approach*, however, a social opinion or *collective preference* (as it will be referred to in the rest of the paper) is determined a priori from individual opinions, and utilized to find a solution for the problem. Regardless of the approach considered, the classical alternative selection process for reaching a solution to GDM problems is composed of two phases [7], as shown in Figure 1:

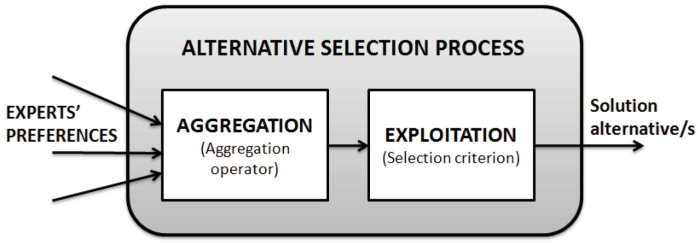

Figure 1: Selection process for the resolution of GDM problems

(i) *Aggregation phase*: the preferences of experts are combined, by using an aggregation operator.

(ii) *Exploitation phase*: This consists of obtaining an alternative or subset of alternatives as the solution to the problem, by means of a selection criterion.

2.2. Consensus in GDM: Consensus Measures and Related Works

The selection process for GDM problems described above does not guarantee the existence of agreement amongst experts before obtaining a solution to the problem. Therefore, it may be that such a solution is not accepted by some experts in the group, because they might consider that their individual opinions have not been taken into account sufficiently [8, 9, 39]. In many real-life GDM problems, obtaining a solution which is highly accepted by the whole group is crucial. In such cases, an additional phase called the consensus phase must be introduced into the resolution process for GDM problems [9]. This phase usually consists of a process of discussion and modification of preferences by experts, with the aim of reaching a high level of collective agreement (further detail regarding this process will be given in Sect. 2.3).

The concept of consensus has been interpreted from different points of view, from total agreement (unanimity), which is usually difficult to achieve in practice, to more flexible interpretations. In [9], Saint et al. defined consensus as "a state of mutual agreement among members of a group, where all legitimate concerns of individuals have been addressed to the satisfaction of the group". Kacprzyk et al. introduced the notion of *soft consensus*, based on the concept of fuzzy majority [2], which states that consensus exists when "most of the important individuals agree as to (their testimonies concerning) almost all of the relevant options" [19].

Flexible notions of consensus imply that it can be measured as different levels of partial agreement in the group, which indicate how far the opinions of experts are from unanimity. Therefore, the definition of appropriate *consensus measures*, which compute the current level of agreement in the group from the individual preferences of experts, has been an important subject of research within the field of consensus in GDM. A large number of consensus measures have been proposed by different authors in the literature [19, 24, 40, 41, 42]. Based on a literature review of different consensus measures proposed by several authors, we have classified them into two categories, depending on the type of computations and information fusion procedures applied to measure consensus:

1. *Consensus measures based on distances to the collective preference*: A collective preference, denoted as P_c, that represents the global opinion of the group is computed by aggregating all individual preferences of experts, P_i, i.e. $P_c = \phi\{P_1, \ldots, P_m\}$, with ϕ being an aggregation operator. Consensus degrees are then obtained by computing the distances between each individual preference and the collective preference, $d(P_i, P_c)$ [24, 40, 41].
2. *Consensus measures based on distances between experts*: For each different pair of experts in the group, (e_i, e_j), $i < j$, the degrees of similarity between their opinions are computed, based on distance metrics. Similarity values $L(P_i, P_j)$ are then aggregated to obtain consensus degrees [19, 22, 25, 42].

Figure 2 shows a general scheme of the computations carried out in both types of consensus measures described above. In the following subsections, some consensus measures belonging to each of these two categories are briefly reviewed.

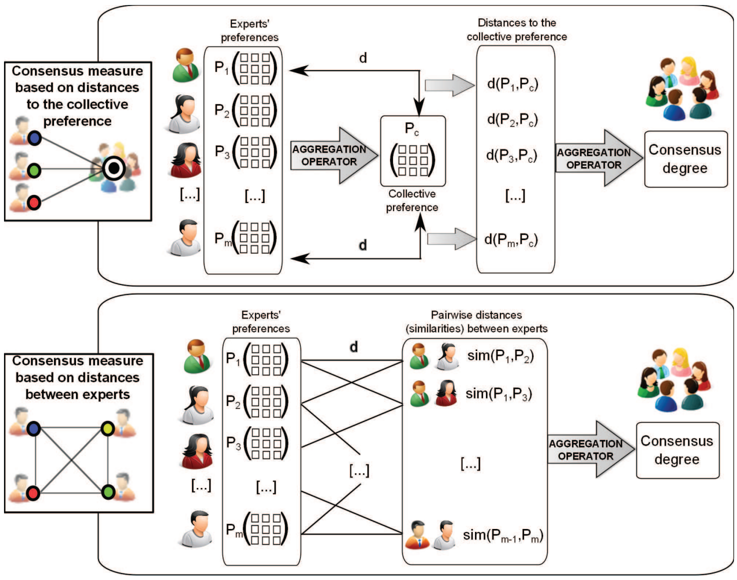

Figure 2: Types of consensus measures

2.2.1. Consensus measures based on distances to the collective preference

Spillman et al. proposed in [40] one of the earliest consensus measures based on mathematical procedures taken from fuzzy set theory [4], thus complying with a notion of consensus which is more flexible and realistic in practice than the idea of consensus as unanimous agreement, as considered in other early works [18]. In their proposal, Spillman et al. measure the degree of consensus for each expert separately, as the distance between his/her reciprocal fuzzy preference relation and an "ideal" consensus matrix with maximum consensus degree, determined a priori by means of matrix calculus. Another complementary measure is the fuzziness degree, whose value is larger if the consensus degree is lower and vice versa, which is also introduced and utilized as a criterion to quantify the level of group agreement.

One of the first consensus measures for linguistic preferences was presented by Herrera et al. in [24], assuming that experts might sometimes have a vague knowledge about the problem and they would prefer to use linguistic assessments instead of numerical ones. Alternatives and experts have fuzzy importance degrees, inspired by Kacprzyk's *soft consensus* approach [2, 19] (which will be revised in Section 2.2.2). Two different consensus measures are calculated: *consensus degrees*, which indicate the current level of agreement; and *linguistic distances*, used to evaluate the distance from each expert's linguistic preference relation to the collective opinion. Both measures are assessed linguistically, by means of linguistic terms s_u belonging to a finite term set $S = \{s_0, \ldots, s_g\}$ defined a priori, and they are calculated at three levels (using the LOWA operator [43] to aggregate information) by applying three steps sequentially: (i) a counting process, (ii) a coincidence process and (iii) a computing process [24].

In [23], Herrera et al. extended the consensus measures described above, by incorporating a process to control the consistency of preferences. The consistency control process is carried out before measuring consensus.

Ben-Arieh et al. studied in [44] the problem of aggregating linguistic preferences, expressed as fuzzy sets in a common linguistic term set by a group of experts who have associated linguistic importance weights. Firstly, they extended the Fuzzy-LOWA operator [41] to consider such importance weights in the aggregation of individual preferences into a collective preference. Then, they defined a consensus measure in which individual preference orderings and a collective preference ordering are compared. Such preference orderings are derived from their corresponding linguistic preferences. The degree of consensus C_l on an alternative x_l is computed as follows:

$$C_l = \sum_{i=1}^{m} \left[\left(1 - \frac{|O_i^l - O_c^l|}{n-1} \right) \times w_i \right] \tag{1}$$

with O_i^l and O_c^l being the ordered position of x_l, for expert e_i and the collective opinion respectively, and w_i the importance weight of e_i. The arithmetic mean operator is then used to compute the global consensus degree from all C_l, $l \in \{1 \ldots n\}$.

2.2.2. Consensus measures based on distances between experts

Kacprzyk et al. conducted extensive research into human-consistent measures of consensus that reflect the human perception of consensus in practice in a better way than consensus as unanimous agreement. As a result, they proposed the notion of *soft consensus*, based on the concept of fuzzy majority [2]. One of the first consensus measures for fuzzy preference relations based on this notion was formalized in [19]. The consensus degree is hierarchically computed at multiple levels, starting by α-degrees of sufficient agreement (with $\alpha \in [0, 1]$) between two experts (e_i, e_j) on a single assessment p_i^{lk}:

$$sim_{ij}^{lk} = \begin{cases} 1 & \text{if } |p_i^{lk} - p_j^{lk}| \leq 1 - \alpha \leq 1, \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

The concept of fuzzy majority is reflected in the consensus measures by applying a fuzzy logic-based calculus of linguistically quantified propositions [2, 45], taking into account the fuzzy importance weights assigned to experts and alternatives. The computation scheme of this "soft" consensus measure was slightly simplified in [42].

A different approach from *soft consensus* was taken into account by Szmidt and Kacprzyk in [46], where they extended the measures for fuzzy preference relations defined by Spillman et al. [40], and developed a consensus measure for reciprocal intuitionistic fuzzy preference relations. Consensus is computed as a scalar value in [0, 1], obtained from a consensus matrix of dimensions $m \times m$, in which each element cm_{ij} represents the degree of agreement between two experts e_i and e_j.

Herrera et al. proposed in [25] some consensus measures for linguistic GDM (linguistic consensus degrees and linguistic proximities, each one at three levels [24]), which pivot on determining degrees of fuzzy coincidence between pairs of experts, by means of a closeness measure between linguistic assessments. Different linguistic term sets can be used for the diverse elements of the GDM problem that are assessed linguistically, e.g. preferences, importance degrees of experts and alternatives, and consensus measures.

Another linguistic consensus measure was presented by Bordogna et al. in [22], being oriented towards MCGDM with linguistic preference matrices. This approach follows the concept of fuzzy majority, and it utilizes OWA operators [47] to aggregate preferences belonging to the different criteria. Such criteria are assessed linguistically by each expert. A linguistic consensus degree is computed for each alternative separately, based on degrees of agreement between pairs of experts.

Korshid et al. [48] presented a consensus measure based on coincidence between the positive and negative ideal degrees of agreement. Experts use linguistic terms to express their preferences by means of a vector of linguistic assessments. Such assessments are associated to triangular fuzzy numbers, and interval judgements are obtained by applying the α-cut operator [4] on fuzzy numbers, thus constructing an $m\times n$ fuzzy judgement matrix from the interval-valued assessments of all experts. Positive and negative agreement matrices are constructed taking into account similarities between pairs of experts, and then the relative closeness degrees to these two matrices are computed for each alternative.

Chen et al. defined in [49] a consensus measure for GDM problems with uncertain linguistic preference relations, with assessments given by uncertain linguistic terms expressed as $p_i^{lk} = [s_u, s_v]$, $s_u, s_v \in S$, $u \leq v$ [50]. They determine the similarity between two experts' assessments upon a deviation measure, $d(p_i^{lk}, p_j^{lk})$, and an overlapping measure, $o(p_i^{lk}, p_j^{lk})$, as follows:

$$sim_{ij}^{lk} = \gamma(1 - d(p_i^{lk}, p_j^{lk})) + (1-\gamma)o(p_i^{lk}, p_j^{lk}) \quad (3)$$

with $\gamma \in [0, 1]$ being the importance given to the deviation measure with respect to the overlapping measure, in the computation of similarity values. Consensus and proximity degrees are then computed at three levels. The Uncertain LOWA operator is utilized to aggregate uncertain linguistic preferences into a collective preference, which is necessary in order to calculate proximity degrees.

2.3. Consensus in GDM: Consensus Reaching Processes (CRPs)

As previously stated, reaching consensus normally implies that experts must modify their initial opinions over the course of a discussion process (i.e a CRP), bringing their positions closer to each other, towards a final collective opinion which satisfies the whole group [8, 9, 51].

Before initiating a CRP, it is important that some a priori assumptions are understood and accepted by the whole decision group [39]:

- Every member of the group *must* understand the process used to achieve an agreement, clarifying any possible doubts or questions before initiating it.
- Conducting a CRP implies that all experts *accept* the search for a common agreed solution, by means of collaboration.
- Experts *should move* from their initial positions, in order to make their preferences closer to each other.

A large number of consensus models have been proposed during recent decades [10, 11, 12, 13, 37, 15, 16, 17]. Consensus models provide groups with the necessary guidelines to support them in CRPs carried out in different GDM frameworks.

The process to reach consensus is iterative and dynamic. Such a process is often coordinated by a human figure known as *moderator*, who is responsible for supervising and guiding the discussion between experts [39]. A general CRP scheme followed by all consensus models revised in the taxonomy (see Section 3), is shown in Figure 3. Its main phases are described below:

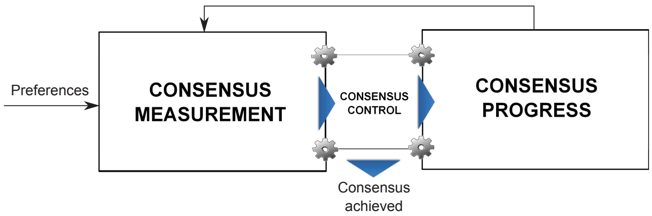

Figure 3: General CRP scheme

1. *Consensus Measurement*: Preferences of all experts, P_i, $i \in \{1, \ldots, m\}$, are gathered to compute the current level of agreement in the group, by using consensus measures (see Sect. 2.2).
2. *Consensus Control*: The consensus degree is compared with a threshold level of agreement μ, defined a priori. If the level of consensus desired has been achieved, the group moves on to the selection process; otherwise, it is necessary to carry out another round of discussion. In order to prevent an excessive number of discussion rounds, a parameter indicating the maximum number of rounds allowed, $Maxround \in \mathbb{N}$, can also be taken into account.
3. *Consensus Progress*: A procedure is applied in order to increase the level of agreement in the following round of the CRP. Traditionally, such a procedure has consisted of applying a *feedback generation* process, in which the moderator identifies the assessments of experts which are farthest from consensus and advises them to modify such assessments [9, 39]. Many existing consensus models incorporate feedback mechanisms based on this process [28, 27, 32, 52]. However, some other proposed models do not incorporate such mechanisms, and instead they implement approaches that update information (e.g. assessments of experts) to increase consensus in the group automatically [41, 53, 54].

3. A taxonomy of consensus approaches in a fuzzy context

In this section, we propose a taxonomy that reviews different consensus models proposed by a variety of authors to support CRPs in GDM problems defined in a fuzzy environment. The main goal of the taxonomy is to categorize such models, so that those with similar characteristics are grouped in the same category.

Figure 4 shows the structure of the taxonomy. In order to categorize the consensus models reviewed, we have considered two different kinds of criteria for constructing the taxonomy:

- *Feedback versus No Feedback*: Many consensus models define a feedback mechanism to support experts in the discussion and modification of their opinions. Such feedback mechanisms generate and provide experts with some advice, indicating to them how to modify their preferences in order to bring them closer to consensus, hence they must *supervise* this advice and decide whether to apply it or not [27, 28, 32, 52]. Some other consensus models do not consider the use of feedback mechanisms, but instead implement other types of mechanisms that *automatically* update the preferences and/or importance weights of those experts whose opinions are not close enough to the rest of the group, thus making the human intervention of experts unnecessary in these models [41, 53, 54].

- *Type of consensus measure*: A key element in all consensus models is the consensus measure utilized to compute the level of agreement in the group. As previously reviewed in

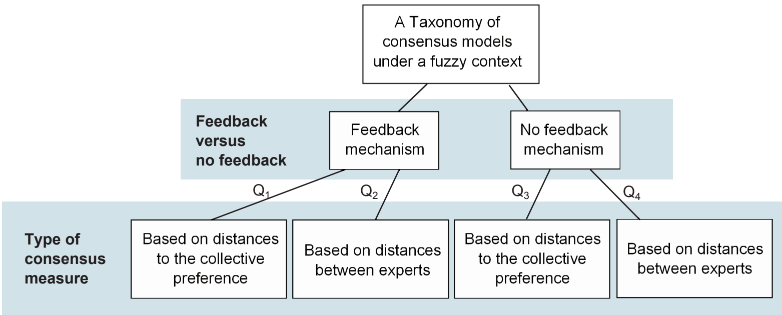

Figure 4: A taxonomy of approaches for consensus reaching

Table 1: Overview of consensus models reviewed in the taxonomy

	Consensus measure based on distances to the collective preference	*Consensus measure based on distances between experts*
Feedback mechanism	(Q_1) Bryson [32, 55] Herrera-Viedma et al. [28] Choudhury et al. [31] Dong et al. [59] Parreiras et al. [12, 29] Jiang et al. [61]	(Q_2) Carlsson et al. [52] Eklund et al. [10, 56] Herrera-Viedma et al. [57, 58] Chiclana et al. [60] Mata et al. [27] Cabrerizo et al. [62] Pérez et al. [63] Alonso et al. [64] Kacprzyk et al. [13, 65, 66] Fu et al. [37, 30, 67, 14]
No feedback mechanism	(Q_3) Lee [68] Ben-Arieh et al. [41] Chen et al. [71] Xia et al. [73], Xu et al. [74, 75] Dong et al. [76], Zhang et al. [53] Gong et al. [15], Xu et al. [21] Wu and Xu [11, 20, 54, 77, 78]	(Q_4) Chen et al. [69] Zhang et al. [70] Palomares et al. [16, 72]

Section 2.2, such measures are normally either based on computing distances to the collective preference (see Sect. 2.2.1) or based on computing distances between experts (see Sect. 2.2.2).

Taking into account the two criteria described above, the classification of consensus models in the taxonomy is based on two axes, so that they are combined into four different quadrants that will categorize the consensus models revised in this paper (see Table 1):

- Q_1: Consensus models with feedback mechanism and a consensus measure based on computing distances to the collective preference, reviewed in Section 3.1.

- Q_2: Consensus models with feedback mechanism and a consensus measure based on computing pairwise similarities, reviewed in Section 3.2.
- Q_3: Consensus models without a feedback mechanism and with a consensus measure based on computing distances to the collective preference, reviewed in Section 3.3.
- Q_4: Consensus models without a feedback mechanism and with a consensus measure based on computing pairwise similarities, reviewed in Section 3.4.

Remark 1. *For several consensus models reviewed throughout the following subsections, some figures with detailed schemes of their phases will be shown. The reason for showing the structure of these specific models in further detail rather than the other ones, is that they are already implemented in the initial version of the simulation-based analysis framework AFRYCA (see Section 4), and they will be utilized in the case study conducted in Section 5.*

3.1. Q_1: Feedback mechanism and consensus measure based on distances to the collective preference

In this section, we briefly review an assortment of consensus models characterized by: (i) the use of a feedback mechanism that provides some guidelines for experts on bringing their preferences closer to the rest of the group, and (ii) consensus measures based on the computation of distances between each expert's preference and the collective preference (see Fig. 5).

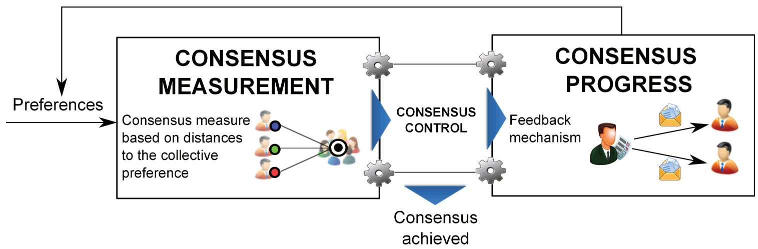

Figure 5: General scheme of consensus models in Q_1

Bryson [32] proposed a model to assess the degree of group consensus and support group discussions under the Analytic Hierarchy Process (AHP) framework [79]. The model gathers, for each expert $e_i \in E$ a normalized numerical preference vector. Individual vectors are aggregated into a collective preference vector. Two thresholds and three consensus indicators are defined to decide whether the degree of consensus is sufficient or not, based on similarities between each individual vector and the collective vector. Bryson stated that the consensus preference vector should reflect an agreement that results from human interaction [32], hence the need for carrying out a negotiation process guided by a moderator [9], encouraging experts to interact with each other. Further guidelines and strategies to support such a negotiation (such as cooperation, communication and so on) by means of decision support tools in different scenarios, were later proposed by Bryson in [55], in which the use of qualitative assessments by experts, associated to numerical ranges (e.g. Poor: [0,40], Good: [60,80], etc.), was also introduced.

The consensus model proposed by Herrera-Viedma et al. in [28] (represented in Figure 6) allows experts to express their preferences by using different preference structures: (i) preference orderings O_i, (ii) utility functions U_i, (iii) fuzzy preference relations P_i and (iv) multiplicative

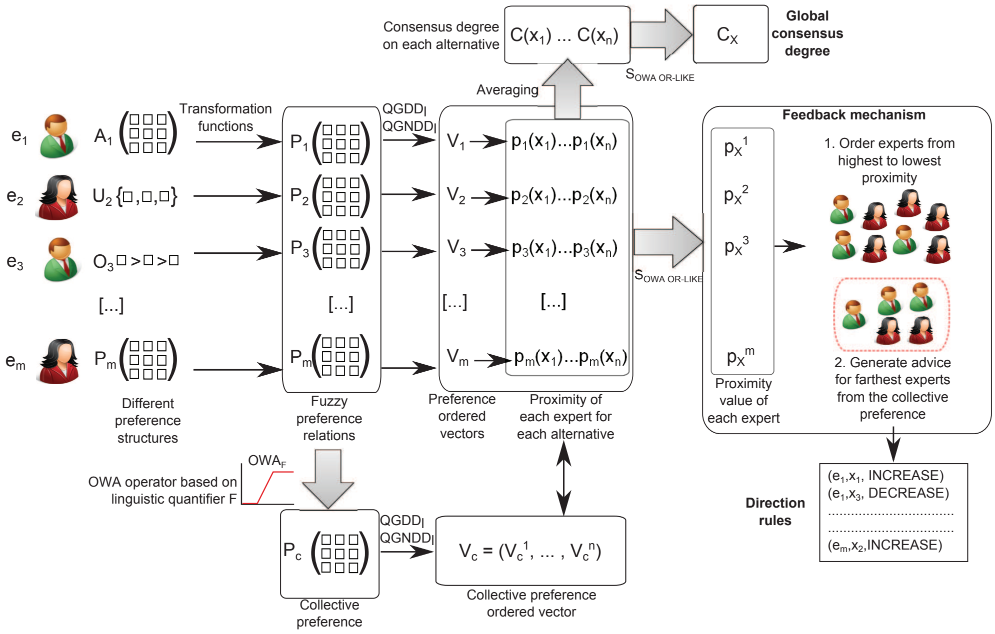

Figure 6: Computation of consensus degrees and feedback mechanism in the model of Herrera-Viedma et al. [28]

preference relations A_i. Each expert chooses his/her most suitable preference structure according to the level of expertise he/she has in the problem. All preferences are conducted into fuzzy preference relations by means of several transformation functions. Furthermore, preference orderings of alternatives are obtained from individual fuzzy preference relations by computing the quantifier-guided dominance and non-dominance degrees for each alternative x_l (denoted in Fig. 6 as $QGDD_l$ and $QGNDD_l$, respectively). Such preference orderings are compared with a collective preference ordering to compute the consensus degrees. The model also introduces a feedback mechanism, based on proximity measures and a set of directions rules to suggest to experts how to increase/decrease some of their assessments.

Inspired by the consensus model with different preference structures proposed in [28], and considering its consensus measures, Choudhury et al. [31] proposed a consensus support system aimed at solving MCGDM problems in the context of advanced technology selection. Its main novelties with respect to previous models include the use of a multi-agent architecture [80] in which software agents with specific roles implement the different phases of the consensus model, as well as the aggregation of proximity degrees between experts and the collective preference, by means of the neat OWA operator, to obtain consensus degrees [81].

Dong et al. presented in [59] two consensus models for AHP-GDM with multiplicative preference relations [79]. The difference between the models is the nature of the consensus measure, which can be either ordinal or cardinal. Furthermore, unlike the above reviewed proposals, consensus measures are characterized by the application of a prioritization method that derives a prioritization vector of alternatives (instead of a preference ordering) from each preference relation. The collective preference is computed by means of the Weighted Geometric Mean operator. The proposed feedback mechanism identifies the expert farthest from consensus, determines some updated values for his/her preferences and shows the updated values to the human expert, who decides whether he/she accepts the changes recommended or not.

Parreiras et al. proposed two consensus models for MCGDM problems. In their first model [12], experts utilize preference matrices with linguistic multi-granular assessments for each alternative and criterion, with the semantics of the linguistic terms given by trapezoidal membership functions. Since experts have importance weights according to their influence or position in the group, the authors suggested two methods to obtain them: either based on a discordance measure or by means of an optimization algorithm. The model presented in [29] introduces a measure of comparability to identify experts who experience difficulties in expressing their preferences (which are given by nonreciprocal fuzzy preference relations). In order to deal with such experts, other group members who are more sure of their opinion are invited to assist them. In both works, when the degree of consensus is insufficient, the moderator analyzes the concordance index of each expert with the collective preference, and suggests that the most discordant expert modifies his/her assessments.

More recently, Jiang et al. [61] defined a compatibility measure between intuitionistic multiplicative preference relations, and proposed two consensus models in which consensus degrees are measured for each expert separately, based on this compatibility measure. As occurred with [12, 29, 59], these models detect the expert farthest from the group opinion and invite him/her to modify his/her assessments. The second consensus model presented in [61] introduces identification rules in the feedback mechanism, in order to identify multiple discordant experts at the same discussion round and make the CRP more efficient.

3.2. Q_2: Feedback mechanism and consensus measure based on distances between experts

A large number of consensus models in the literature calculate the closeness between all the different pairs of experts in the group for the measurement of consensus [19, 22, 23, 57, 82]. This section revises some consensus models that present this type of consensus measure and incorporate a feedback mechanism to guide experts across the CRP (Fig. 7).

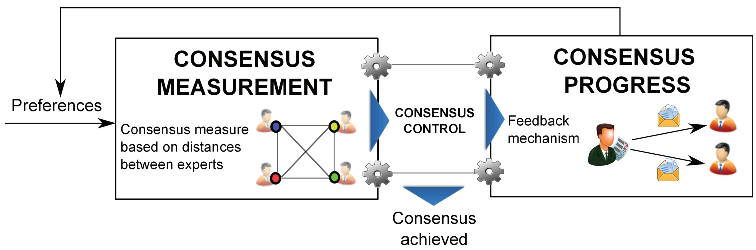

Figure 7: General scheme of consensus models in Q_2

Carlsson et al. [52] developed one of the first distributed consensus support systems to assist a group of experts connected to a local computer network. Its underlying consensus model follows an AHP framework for MCGDM problems in which experts provide preference matrices with assessments for each alternative and criterion, as well as the subjective importance weights they want to consider for each criterion. The consensus degree in the group is given by the maximum pairwise geometric distance between experts, i.e. $\max_{ij} d(P_i, P_j)$. The feedback mechanism

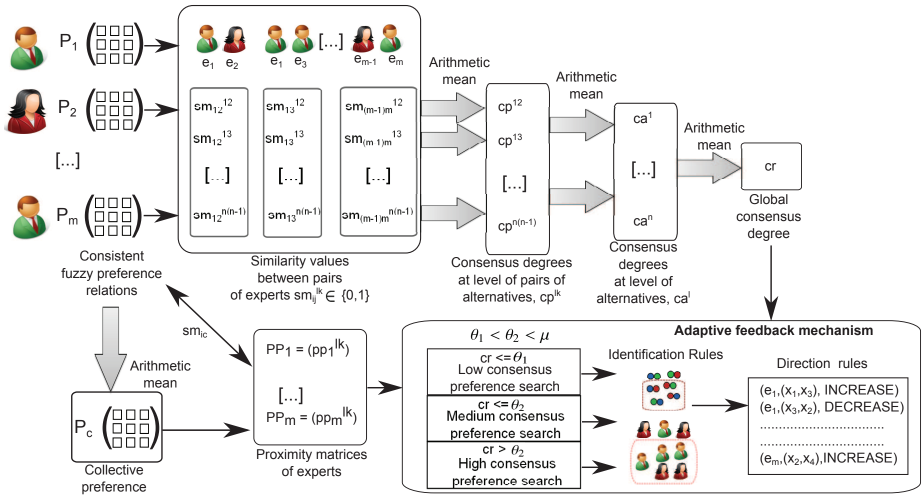

Figure 8: Adaptive consensus model of Chiclana et al. [60]

finds the expert farthest from consensus and suggests to him/her how to bring his/her preferences towards a central point between the rest of the experts' preferences. Based on the consensus measure defined by Carlsson et al. in which consensus is given by the maximum distance between two experts, Eklund et al. developed some models for consensus reaching in committees [56] and dynamic political contexts with coalition formation [10]. Their works include a detailed comparison between their consensus model and several voting schemes and rules, e.g. majority vote, plurality vote and Borda rule [83].

Herrera-Viedma et al. presented the first model aimed at letting experts with diverse levels of expertise express their preferences by means of different linguistic term sets (multi-granular linguistic preference relations) [57]. In order to deal with multi-granular linguistic information, they introduced a unification phase to conduct preferences into fuzzy sets in a common linguistic term set. This consensus model adopted some features which have been later considered by the authors in several works, such as: (i) a scheme for the computation of consensus degree at three levels (assessment, alternative and preference relation) upon pairwise similarities of experts, and (ii) a feedback mechanism consisting of identification and direction rules for experts, based on the computation of proximity degrees with the collective preference.

Several works have since been proposed, based on the consensus measure and feedback mechanism defined in Herrera-Viedma et al.'s model [57]. Their work in [58] is characterized by dealing with incomplete fuzzy preference relations whose missing assessments are computed by applying an estimation procedure. The model of Chiclana et al. [60] (see Fig. 8), incorporates a consistency control process applied before beginning the CRP to ensure consistency in individual fuzzy preference relations, and proposes an adaptive feedback mechanism in which the direction rules generated for experts depend on the level of agreement achieved at each round, which is compared with several consensus thresholds, $\theta_1 < \theta_2 < \mu$. The adaptive consensus model proposed by Mata et al. [27] considers the use of multi-granular linguistic information [58], and implements the adaptive feedback mechanism proposed in [60]. The consensus model of Cabrerizo et al. [62] is capable of dealing with unbalanced fuzzy linguistic information, given

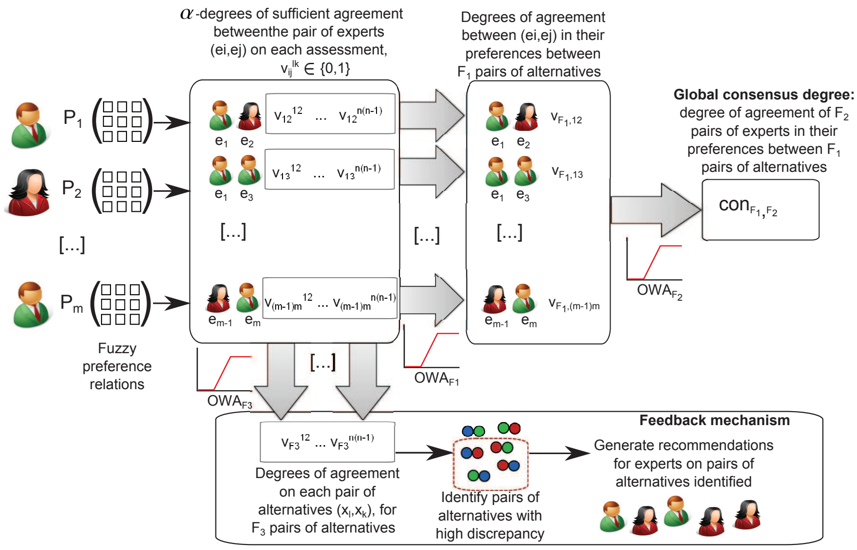

Figure 9: Computation of consensus degree based on the concept of fuzzy majority, and feedback mechanism proposed by Kacprzyk et al. in [65] and [13], respectively.

by linguistic terms distributed in a non-symmetrical and non-uniform way around a central term. Computational processes on unbalanced linguistic information are carried out my means of the 2-tuple linguistic model [84, 85]. A mobile consensus support system model for dynamic GDM, was presented by Pérez et al. in [63]. The system allows experts connected to their own mobile device to use different preference structures to provide their opinions [28], and it considers dynamic problems in which the set of alternatives X may vary over time. Finally Alonso et al. proposed in [64] a linguistic consensus model for Web 2.0 communities, in which the set of experts might vary during the CRP. A delegation scheme based on trust weights between similar experts is defined to simplify GDM processes with large groups.

Kacprzyk et al. developed several consensus models based on their notion of *soft consensus* and fuzzy majority (see Sect. 2.2.2). In [65], they proposed a consensus model in which the moderator identifies experts and alternatives with difficulties in achieving a consensus by means of linguistic data summaries [86]. This proposal does not assign importance weights to experts and alternatives. Instead, two linguistic quantifiers F_1 and F_2 are utilized to capture the concept of fuzzy majority in the computation of consensus degrees at multiple levels [65], as illustrated in Figure 9. The authors also proposed some models of consensus support systems that implement their previous ideas. For instance, in [13, 66] a concept of Web-based consensus support system that not only implements previous models, but also includes a guidance system based on several approaches, such as rule generation and collaborative filtering, is shown. In [13], ontologies are utilized to formalize knowledge managed by the system with regard to the consensus reaching processes and each particular GDM problem. In addition, the system incorporates a feedback mechanism consisting of computing quantifier-guided degrees of agreement over pairs of alternatives, identifying the pairs of alternatives in which the experts present a higher degree of discrepancy, and providing recommendations to experts, based on several rules (see feedback mechanism in Figure 9).

In [37, 30, 67, 14], Fu et al. developed four consensus models for MCGDM problems in evidential reasoning contexts, where assessments of alternatives according to different criteria are given by distributed vectors of belief degrees, based on Dempster-Shafer evidence theory [87]. Such belief degrees can be either numerical [30, 67] or interval-valued [37]. Assessments of pairs of experts are compared by means of a compatibility measure. Consensus degrees are then computed at three levels, similarly to [57]. In [37, 67], they introduce a feedback mechanism consisting of identification rules and direction rules for experts, taking into account assessments related to criteria with the highest importance weights only. In [14], they extend the feedback mechanism, so that if consensus is not reached after some consecutive rounds of generating feedback, weights of experts are adjusted based on an optimization algorithm to ensure convergence to consensus.

3.3. Q_3: No feedback mechanism and consensus measure based on distances to the collective preference

Some consensus models do not incorporate a feedback mechanism and are designed to carry out the whole CRP automatically, so that the preferences and/or importance weights of experts are adjusted in order to reach a high level of agreement without the need for human intervention. This section revises several consensus models characterized by: (i) not incorporating any feedback mechanism, and (ii) defining consensus measures based on the computation of distances to the collective preference (see Fig. 10).

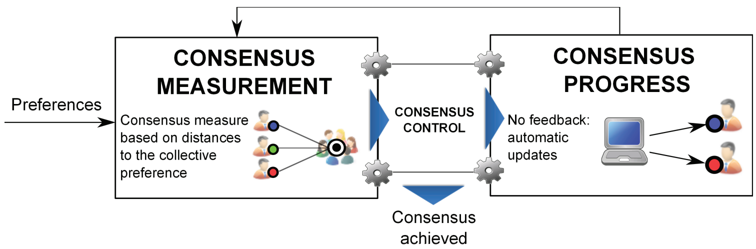

Figure 10: General scheme of consensus models in Q_3

In [68], Lee developed an iterative algorithmic approach to finding an optimal level of group consensus by adjusting the importance weights of experts and computing a collective preference based on them, so that the weighted sum of distances to the collective preference becomes minimal. The collective preference is given by the weighted average of individual preferences, which are expressed as trapezoidal fuzzy numbers. The consensus reaching algorithm is applied for each alternative separately.

Ben-Arieh et al. presented a consensus model for autocratic GDM [41] in a linguistic framework. Experts use linguistic preference relations, from which preference orderings are obtained to compute distances to the collective preference. Then consensus degrees are computed at the alternative and global level. If consensus is not enough, the degree of contribution of each expert towards consensus is determined, and weights of the least cooperating experts are penalized. More recently, Chen et al. defined in [71] an aggregation operator called ILLOWA

(Interval Linguistic Labels Ordered Weighted Averaging) to facilitate the management of preferences expressed as interval linguistic labels, together with a consensus model that extends the one presented in [41] to manage this type of information.

Xu [74] considered the problem of consensus reaching in MCGDM, and developed a model that automatically updates all experts' assessments at the end of each consensus round if the level of agreement is not sufficient. To do so, an update coefficient $\eta \in (0, 1)$, which partially takes into account values of the collective preference to update experts' assessments, is defined and utilized. A convergent iterative algorithm that automates the whole CRP is proposed. Unlike previous automatic consensus approaches, the importance weights of the experts remain fixed across the CRP. They are utilized to compute the collective preference. Consensus is only achieved when *all* distances between experts and the collective preference fall below a threshold, i.e. $d(P_i, P_c) \leq \mu$, $\forall e_i \in E$. An extension of this work was proposed by Xia et al. in [73], in which an automatic consistency improvement algorithm on reciprocal fuzzy preference relations is also defined.

The work of Xu et al. [75] proposes a number of goal and quadratic programming models oriented towards the maximization of consensus in groups of experts whose preferences are given in the form of fuzzy and multiplicative preference relations. Such programming models aim to find the optimal weights of experts that minimize their deviation with respect to the collective preference.

Wu and Xu have proposed several automatic consensus models in the last few years [11, 20, 54, 77, 78], in which the process used to compute and control individual consensus degrees similar to [74] in all of them. The model in [77] is aimed at the resolution of MCGDM problems with cost/benefit criteria, hence a normalization of assessments in the unit interval is applied before proceeding to measure consensus. Its mechanism to bring preferences closer to each other consists of obtaining at each CRP round a weighted distance matrix DM. Then its maximum element is identified, and the corresponding assessment is updated by assigning the value of the collective assessment to the preferences of those experts with the largest distance from the group preference. Their subsequent works [11, 20, 54, 78] utilize a simpler mechanism that updates the preferences of all experts whose distance to consensus exceeds a specified threshold. The updating of assessments is based on the updating coefficient, η [74]. Each of these proposals is characterized by the use of a different preference structure: linguistic preference relations [11], multiplicative preference relations [54], uncertain linguistic preference relations [78], and reciprocal fuzzy preference relations [20]. Figure 11 shows the procedure used to compute consensus degrees and update preferences, corresponding to the consensus model based on reciprocal fuzzy preference relations.

The work of Dong et al. [76] focuses on the use of two different representational models to deal with linguistic preferences (continuous linguistic model [88] and 2-tuple fuzzy linguistic model [84, 85]). They define a consensus measure based on an aggregation operator called Extended-OWA, to obtain the collective preference from continuous linguistic information. As stated in [74], all the experts must be close enough to the collective preference in order to reach a consensus, otherwise a quadratic programming algorithm that seeks the minimum required changes to individual preferences to find an agreement, is applied. Such an algorithm has since been considered by Zhang et al. in [53], in which a more generic consensus model under numerical preferences and the use of OWA operators is proposed.

Gong et al. formulated in [15] an optimization algorithm that, given a set of experts with associated weights and preferences expressed as 2-tuple linguistic preference relations, minimizes the deviation between all individual preferences and the collective preference. The optimization technique is applied to the values of experts' weights only, and no consensus thresholds are de-

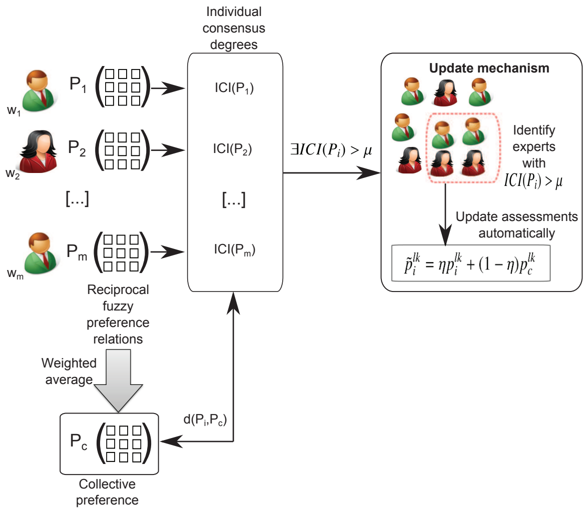

Figure 11: Consensus model of Wu et al. [20]

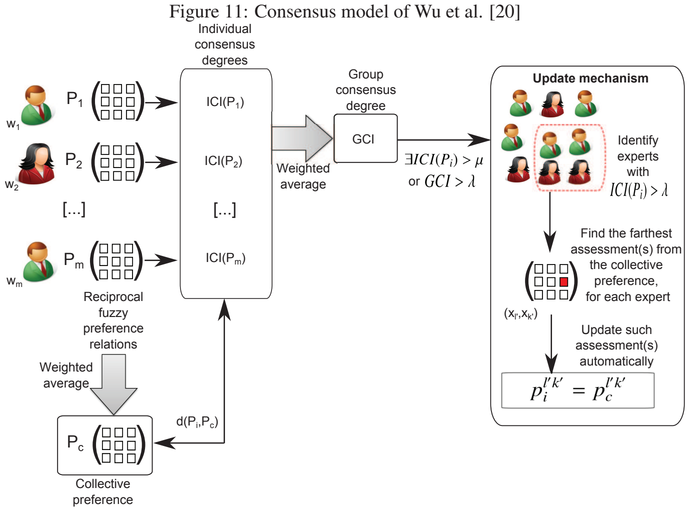

Figure 12: Consensus model of Xu et al. [21]

fined to decide on the existence of sufficient agreement, therefore the process ends when optimal weights are found. The additive consistency of preferences is also controlled.

The work of Xu et al. in [21] (see Figure 12) proposes two distance-based consensus models for fuzzy and multiplicative preference relations, respectively. Two consensus measures are used in both models: Individual Consensus Indices $ICI(P_i) = d(P_i, P_c)$ for each $e_i \in E$, and a Group Consensus Index GCI for the whole group. The feedback mechanism to update preferences must be applied if $ICI(P_i) > \mu$ for at least one $e_i \in E$, or $GCI > \lambda$, with μ and λ being the individual and group consensus threshold, respectively, with $\lambda \leq \mu$. In such a case, the assessments of discordant experts with the greatest differences among them are updated by assigning the corresponding value of the collective preference to them. This procedure is similar to the one previously shown in [77].

3.4. Q_4: No feedback mechanism and consensus measure based on distances between experts

Most automatic consensus models compute consensus degrees based on distances to the collective preference (see Section 3.3), but a small number of them carry out computations of similarities between pairs of experts to measure consensus. Some automatic and semi-automatic models based on computing distances between experts (Fig. 13) are reviewed in this section, corresponding to the fourth quadrant of the taxonomy presented in this paper.

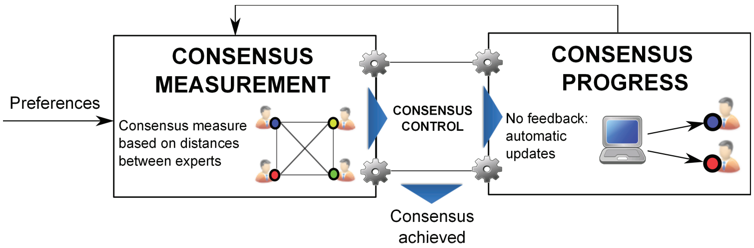

Figure 13: General scheme of consensus models in Q_4

An adaptive consensus support system model inspired by the ideas of [27] was proposed by Chen et al. in [69]. Its main novelties with respect to the work of Mata et al. are: (i) preferences are given by intervals of linguistic 2-tuples, (ii) the system modifies preferences of experts by adjusting interval-valued assessments, and (iii) despite the underlying consensus model being automatic, the human expert can optionally decide to revise the changes applied to the preferences and accept them or not.

Zhang et al. extended in [70] the consistency-driven consensus model of Chiclana et al. [60], by introducing a linear optimization model to update preferences that ensures a minimum cost of modifying preferences, expressed as fuzzy preference relations. The main advantage of applying a linear optimization model is its low computational cost. Therefore, such a technique is utilized not only to conduct the CRP, but also to reach a high level of consistency for each individual preference relation.

In [16], Palomares et al. developed and presented a consensus support system based on a multi-agent architecture [80]. The main novelty of such a system is its capacity to automate the CRP completely, not only for the human moderator, but also for experts. To do this, experts provide their initial preferences (expressed as fuzzy preference relations) and delegate to autonomous software agents the revision of the advice received and the application of changes to preferences throughout the overall CRP. The underlying consensus model (see Fig. 14) follows some of the guidelines proposed in [27, 57], such as: (i) the computation of pairwise similarities between experts by using the euclidean distance, (ii) the computation of consensus degrees at three levels, and (iii) although there is no real feedback for human experts, an agent-oriented feedback scheme consisting of identification and direction rules is implemented. Software agents are responsible for checking and applying direction rules on experts' preferences automatically. Moreover, two ontologies are defined and integrated in the model to facilitate communication

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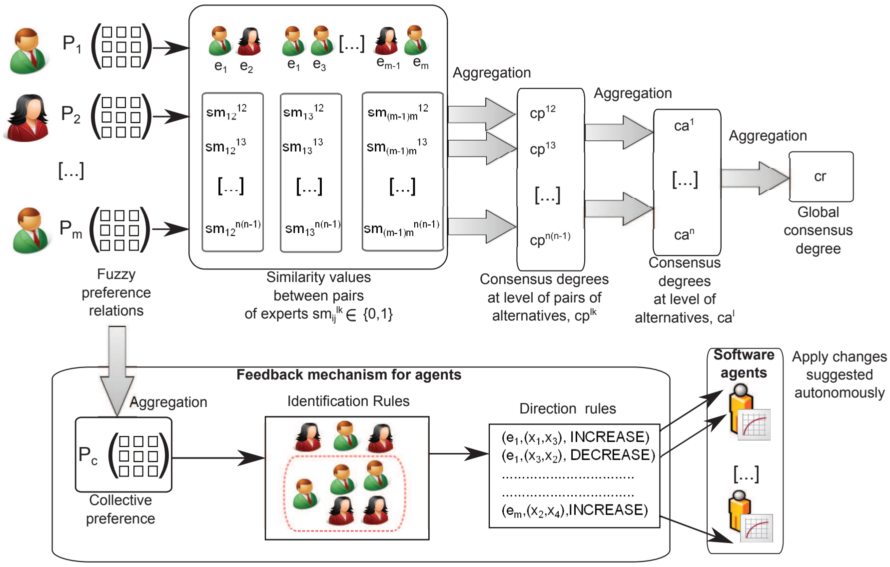

Figure 14: Agent-based consensus model of Palomares et al. [16]

and exchange of information amongst agents, based on the ideas propounded by Kacprzyk and Zadrozny in [13]. Palomares et al. suggested the implementation and flexible use of different aggregation operators to measure consensus.

The system presented by Palomares et al. allows a full automation of human experts, regarding the process of supervising and modifying preferences. However, in [72], they argued that in some specific situations, it might be desirable that the human expert supervises the advice generated on an assessment p_i^{lk}, e.g. if such advice implies an important change to his/her preference. Based on this idea, they propose an agent-based semi-supervised approach that allows software agents to carry out most revisions of preferences by themselves, so that they only request human intervention when critical changes must be applied. Such an approach is based on the definition of several behavioral profiles that define how agents apply changes autonomously, as well as a rule-based mechanism to indicate the situations in which the human expert must revise his/her opinions. Its main advantage is the capacity of automating the CRP for human experts to a high degree, while preserving their sovereignty.

4. AFRYCA: A FRamework for the analYsis of Consensus Approaches

This section introduces a novel software framework called AFRYCA to simulate the resolution of GDM problems by using different consensus models proposed in the literature, many of which have been categorized and reviewed in the taxonomy previously presented. AFRYCA is mainly oriented towards a practical study of consensus models, for discovering the advantages and weaknesses of each model, analyzing the performance of a model under different settings, etc. The framework also aims at: (i) providing a better understanding of which models would be the most suitable to solve a specific type of GDM problem, and (ii) enabling comparisons between different consensus models, which could be useful to find out the main contributions of new proposals with respect to other existing works, for instance.

Firstly, we present the architecture and technologies of the framework (Section 4.1). A methodology for the use of the framework is then briefly described. Finally, we undertake a case study to show the performance of several consensus models implemented in the framework, for the resolution of several GDM problems (Section 5).

4.1. Architecture of AFRYCA

Here, the architecture of AFRYCA and the technologies that have been utilized in the analysis framework are presented.

AFRYCA has been developed under Java language, by means of the set of plugins *Rich Client Platform* (RCP), which enables the development of client desktop applications with rich functionality. One of the main advantages of RCP is its appropriateness for building component-based software applications based on high quality components that are easy to maintain and extend, due to the high cohesion degree within each component and the low coupling between different components. Additionally, the software suite R[1] for statistical computing and graphics has been utilized to develop some components of the framework.

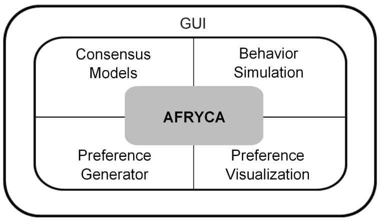

Figure 15: Architecture of AFRYCA

The framework is divided into five modules, as shown in Figure 15. Such modules implement the functionalities and tools included in AFRYCA for the simulation and analysis of GDM problems based on consensus models, and they are described below:

- *Consensus Models*: Libraries that develop several existing consensus models. Each library corresponding to an existing consensus model is implemented in Java, and it includes the different phases (e.g. computation of consensus degrees, advice generation, etc.), operators (e.g. OWA, weighted mean, etc.) and parameters (e.g. consensus thresholds, linguistic quantifiers, etc.) necessary to apply such a model in practice. The flexible, loosely coupled architecture of AFRYCA facilitates the introduction of new libraries that implement additional consensus models easily. The current version of the framework incorporates the necessary libraries for using six consensus models based on the use of fuzzy preference relations:
  - Three consensus models with feedback mechanism: Herrera-Viedma et al. [28] (see Fig.6), Chiclana et al. [60] (see Fig. 8), and Kacprzyk et al. [65, 13] (see Fig. 9).

[1] http://www.r-project.org

- Three consensus models without feedback mechanism: Wu et al. [20] (see Fig.11), Xu et al. [21] (see Fig. 12), and Palomares et al. [16] (see Fig. 14).

  Remark 2. *In AFRYCA, the current implementation of Herrera-Viedma et al.'s consensus model [28] omits the initial phase of unifying different preference structures, because the model deals with fuzzy preference relations only. Besides, in the model of Kacpryzk et al. in [65], the feedback mechanism based on linguistic summaries has been replaced by a feedback mechanism based on the criterion of "lack of arguments" suggested in [13].*

- *Behavior Simulation*: This module has been designed to choose and simulate different patterns of behavior adopted by experts when accepting/ignoring feedback and modifying their assessments across the CRP. Such behavior patterns are utilized by the consensus models that have a feedback mechanism (see Sections 3.1 and 3.2). Two key aspects must be taken into account to define a behavioral pattern in AFRYCA. These two aspects are modeled by generating values belonging to different probability distributions, as follows:

  - The amount of recommendations on assessments that an expert e_i may accept or ignore. This feature can be modeled by means of a generator of discrete random values (e.g. 1 for *accept* or 0 for *ignore*) belonging to a probability distribution (e.g. binomial), whose parameter values (e.g. probability of success p in binomial distribution) can be fixed by the developer.
  - The degree of change that e_i may apply to the assessment p_i^{lk}, the modification of which he/she has accepted. This feature can be modeled with either a discrete or continuous probability distribution (e.g. Normal or Negative Binomial), so that values generated with R under this distribution represent the degree of change applied to the assessment.

  A number of built-in *R* functions for the generation of random values under different probability distributions are utilized. R functions are invoked from Java code, by means of a third-party Java-R interface library. As occurred with consensus model libraries, this component can also be extended in the future. Moreover, such patterns can be used by different consensus models flexibly, in the sense that the user of AFRYCA may configure which behavioral pattern may be utilized with a specific consensus model at a given moment.

- *Preference Generator*: A Java implementation of the method proposed in [89] to construct consistent reciprocal fuzzy preference relations P_i from a set of $n-1$ values of assessments $p_i^{l(l+1)}$, $l \in \{1, \ldots, n-1\}$. Although such $n-1$ assessments are initialized randomly, the rest of the assessments are constructed taking into account the method mentioned above, thus ensuring consistency in preferences. This module allows the generation of data sets of experts' preferences. Each data set contains a specified number m of preference relations, as well as the formulation of a GDM problem, alternatives, etc. Such information is specified a priori, through the AFRYCA user interface. Data sets can be stored on a disk for future use.

- *Preference Visualization*: This module, inspired by the graphical monitoring tool of preferences presented in [90], provides a graphical 2-D representation of experts' preferences and the group preference, P_c, obtained after having conducted a CRP during the resolution of a GDM problem. Such a visualization is shown to the user of AFRYCA, together with the results of the GDM problem resolution. Some built-in R multi-dimensional scaling functions have been considered for the implementation of this module.

- *Graphical User Interface* (GUI): This allows users to interact with the rest of the modules in the framework. The GUI of AFRYCA has been implemented with the SWT (*Standard Widget Toolkit*) library, and it includes the necessary interfaces to: (i) choose the GDM problem and consensus model to utilize, (ii) configure the consensus model and select the behavioral pattern to simulate experts' behavior, (iii) visualize a summary of results after having applied the consensus model. It is also possible to generate a log file with more detailed results of the CRP conducted.

The architecture of AFRYCA offers several advantages, some of which are:

- Since it has been developed as a Java-based RCP, the framework can be used on any platform provided with a Java Virtual Machine, regardless of the operating system.
- The structure of AFRYCA, which is divided into separated modules, makes it possible to upgrade or extend some of its components (e.g. consensus model libraries and behavioral patterns, as mentioned above) without having to carry out changes that affect the whole framework.

A downloadable version of AFRYCA, as well as further details and documentation about the framework and its modules, can be found on the AFRYCA website[2].

4.2. Methodology for using AFRYCA to simulate the resolution of GDM problems

Here, we describe the methodology for using AFRYCA to simulate the resolution of a GDM problem by using a consensus model implemented in the framework, and analyze different aspects of such a model, e.g. determining the strong points, weaknesses and types of GDM problems that can be solved with such a model, studying its performance with respect to other models, etc. The methodology is divided into the following steps, as depicted in Figure 16:

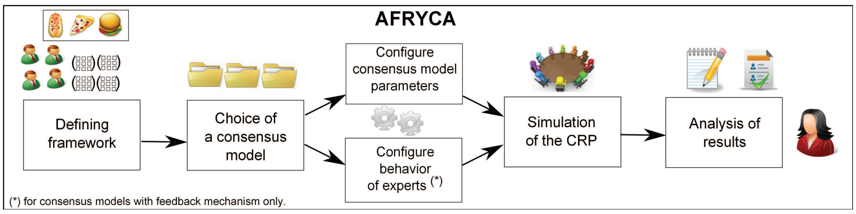

Figure 16: Methodology for the analysis of consensus models based on simulation of GDM problems

1. *Defining Framework*: An instance of a GDM problem is chosen, to be solved by applying the consensus model previously chosen. To do so, the user can either select a data set file with an already existing GDM problem, or he/she can use the *Preference Generator* module to create a data set for a new GDM problem with m experts.

[2]The AFRYCA website can be found at: http://sinbad2.ujaen.es/afryca

2. *Choice of a consensus model*: A consensus model is chosen from amongst those included in the framework. The GUI of the framework provides a description and the main features of each model, as shown in Figure 17.

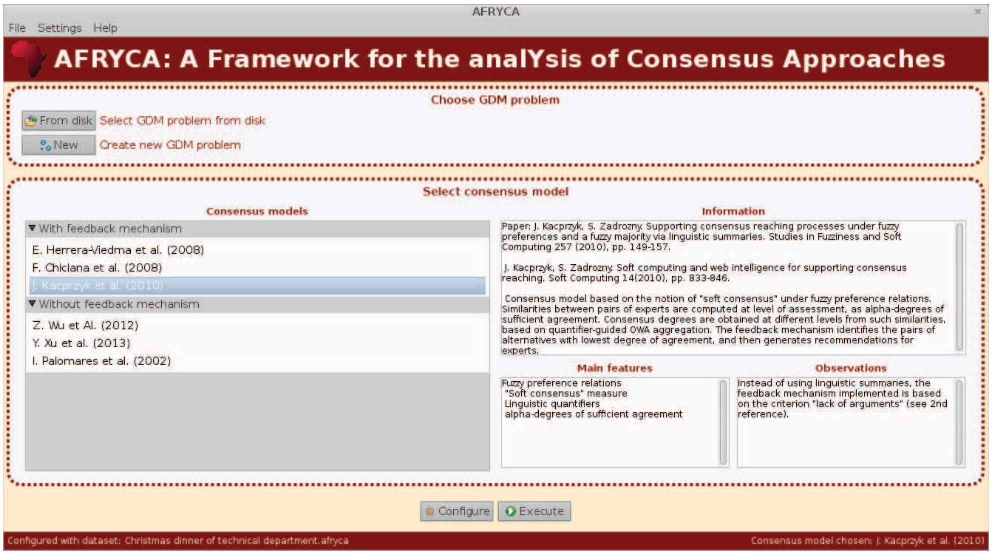

Figure 17: Main interface of AFRYCA for the selection of a GDM problem and consensus model

3. *Configure parameters of the consensus model and behavior of experts*: Before proceeding to carry out the CRP, it is necessary to configure the values of parameters in the consensus model chosen (e.g. consensus thresholds, aggregation operators, etc.). For consensus models with a feedback mechanism, it is also necessary to specify the pattern of behavior adopted by experts when they receive recommendations and apply changes to their preferences (see *behavior simulation* module, Section 4.1).
4. *Simulation of the CRP*: Once the consensus model settings are fixed, the CRP is carried out.
5. *Analysis of results*: When consensus is achieved, an alternative selection process based on fuzzy non-dominance degrees of alternatives is applied [36], and the results of the GDM problem resolution are shown, in order to allow the user to analyze them. Results shown in the GUI include: (i) the initial consensus degree in the group and the final consensus degree achieved, (ii) the number of discussion rounds required, (iii) the ranking of alternatives and alternative/s chosen as the solution, and (iv) a visualization of experts' preferences and the group preference at the end of the CRP (see Figure 18). AFRYCA also offers the possibility of storing a log file with more detailed results of the CRP performance.

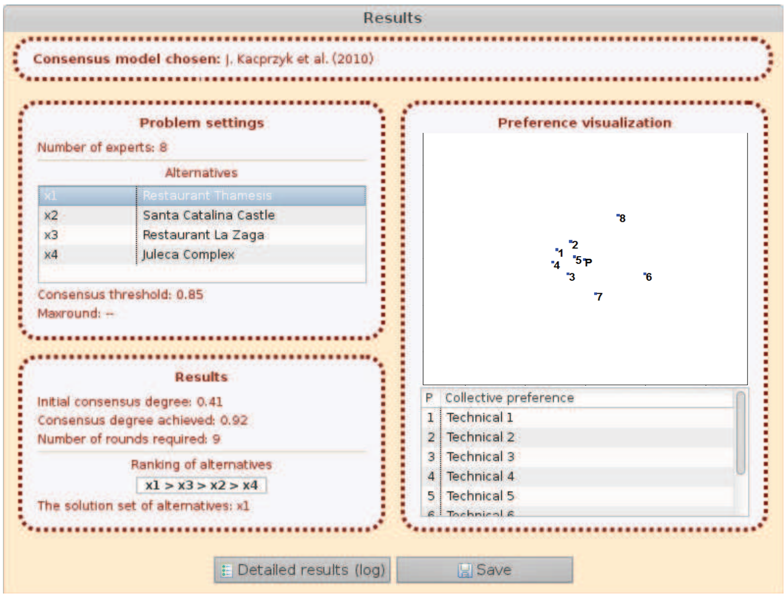

Figure 18: Interface of results in AFRYCA

5. Experimental Study

In order to illustrate the purpose of AFRYCA, in this section we show an experimental study conducted to study the performance of the consensus models integrated in the analysis framework [13, 16, 20, 21, 28, 60, 65], during the resolution of GDM problems with four different groups of experts.

Let us suppose a company composed of 32 employees, divided into four departments of equal size: Technical Department, $E_T = \{e_{T1}, \ldots, e_{T8}\}$, Human Resources Department, $E_H = \{e_{H1}, \ldots, e_{H8}\}$, Marketing Department, $E_M = \{e_{M1}, \ldots, e_{M8}\}$ and Sales Department, $E_S = \{e_{S1}, \ldots, e_{S8}\}$. Each department plans to celebrate a Christmas dinner separately, hence each group must make a common decision about the choice of a restaurant to celebrate their dinner, from amongst four possible alternatives for all of them: $X = \{x_1 : Restaurant\ Thamesis, x_2 : St.\ Catalina\ Castle, x_3 : Restaurant\ La\ Zaga, x_4 : Juleca\ Complex\}$.

All experts express their preferences as fuzzy preference relations. These preferences are included in four data sets that will be used in this case study: one for each department. The minimum level of agreement required is $\mu = 0.85$ for all groups, and the maximum number of discussion rounds, *Maxround*, will not be taken into account in this case study, therefore all simulations will be carried out without the CRPs ending due to having exceeded the number of discussion rounds permitted.

The case study is divided into three parts: (i) simulation of consensus models with a feedback mechanism, (ii) simulation of consensus models without a feedback mechanism, and (iii) discussion of results. At each stage, the four GDM problems defined above are solved by means of three different consensus models. Then, the results obtained are analyzed and compared.

5.1. Consensus Models with a Feedback Mechanism

The phases of the methodology shown in Section 4.2 to simulate CRPs and analyze the performance of consensus models are carried out for each GDM problem and consensus model separately:

(1) *Defining Framework.*
(2) *Choice of a consensus model.*
(3) *Configure parameters of the consensus model and behavior of experts:* Table 2 summarizes the values chosen for parameters that need to be configured by the user of AFRYCA for each consensus model. Further information about such parameters, as well as the rules of the feedback mechanism and operations carried out during the different phases of the CRP, can be found in the reference associated to each model.

Table 2: Parameters of consensus models with a feedback mechanism

	Herrera-Viedma et al. [28]	Chiclana et al. [60]	Kacprzyk et al. [65]
Consensus threshold	$\mu = 0.85$	$\mu = 0.85, \theta_1 = 0.75, \theta_2 = 0.8$	$\mu = 0.85$
Quantifier for aggregating information	F_{most}	-	$F_1 = F_2 = F_3 = F_{most}$
Quantifier for *QGNDD*$_l$	$F_{as\ many\ as\ possible}$	-	-
S_{OWA} OR-LIKE behavior	$\beta = 0.8$	-	-
S_{OWA} OR-LIKE behavior	$\beta = 0.8$	-	-
Recommendation rule in feedback mechanism	-	-	Lack of arguments [13]

Regarding the pattern utilized to simulate the behavior of experts in this case study, the degree of acceptance or rejection of recommendations to modify preferences is modeled by means of a Binomial Distribution, and the degree of change applied to accepted recommendations is modeled by means of Negative Binomial Distribution.
(4) *Simulation of the CRP.*
(5) *Analysis of Results:* The results of the performance of the CRP and the solution set of alternatives obtained with each consensus model, are summarized in Table 3. They will be discussed in Sect. 5.3.

5.2. Consensus Models without a Feedback Mechanism

The previous methodology is applied again to solve the four GDM problems by means of each of the three consensus models without a feedback mechanism, with the only difference being that no experts' behavior needs to be configured for its simulation in the third phase.

(1) *Defining Framework.*
(2) *Choice of a consensus model.*
(3) *Configure parameters of the consensus model:* The values chosen for parameters that require configuration in AFRYCA for each consensus model are shown in Table 4. Notice

Table 3: Results of the GDM problem resolution for consensus models with feedback mechanism

	Herrera-Viedma et al. [28]	Chiclana et al. [60]	Kacprzyk et al. [65]
Technical Dept. (E_T)			
- Initial consensus degree	0.79	0.77	0.41
- Number of rounds	2	15	9
- Final consensus degree	0.85	0.85	0.92
- Ranking	$x_1 \succ x_3 \succ x_2 \succ x_4$	$x_1 \succ x_3 \succ x_2 \succ x_4$	$x_1 \succ x_3 \succ x_2 \succ x_4$
- Alternative/s chosen	x_1	x_1	x_1
Human Res. Dept. (E_H)			
- Initial consensus degree	0.76	0.69	0.1
- Number of rounds	4	15	20
- Final consensus degree	0.88	0.86	0.92
- Ranking	$x_1 \succ x_3 \succ x_2 \succ x_4$	$x_1 \succ x_2 \sim x_3 \succ x_4$	$x_1 \succ x_2 \succ x_3 \succ x_4$
- Alternative/s chosen	x_1	x_1	x_1
Marketing Dept. (E_M)			
- Initial consensus degree	0.78	0.63	0.11
- Number of rounds	7	24	26
- Final consensus degree	0.86	0.85	0.86
- Ranking	$x_3 \succ x_1 \succ x_2 \succ x_4$	$x_1 \sim x_3 \succ x_2 \succ x_4$	$x_1 \succ x_3 \succ x_2 \succ x_4$
- Alternative/s chosen	x_3	x_1, x_3	x_1
Sales Dept. (E_S)			
- Initial consensus degree	0.71	0.61	0.09
- Number of rounds	7	26	25
- Final consensus degree	0.88	0.85	0.89
- Ranking	$x_3 \succ x_1 \succ x_2 \succ x_4$	$x_1 \sim x_3 \succ x_2 \succ x_4$	$x_3 \succ x_2 \succ x_1 \succ x_4$
- Alternative/s chosen	x_3	x_1, x_3	x_3

that the consensus thresholds in [20, 21] are distance-based thresholds, i.e. in this case consensus indices *below* these thresholds represent a satisfactory level of agreement, hence the values assigned to them are equal to $1 - \mu = 0.15$.

(4) *Simulation of the CRP.*

(5) *Analysis of Results:* Table 5 shows the results obtained from conducting the CRP with each consensus model and applying an alternatives selection process. In order to facilitate the comparison of consensus models, the consensus degrees shown in the table for the models of Wu et al. and Xu et al. are given by $1-GCI$, because these models utilize group and individual distance-based consensus indices (denoted as GCI and ICI respectively, as shown in Sect. 3.3). The consensus degrees depicted in the table for the model of Wu et al. correspond to the ICI of the most distant expert in the group, i.e. $1 - \max_i ICI(P_i)$. The results are described in Section 5.3.

5.3. Discussion of the Experimental Study

Once the results of the experimental study have been set out, they are briefly discussed and analyzed, regarding their convergence towards agreement and the solution achieved.

From results of simulation with the consensus models with feedback mechanism (Sect. 5.1, Table 3), it can be observed that:

Table 4: Parameters of consensus models without a feedback mechanism

	Wu et al. [20]	Xu et al. [21]	Palomares et al. [16]
Consensus threshold	$\mu = 0.15$	$\mu = 0.2, \lambda = 0.15$	$\mu = 0.85$
Normalized weights of experts	$w_i = 1/8, i = 1, \ldots, 8$	$w_i = 1/8, i = 1, \ldots, 8$	-
Updating coefficient	$\eta = 0.8$	-	-
Choice of aggregation operator	-	-	Arithmetic mean
Degree of change on assessments	-	-	0.05

Table 5: Results of the GDM problem resolution for consensus models without feedback mechanism

	Wu et al. [20]	Xu et al. [21]	Palomares et al. [16]
Technical Dept. (E_T)			
- Initial consensus degree	0.7	0.84	0.77
- Number of rounds	10	3	6
- Final consensus degree	0.86	0.9	0.85
- Ranking	$x_1 \succ x_3 \succ x_2 \succ x_4$	$x_1 \succ x_3 \succ x_2 \succ x_4$	$x_1 \succ x_3 \succ x_2 \succ x_4$
- Alternative/s chosen	x_1	x_1	x_1
Human Res. Dept. (E_H)			
- Initial consensus degree	0.67	0.79	0.69
- Number of rounds	16	3	10
- Final consensus degree	0.85	0.87	0.85
- Ranking	$x_1 \succ x_3 \succ x_2 \succ x_4$	$x_1 \succ x_3 \succ x_2 \succ x_4$	$x_1 \succ x_2 \succ x_3 \succ x_4$
- Alternative/s chosen	x_1	x_1	x_1
Marketing Dept. (E_M)			
- Initial consensus degree	0.41	0.75	0.63
- Number of rounds	19	4	14
- Final consensus degree	0.86	0.89	0.86
- Ranking	$x_1 \succ x_3 \succ x_2 \succ x_4$	$x_3 \succ x_1 \succ x_2 \succ x_4$	$x_3 \succ x_1 \succ x_2 \succ x_4$
- Alternative/s chosen	x_1	x_3	x_3
Sales Dept. (E_S)			
- Initial consensus degree	0.46	0.73	0.60
- Number of rounds	20	4	12
- Final consensus degree	0.85	0.87	0.86
- Ranking	$x_3 \succ x_1 \succ x_2 \succ x_4$	$x_3 \sim x_2 \succ x_1 \succ x_4$	$x_3 \succ x_1 \succ x_2 \succ x_4$
- Alternative/s chosen	x_3	x_3	x_3

1. **Convergence**
   a) The consensus model of Herrera-Viedma et al. presents a significantly higher convergence towards consensus for all the GDM problems, i.e. a lower number of consensus rounds are necessary to achieve the required level of agreement, $\mu = 0.85$.
   b) The consensus model of Chiclana requires a large number of rounds to reach consensus, due to the values chosen for intermediate consensus thresholds θ_1 and θ_2, and the nature of its adaptive feedback mechanism, which generates a much lower amount of

advice when the consensus degree exceeds θ_1.

c) Consensus degrees are much lower in the model of Kacprzyk et al., due to its similarity measure being based on α-degrees of sufficient agreement (see Eq. (2)), which is a rather strict measure.

2. **Solution**: The ranking of alternatives is very similar in the groups of experts belonging to the Technical and Human Resources Departments, with x_1 being the alternative chosen in both of them, regardless of the consensus model utilized. In the Marketing and Sales departments, either x_1 or x_3, or both of them, can be chosen as the solution to the GDM problem, depending on the model used.

Regarding the results of simulation with the consensus models without feedback mechanism (Sect. 5.2, Table 5), it can be observed that:

1. **Convergence**
   a) The convergence towards consensus is higher in the model of Xu et al., due to the fact that the identified assessments are directly updated with the value of the collective preference (see Fig. 12), therefore experts' preferences may experience significant changes in a single round.
   b) The consensus model of Wu et al. applies small changes to preferences at each round (since $\eta = 0.8$ and the closer η is to 1, the smaller the changes applied [20]), hence its lower convergence.
   c) The model of Palomares et al. also presents a lower convergence, because it has been applied with a low degree of autonomous change (increase/decrease) to assessments, 0.05.
2. **Solution**: x_1 is the best alternative at the Technical and Human Resources Departments, x_3 is the best alternative at the Sales Department, and either x_1 or x_3 could be the chosen alternative at the Marketing Department, depending on the consensus model.

We draw the following conclusions from the experimental case of study conducted:

- A similar solution is obtained at each group, regardless of the consensus model used for simulation: similar consensus degrees have been achieved, with slight differences in the alternative/s chosen as solution to the GDM problem.

- The main distinguishing element amongst the performances of consensus models, is the convergence that each one presents. Such a convergence is evaluated as the number of iterations or discussion rounds carried out before reaching a sufficient consensus degree. This could be an important factor for groups of experts, when they have to choose the most suitable consensus model in terms of usability.

6. Lessons Learnt and Future Directions

The simulation of CRPs with AFRYCA provides multiples advantages and possibilities, some of which are:

- The framework makes it possible to simulate the resolution of a GDM problem under different consensus models, provided that they are suitable for dealing with such types of problems (e.g. consensus models for GDM problems with fuzzy preference relations). Thus, a decision maker, i.e. a person responsible for making the group decision, is able to study the performance and results obtained with each model.

- For a specific problem and consensus model, AFRYCA offers the possibility of investigating the different settings of such a model, based on the parameters or operators defined in it. Moreover, for those models with a feedback mechanism, the problem might be simulated under different patterns of expert behavior, in order to observe the effect of considering different types of behavior in the simulation.
- Although the decision group may prefer to conduct a real CRP, AFRYCA could provide them with a rough idea a priori about the performance of results that would be obtained, taking into account the initial preferences of experts and defining the appropriate problem settings that would reflect the real context of the problem.
- The experimental study presented has not focused on the use of different representational formats (e.g. linguistic preferences) to assess alternatives, but it is possible to implement and utilize any other existing types of preferences or representational formats in AFRYCA, for simulation purposes.

Six consensus models have been implemented in AFRYCA so far. Nevertheless, we note again that the architecture of the framework is designed to allow the inclusion of new consensus models (based on other types of preferences, information domains or even focused on MCGDM problems), as well as the further comparison between new models introduced and the existing ones.

Multiple proposals of consensus models have been presented in the specialized literature without showing a comparison with other existing models, hence their usefulness and main contributions are not justified properly. AFRYCA enables the implementation and analysis of these new proposals to find out their main contributions, with respect to the already existing ones.

Future work on extending the functionalities of AFRYCA, will mainly be oriented towards the definition of new metrics to measure the performance of a CRP. Such metrics would evaluate not only the discussion process itself, but also the quality of the collective solution achieved (in terms of its degree of acceptance by each member of the group, for instance), with the aim of facilitating a more comprehensive comparative study amongst different consensus models. This is currently one of the most important challenges in consensus: defining good performance measures would make it possible to evaluate the real usefulness of new proposals in the future.

7. Concluding remarks

Consensus has become a prominent research area in the field of group decision making. A large number of approaches to support consensus reaching have been proposed - and continue to be proposed - by a variety of authors.

In this paper, we have presented a taxonomy of existing consensus models for group decision making problems defined in a fuzzy context, which categorizes a number of consensus models based on their main characteristics, e.g. the type of information fusion techniques utilized to measure consensus in the group, or the procedures applied to increase the level of agreement throughout the discussion process. Besides characterizing a large number of existing consensus models, the taxonomy would also be useful to determine which could be considered for comparison with a new proposal, based on its characteristics and taking into account the taxonomy structure. Comparative studies are necessary to analyze the real capabilities of new proposals, instead of undertaking straightforward consensus exercises with them directly.

We have also presented a prototype of simulation-based analysis framework called AFRYCA, for the simulation of group decision making problems under consensus, by means of implementations of different existing consensus models in the literature. An experimental study has been shown to illustrate the usefulness of AFRYCA. To do this, six consensus models have been implemented and utilized in the study, based on the use of fuzzy preference relations to represent and manage preferences. As a result of the study conducted with AFRYCA, we suggest some future directions in the research topic of consensus: (i) the importance of comparing new proposals with existing ones, in order to show their contributions, and (ii) the definition of new performance measures for consensus reaching processes, as a major challenge in the topic.

Finally, some recent approximations for consensus reaching consider different perspectives, e.g. agent-based consensus support systems [72], consensus models for large-scale group decision making problems [91, 92], etc. These works could also be considered for their simulation in the framework.

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4.2. A Semi-Supervised Multi-Agent System Model to support Consensus Reaching Processes

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A Semi-Supervised Multi-Agent System Model to support Consensus Reaching Processes

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***Abstract*—Consensus reaching processes as part of solving group decision making problems attempt to reach a mutual agreement in the group before making a decision. Most consensus models and consensus support systems proposed in the literature present some noticeable drawbacks: the need for constant human supervision by experts to guarantee an effective process, and the difficulty to manage large groups of experts, which are increasingly common in nowadays decisions and may imply a higher cost and complexity to carry out such processes. In order to overcome these problems, this paper presents a novel consensus support system based on the multi-agent system paradigm, which automates and supports consensus reaching processes by providing agents with the necessary degree of autonomy to conduct discussion processes by themselves, with a semi-supervised methodology. The main novelty of such a system is the agent semi-supervised autonomy approach it incorporates, which lets agents conduct most of the discussion process by themselves, and also allows them to interact with their corresponding human experts in certain circumstances that human supervision might be convenient and necessary.**

***Index Terms*—Group decision making, consensus reaching process, multi-agent system, fuzzy preference relation.**

I. INTRODUCTION

Decision Making is a usual mankind process in daily life. In a Group Decision Making (GDM) problem, two or more decision makers or experts try to achieve a common solution to a problem consisting of several alternatives or possible solutions to such a problem [1]–[3]. In many real situations, the resolution of GDM problems requires dealing with vague and imprecise information given by experts, i.e. the GDM problem is defined under uncertainty [4]. Such an uncertainty implies that experts may not show a clear preference about an alternative with respect to the other ones, therefore they might need an adequate expression domain and preference structure (e.g. fuzzy preference relations, multiplicative preference relations, etc. [5], [6]) to express partial degrees of preference between alternatives.

Traditionally, GDM problems have been solved by applying a selection process to choose the best alternative/s, without taking into account the level of agreement amongst experts [7]. This process can lead sometimes to solutions that are not well accepted by some experts in the group [8], because they might think that their own opinions have not been considered properly to make the decision. In order to prevent such situations, it is advisable that experts carry out a *consensus reaching process* (CRP), so that they discuss and modify their preferences gradually to achieve a high level of agreement before making a decision [1]. CRPs normally consist of several rounds of discussion supervised by a human moderator, who helps experts to move their opinions closer to each other [8], [9].

As a result of a thorough study on CRPs over the last decades, many theoretical consensus models have been proposed in the literature to conduct them [10]–[15]. On the other hand, in order to provide groups with computer-based decision support systems focused on supporting CRPs, some research has been done in the development of *Consensus Support Systems* (CSSs) [14], based on the implementation of different consensus models.

Despite the great amount of research conducted on CRPs, there are still some weaknesses and aspects that require improvement. One of them is the need for managing large groups in such processes. New paradigms and means of making large-scale group decisions (such as e-democracy [16], social networks [17] and marketplace selection for group shopping [18], for instance) have arisen in the last few years. As a result, the so-called large-scale GDM problems have become increasingly frequent in the last few years. Managing large groups in GDM makes more frequent the existence of strong disagreement positions between some experts in the group, hence the higher necessity of applying a CRP in these circumstances. Additionally, large-scale CRPs imply a considerable cost, complexity and time invested in reaching a collective agreement. For this reason, some experts might eventually abandon the discussion process because no consensus is reached after having invested much time in the discussion process [9].

Other challenges and difficulties, that attain a greater importance when a large-scale GDM problem must be solved under consensus, are the following ones:

1) The necessity of organizing physical meetings to deal with CRPs. In some real-life environments that require large-scale GDM, such as multi-regional or multinational organizations, experts may be physically located in many different geographical places. Therefore, a CSS based on distributed and Internet technologies would be highly convenient to make agreed decisions that involve all of them [14].
2) The need for a constant human supervision by the human moderator, who must guide and advice experts across the CRP and control its right development. Such supervision becomes much more complex and costly if the moderator has to deal with a large group. Consequently, it would be convenient to replace the human moderator by means of a CSS that automates all (or most of) his/her tasks [15], [19], [20].

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3) The possible biasness presented by the human moderator, due to subjective factors. This situation would be more apparent in large-scale decisions, in which the moderator might decide to consider the opinions and concerns of his/her interest only (to save time and cost, for instance), which implies that no true consensus is reached by the group as a whole [9]. Again, the development and use of CSSs that replace the human moderator by automating his/her tasks could be considered to overcome this problem.

Although some proposed CSSs have eliminated the need for constant supervision by the human moderator, prevented his/her possible subjectivity by automating his/her tasks and made it possible to conduct non-physical meetings [12], [15], [19], [20], dealing with large-scale CRPs still requires the development of an appropriate architecture that manages the high amount of information and communication flow present in such processes efficiently. In this sense, the *Multi-Agent System* (MAS) paradigm [21], [22], which is characterized by its scalability and distributed computing capabilities, can be a reasonable choice to develop a CSS that supports large groups effectively.

Another important challenge that has not been addressed properly yet in spite of the achievements made with current CSSs, is the constant supervision of preferences by experts, who must reconsider and modify their opinions repeatedly throughout the overall CRP [8]. An excessive amount of experts' supervisions may often lead to some undesired consequences, especially if a large number of them take part in the GDM problem:

1) The amount of time invested by experts to supervise and modify their opinions manually based on feedback received might increase the CRP's length considerably.
2) Some experts may experience an eventual loss of motivation and interest on the problem addressed, if the group has not reached a consensus after having carried out the supervision suggested by the CSS at several discussion rounds.

Although some approaches have been recently proposed to fully automate experts' behavior in CRPs (see [15] for instance), a total experts' automation would not be desirable in some real-life problems. In some specific cases in which experts are suggested to apply a substantial change on their preferences, they may prefer to revise preferences manually because they might think that their own concerns should be considered in such cases. We attempt to overcome this problem by developing a novel agent-based approach capable of minimizing human supervision, without eliminating it completely, thus modeling experts' behavior by means of software agents that carry out most of the supervision tasks assigned to human experts autonomously, and let them supervise their preferences manually in some specific cases that it would be convenient and necessary.

This paper presents a novel semi-supervised CSS based on the MAS paradigm, that automates all the human moderator tasks, removing his/her inherent subjective biasness, and helps experts conducting CRPs to solve real-life large-scale GDM problems defined under uncertainty. Human expert supervision is only necessary in those cases that they are requested to apply critical changes in their opinions to increase the agreement, otherwise agents carry out the necessary tasks to make experts' opinions closer autonomously. The system is characterized by providing users a set of agents that implement a semi-supervised autonomy approach capable of emulating different behavioral profiles based on experts' requirements, thus providing agents with a high autonomy degree. Such a semi-supervised approach can be applied irrespective of the specific underlying consensus model considered. In addition, the multi-agent architecture provides the necessary scalability to deal with large-scale GDM problems effectively.

This paper is set out as follows. In Section II, preliminaries about CRPs, multi-agent technologies and some related work are reviewed. The multi-agent CSS components, its underlying consensus model and the agent semi-supervised autonomy approach proposed are presented in Section III. A case study that shows the system's performance is given in Section IV. Finally, some concluding remarks are expounded in Section V.

II. Preliminaries

In this section, we review GDM problems and the main concepts related to CRPs. Then, we briefly revise MAS technologies and some related works on consensus models, CSSs and some existing multi-agent based proposals for GDM in the literature.

A. Consensus Reaching Processes in GDM

Group Decision Making (GDM) problems are defined as decision situations where two or more individuals or experts participate in a problem consisting of a set of alternatives or possible solutions to the problem [1], [2]. Formally, the main elements found in any GDM problem are:

- A set X of two or more feasible *alternatives*:

$$X = \{x_1, \ldots, x_n\} (n \geq 2) \tag{1}$$

- A set E of *experts* who express their judgements on the alternatives in X:

$$E = \{e_1, \ldots, e_m\} (m \geq 2) \tag{2}$$

Each expert e_i provides his/her opinion over alternatives in X by means of a preference structure. One of the most widely used preference structures in GDM problems defined under uncertainty, is the so-called fuzzy preference relation.

Definition 1. *[5], [23] Given a finite set of alternatives X, a fuzzy preference relation P_i associated to expert e_i is a fuzzy set on $X \times X$, characterized by a membership function $\mu_{P_i} : X \times X \to [0,1]$, and represented by a square matrix as follows:*

$$P_i = \begin{pmatrix} - & \ldots & p_i^{1n} \\ \vdots & \ddots & \vdots \\ p_i^{n1} & \ldots & - \end{pmatrix}$$

where each assessment $p_i^{lk} = \mu_{P_i}(x_l, x_k)$ $\forall l, k \in \{1, \ldots, n\}$, *($l \neq k$), represents the degree of preference of alternative x_l*

over x_k, for expert e_i, so that $p_i^{lk} > 0.5$ indicates preference of x_l over x_k, $p_i^{lk} < 0.5$ indicates preference of x_k over x_l, and $p_i^{lk} = 0.5$ indicates indifference between both alternatives [23], [24].

Remark 1. *Assessments p_i^{ll}, $l \in \{1, \dots, n\}$, situated in the diagonal of the matrix, are not defined, since an alternative x_l is not assessed respect to itself.*

Besides fuzzy preference relations, several types of preference structures based on different information domains have been proposed in the literature to deal with uncertain information [7], [25]–[27]. The nature of the GDM problem or the level of experts' background knowledge might sometimes determine the most suitable preference structure/s to be used. Some examples of them, based on preference relations, are the following:

- *Multiplicative preference relation*, $A_i = (a_i^{lk})^{n \times n}$, where an assessment a_i^{lk} indicates a ratio of preference intensity of x_l respect to x_k, measured in Saaty's 1 to 9 discrete scale [25].
- *Linguistic preference relation*, $T_i = (t_i^{lk})^{n \times n}$, where $t_i^{lk} = s_u$, $S = \{s_0, \dots, s_g\}$, where $s_u \in S$, $u = 0, \dots, g$ is a linguistic term belonging to a term set S with granularity g [7].

Some approaches have been proposed to ease the resolution process of GDM problems where several types of preference structures could be used by experts, for example the one in [6], which unifies them into fuzzy preference relations. Although the consensus model described in this paper (see Section III-A) will focus, without loss of generality, on the use of fuzzy preference relations exclusively (due to their appropriateness in many situations [12], [28]–[30]), it must be pointed out that the flexibility of the proposed CSS allows the integration of different consensus models and, consequently, their extension to manage different types of preferences.

The solution to a GDM problem may be obtained using either a *direct approach*, where the solution is directly obtained from experts' preferences; or an *indirect approach*, where a social opinion is computed before determining the chosen alternative/s [7], [31]. Regardless of the approach considered, two phases are conducted to solve a GDM problem: (i) an *Aggregation phase*, which consists in combining experts' preferences; and (ii) an *Exploitation phase*, where an alternative or subset of alternatives is obtained as the solution to the problem [32].

Different classic rules have been suggested to find the solution for a GDM problem, some of which are listed below [3], [8], [9]:

- *Majority Rule*: The decision is made according to the majority opinion. This rule admits two modalities: absolute majority, when the predominant opinion has been considered by more than half of the group, and relative majority, otherwise.
- *Minority Rule*: The decision is delegated to a reduced subgroup of people, due to their level of expertise on the problem.
- *Authority Rule*: A group's leader is given the authority to make the final decision for the group.
- *Unanimity*: All members must agree with the decision made.

One of the main shortcomings found in these rules is the possible disagreement shown by some experts with the solution achieved, because they might consider that their opinions have not been taken into account sufficiently [8]. Given the importance of obtaining an accepted solution by the whole group, CRPs as part of the decision process have attained a great attention.

The term *consensus* can be defined as a state of mutual agreement among members of a group, where the decision made satisfies all of them [8]. Reaching a consensus normally implies that experts change their initial opinions in a discussion process, tending to make them closer to each other, towards a final collective opinion which satisfies the whole group.

The concept of consensus can be interpreted in several ways, from a strict view of consensus as total agreement, which is usually difficult to achieve in practice, to a more feasible and flexible approach considering different degrees of partial agreement [1], [11]. One of the most accepted approaches in the literature to soften the concept of consensus is the so-called notion of *soft consensus*, proposed by Kacprzyk in [2]. This approach, which has been successfully applied to different GDM problems [14], [33], is based on the concept of fuzzy linguistic majority. Such a concept states that there exists consensus in a group when "*most experts participating in a problem agree with their opinion on the most important alternatives*". Consensus measures based on *soft consensus* are more human-consistent and suitable for reflecting human perceptions of the meaning of consensus [34], therefore this idea is considered in the consensus model proposed in this paper.

The process to reach a consensus is an iterative and dynamic process, frequently coordinated by a *moderator*, a human figure responsible for supervising the overall discussion process and guiding experts throughout it [9]. A general scheme for conducting CRPs, based on a flexible notion of consensus and followed by different authors to propose consensus models [12], [13], [19], is shown in Figure 1. The phases shown in this scheme are described below:

1) Define the decision making problem and the set of possible alternatives.
2) Identify the format to represent preferences, and consensus measures used to determine the level of agreement based on these preferences.
3) Discussion process and gathering experts' preferences.
4) Compute the current level of agreement. If the level achieved is enough, then the process ends and the group moves onto the selection of alternatives; otherwise, go to step (5).
5) Feedback generation for experts. The moderator identifies alternatives that hamper reaching a consensus and suggests experts modifying preferences on such alternatives, in order to make their opinions closer to each other in the following rounds.
6) Go back to step (3) and continue discussion process. A parameter indicating a limit of discussion rounds can be

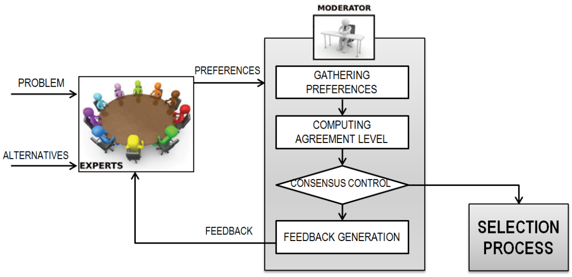


Fig. 1: General CRP scheme

used to stop the process when consensus is not achieved after several discussion rounds.

In order to support CRPs computationally and overcome the difficulties of gathering experts together into physical meetings, several CSSs based on intelligent techniques have been proposed by different authors and implemented to be put in practice [14], [20]. Such CSSs have been developed upon different theoretical consensus models, some of which allow an automation of the tasks carried out by the human moderator [12], [15].

B. *Multi-Agent Systems*

Amongst the current challenges and difficulties of CRPs stated in the introduction, it was pointed out the importance of selecting a CSS architecture suitable to deal with large-scale GDM problems efficiently, and the necessity of an approach that minimizes human experts' supervision of preferences, without eliminating their sovereignty completely.

The MAS paradigm would be a convenient choice to develop a CSS that overcomes the above mentioned difficulties, due to its scalability, distributed computing capabilities and the possibilities it offers to model different types of behavior by means of software agents. In MAS technology, the term *agent* refers to a software entity capable of achieving a goal in an autonomous and intelligent way, exchanging information with its environment or with other agents [21]. An agent in a MAS is independent and capable of making its own decisions [21], [22]. A MAS can be defined as a system composed by a number of agents with different roles and responsibilities, that operate in an organized and coordinate way to achieve an individual or collective goal [22].

Different standards have been proposed to support the development of MAS, such as FIPA and RETSINA, amongst others. FIPA[1] is one of the most extended architectural standards, characterized by defining a collection of specifications aimed to guarantee the inter-operability of heterogeneous MAS with each other, and with other technologies as well. Some of the main FIPA specifications are:

- A language so-called FIPA-ACL (*Agent Communication Language*) to enable an effective agent communication based on the exchange of messages.
- A set of interaction protocols to emulate different communicative acts between agents (e.g. requests, proposals, queries, etc.)
- A content language that facilitates the use of ontologies in the content of messages exchanged between agents, with the purpose of representing and managing knowledge about a domain in a structured way, and enabling a comprehensive agent communication under a common language and semantics [35].

FIPA standards have been utilized in a large number of MAS proposed in the literature [36]–[38].

In order to support developers in the implementation of agent-based applications which are compliant with FIPA standards, some development frameworks and platforms have arisen, being JADE[2] one of the most utilized ones. JADE (Java Agent DEvelopment Framework) is an open source middleware platform implemented in Java language [39], that incorporates a library of FIPA interaction protocols, allows the use of content languages and ontologies in agent communication and, most importantly, simplifies the overall development of highly portable and distributed MAS, while guaranteeing a full compliance with standards. JADE also makes it possible to develop mobile agent-based applications [40] and the integration of MAS with Web Services [41], amongst other interesting features [39]. JADE has been extensively used in the development of MAS in a variety of research fields [36], [37], [40]–[44].

A number of approaches based on MAS to support group decisions have been proposed in the last few years. They are briefly revised, together with some related work in consensus reaching, in the following subsection.

C. *Related Work*

In the following, it is revised some related work on consensus models for GDM and proposals of CSSs that implement such models. Then, some examples of MAS focused on supporting different types of negotiation processes to seek agreements in group decisions are briefly reviewed.

Saint et. al proposed in [8] a theoretical consensus model that describes CRPs as they usually occur in real organizations and companies. The model considers diverse social aspects, including the initial proposal's presentation and acceptation, resolution of concerns and alternative actions to perform when failing to reach a consensus; and introduces some roles to support the consensus reaching process.

Classic consensus models are aimed to reach a consensus as unanimous agreement, which is sometimes difficult to achieve in practice. Therefore, some authors proposed consensus models based on more flexible notions of consensus. For instance, Kacprzyk et al. proposed several consensus models inspired by the concept of *soft consensus* [11], [45], establishing a fuzzy consensus measure based on the use of linguistic quantifiers to apply the concept of fuzzy majority [46], which permits to measure the level of agreement in a consistent way, similarly

[1] FIPA (Foundation for Intelligent Physical Agents): http://www.fipa.org

[2] Latest JADE version (released on 29/03/2013) can be found at: http://jade.tilab.com

to human reasoning. One of these models, proposed in [14], has been implemented into a Web-based CSS.

Another key issue that had not been studied yet in the definition of new consensus models is the automation of the human moderator in CRPs. An example of model that addressed such an issue was the work of Mata et al. in [12], which presented an adaptive consensus model, which adapts its behavior to the level of agreement achieved in each discussion round. Thus, once a global consensus degree is computed in each discussion round, its closeness to a consensus threshold (the minimum level of agreement desired) is determined in order to choose the most appropriate policy to generate feedback. This model automates most of the the human moderator's tasks, which makes it suitable to develop CSSs based on it. More recently, some consensus models incorporating additional techniques to manage knowledge, such as the use of ontologies, have been proposed by Kacpzryk and Zadrozny in [20].

Parreiras et al. presented in [10], [13] some flexible consensus schemes to deal with multi-criteria GDM problems in a multi-granular linguistic framework, where the aggregation of experts' preferences and the process to assign them weights can be conducted in different ways: either based on a discordance measure, or by means of an optimization algorithm. The proposed aggregation processes in these models guarantee obtaining a consistent collective opinion.

One of the most important aspects to consider when developing a CSS consists in achieving a high automation degree and minimizing the need for human supervision, but not many consensus models present in literature are designed to apply a direct automation on them. The model proposed by Xu in [15] addresses this problem by developing an automatic approach to reach consensus in multi-criteria GDM problems, characterized by iteratively modifying the (initially diverging) experts' opinions, to reach consensus amongst them. The underlying algorithm in this model has proved to converge towards consensus, thus guaranteing its effectiveness.

Despite Xu's approach clearly addresses the problem of cost and time consumption due to constant supervision in CRPs, in real situations it would be sometimes desirable that experts have the opportunity to revise and accept/reject the modifications proposed on their preferences before they are applied, especially in the cases that such modifications imply a substantial change in their overall opinion. The compromise between automating CRPs to reduce the cost invested in them, and preserving experts' sovereignty in the above mentioned situations, is one of the main goals achieved with the semi-supervised CSS proposed in this paper.

Regarding proposals based on MAS for group decisions, several authors have focused their research on multi-agent architectures applied to negotiation frameworks. A preliminary discussion on the use of MAS for supporting distributed negotiation processes can be found in [47]. A review of different group negotiation protocols (e.g. voting methods, bargaining, auctions, etc.) is given in this work, together with the basic guidelines to model such protocols by means of software agents.

Hindriks et al. proposed in [48] an agent-based architecture for negotiation processes. In [49], they instantiated such an architecture and put it in practice to conduct bilateral multi-issue negotiations in e-commerce, by modeling different buyer and seller tactics to be adopted by each agent.

More recently, Sánchez-Anguix et al. presented in [50] an agent-based negotiation model aimed to automate purchases in e-markets, in which a decision group coordinated by a mediator (collective buyer) must negotiate a deal with an opponent (seller) before proceeding to purchase a product. A thorough research on different team strategies and agreement technologies to be considered in such a model was later presented by the same authors in [51].

A multi-agent approach for large-scale group decisions was proposed by Okumura et al. in [52]. They presented a MAS for collaborative park-design support, characterized by gathering opinions from human experts, estimating utility functions upon such preferences and applying an automated agent-based negotiation protocol to find optimal agreements. The negotiation process to find a consensus is carried out in a completely autonomous way, therefore human experts provide their preferences to the system only at the beginning of the process.

The works revised above utilize the MAS paradigm to support group decisions that require a high level of agreement by means of specific negotiation frameworks and protocols (e.g. auctions and bargaining) but, as far as we know, there are still no proposals based on MAS to support CRPs in GDM problems under uncertainty considered in our research field [9], [13], [20], [30]. The development of a MAS-based CSS (such as the one presented in this paper), would be particularly convenient when it comes to deal with large-scale GDM problems, due to the considerable computational cost and scalability required.

III. MULTI-AGENT SYSTEM TO SUPPORT CONSENSUS REACHING PROCESSES

In this section, our proposal for a semi-supervised multi-agent based CSS is presented. This system is aimed to facilitate, guide and automate CRPs in large-scale GDM problems defined under uncertainty, replacing the human moderator and providing intelligent agents with as much autonomy as possible to minimize the need for human supervision by experts. Its highly scalable multi-agent architecture is suitable for dealing with GDM problems in which a large number of experts must take part. The theoretical consensus model considered is first described. The agent semi-supervised autonomy approach proposed, which is the main novelty of the CSS developed, is then presented. Afterwards, the main aspects of the multi-agent architecture, including agents implemented, communication mechanisms and ontologies used to exchange information, are briefly described.

A. Consensus Model

Our proposal for a CSS allows the inclusion and use of different consensus models proposed in the literature. In this paper, we will consider a consensus model that extends the main ideas of some models presented in [12], [19], and is characterized by the use of flexible consensus measures to determine the level of agreement as a value in [0,1]. This model is aimed at the resolution of GDM problems under uncertainty in which fuzzy preference relations are the preference structure used by experts to express their opinions.

This model attempts to facilitate a full automation degree on the moderator's responsibilities, and a high automation of experts' behavior. Therefore, a novel semi-supervised autonomy mechanism for agents is proposed in this paper as an additional feature of the CSS aimed to complement the consensus model and increase the system autonomy. Such an approach will be further described in Sect. III-B.

The context to define GDM problems addressed in this model is as follows: consider a set E of m experts, in which each $e_i \in E$ expresses his/her preferences over a set X of n alternatives by means of a fuzzy preference relation $P_i = (p_i^{lk})^{n\times n}$, being $p_i^{lk} \in [0,1]$ the assessment given by e_i to the pair of alternatives (x_l, x_k).

Following, we describe in detail the phases composing the model:

- *Call to participate in a problem*: The moderator invites experts to participate in a problem, informing them about the set of existing alternatives to solve it. Each expert must decide whether he/she participates or not within a defined time interval. Before the process begins (provided that at least two experts participate), the initial problem parameters are fixed, including a consensus threshold, $\mu \in [0,1]$, and the maximum number of rounds permitted, *Maxrounds*.
  In certain circumstances where some experts might be more familiar with the GDM problem than others, present different degrees of knowledge about it or have different roles/positions in the group, it would be reasonable that the moderator assigns them different importance weights $\lambda = [\lambda_1 \dots \lambda_m]$, being $\lambda_i \in [0,1]$ the importance weight assigned to expert e_i [3], [13]. Such importance weights will be taken into account in a latter phase of the model that computes a collective preference for the group [3]. Weights can be determined by using different existing methods, for instance they can be explicitly assigned by a moderator of the group, based on the role and/or degree of expertise of each expert [3], [15], or it can be applied an optimization technique to determine them [13].
- *Gathering Preferences*: As a result of a discussion process, experts provide their preferences P_i to the moderator by means of fuzzy preference relations. It is advisable that experts' opinions would be consistent [53], [54], which could be easier to accomplish if assessments are reciprocal, i.e. if $p_i^{lk} = x$, $x \in [0,1]$, $l \neq k$, then $p_i^{kl} = 1 - x$.
- *Compute Consensus Degree*: The moderator computes the level of agreement between experts, by means of the following steps:
  1) For each pair of experts e_i, e_j, $(i < j)$ a similarity matrix $SM_{ij} = (sm_{ij}^{lk})^{n\times n}$, defined by
  $$SM_{ij} = \begin{pmatrix} - & \dots & sm_{ij}^{1n} \\ \vdots & \ddots & \vdots \\ sm_{ij}^{n1} & \dots & - \end{pmatrix}$$
  is computed as follows [19]:
  $$sm_{ij}^{lk} = 1 - |(p_i^{lk} - p_j^{lk})| \quad (3)$$
  where $sm_{ij}^{lk} \in [0,1]$ is the similarity degree between experts e_i and e_j in their assessments p_i^{lk}, p_j^{lk}.
  2) A consensus matrix $CM = (cm^{lk})^{n\times n}$ is computed by aggregation of similarity matrices. Each element cm^{lk} is computed as [12]:
  $$cm^{lk} = \phi(sm_{12}^{lk}, \dots, sm_{1m}^{lk}, sm_{23}^{lk}, \dots, sm_{2m}^{lk}, \dots, sm_{(m-1)m}^{lk}) \quad (4)$$
  where ϕ is the aggregation operator used. Different aggregation operators can be used in our system to reflect a flexible notion of consensus [30], thus obtaining partial degrees of agreement in the unit interval.
  3) In order to obtain the level of agreement achieved between experts not only about a given assessment, but also about each alternative and the GDM problem as a whole, a consensus degree is computed at three different levels:
     a) Level of pairs of alternatives (cp^{lk}): Obtained from CM as $cp^{lk} = cm^{lk}, l,k \in \{1,\dots,n\}, l \neq k$.
     b) Level of alternatives (ca^l): The level of agreement on each alternative $x_l \in X$ is computed as $ca^l = \varphi(cp^{l1}, \dots, cp^{l(l-1)}, cp^{l(l+1)}, \dots, cp^{ln})$.
     c) Level of preference relation (overall consensus degree, cr): The global agreement achieved in the current round is computed as $cr = \nu(ca^1, \dots, ca^n)$.

  Being φ, ν aggregation operators. Notice that we do not necessarily use the same operator for all the steps involving aggregation of information throughout the process, which gives a higher degree of flexibility to the consensus model proposed.
- *Consensus Control*: The consensus degree cr is compared with a consensus threshold μ. If $cr \geq \mu$, the consensus process ends successfully and the group moves on to the alternatives selection process; otherwise, the CRP requires further discussion. $Maxrounds$ controls the maximum number of discussion rounds allowed. If this parameter is exceeded, an alternate strategy might be adopted, such as applying a classic GDM rule (see Sect. II-A). Some examples of such alternate strategies to be adopted in these situations could be [8]:
  – Delegate the decision to a subgroup, either due to the major degree of importance they present compared to the rest of the group, or because their opinions are closer to each other.
  – If some experts with clearly conflicting opinions are found, conduct a community building session, consisting in involving formal mediation from experts whose opinions are outside the conflict.
  – Conduct a simple majority vote.
  – Exclude group members who did not contribute to achieve a consensus, i.e. their opinion is far from the

collective opinion and they also rejected applying some changes suggested on their preferences (see *Generate Recommendations* phase below).

- *Generate Recommendations*: If $cr < \mu$, experts are advised to modify their preferences in order to increase the level of agreement in the following rounds. Three steps are considered in this phase:
  1) Compute a collective preference and proximity values for experts: A collective preference $P_c = (p_c^{lk})^{n \times n}$ is computed for each pair of alternatives by aggregating experts' preference relations:
  $$p_c^{lk} = \psi_\lambda(p_1^{lk}, \ldots, p_m^{lk}) \quad (5)$$
  where ψ is an aggregation operator and $\lambda = [\lambda_1 \ldots \lambda_m]$ is the vector of experts' importance weights [3], [13], [15], which can be taken into account in this step if a weighted aggregation operator is chosen to compute P_c. Afterwards, a proximity matrix $PP_i = (pp_i^{lk})^{n \times n}$ between each expert's preference relation and P_c is obtained:
  $$PP_i = \begin{pmatrix} - & \ldots & pp_i^{1n} \\ \vdots & \ddots & \vdots \\ pp_i^{n1} & \ldots & - \end{pmatrix}$$
  Proximity values pp_i^{lk} are obtained for each pair (x_l, x_k) as follows:
  $$pp_i^{lk} = 1 - |(p_i^{lk} - p_c^{lk})| \quad (6)$$
  Proximity values are used to identify the furthest preferences from the collective opinion and, therefore, those preferences to be changed.
  2) Identify preferences to be changed (CC): Pairs of alternatives (x_l, x_k) whose consensus degrees ca^l and cp^{lk} are not enough, are identified:
  $$CC = \{(x_l, x_k) | ca^l < cr \wedge cp^{lk} < cr\} \quad (7)$$
  Afterwards, the model identifies experts who should change their opinion on each of these pairs, i.e. those experts e_i whose preference p_i^{lk} on the pair $(x_l, x_k) \in CC$ is furthest to p_c^{lk}. An average proximity $\overline{pp}^{lk}$ is calculated to identify them, by means of an averaging aggregation operator Γ, as follows:
  $$\overline{pp}^{lk} = \Gamma(pp_1^{lk}, \ldots, pp_m^{lk}) \quad (8)$$
  As a result, experts e_i whose $pp_i^{lk} < \overline{pp}^{lk}$ are advised to modify their assessment on pair (x_l, x_k).
  3) Establish change directions: Several direction rules are applied to suggest the direction of changes proposed to experts, in order to increase the level of agreement in the following rounds. In [12], an approach to generate direction rules was proposed. However, such an approach is too strict, in the sense that an excessive number of changes is suggested, even when the expert's opinion is very close to the collective opinion. Therefore, we propose extending it by introducing an acceptability threshold for the whole group, $\varepsilon \geq 0$, which should take a positive value close to zero (usually $\varepsilon \in [0, 0.1]$), in order to allow a margin of acceptability when p_i^{lk} and p_c^{lk} are close enough to each other.
     - DIR.1: If $(p_i^{lk} - p_c^{lk}) < -\varepsilon$, then expert e_i should *increase* the assessment associated to the pair of alternatives (x_l, x_k).
     - DIR.2: If $(p_i^{lk} - p_c^{lk}) > \varepsilon$, then expert e_i should *decrease* the assessment associated to the pair of alternatives (x_l, x_k).
     - DIR.3: If $-\varepsilon \leq (p_i^{lk} - p_c^{lk}) \leq \varepsilon$ then expert e_i should not modify the assessment associated to the pair of alternatives (x_l, x_k).

     The degree of increase/decrease in assessment p_i^{lk} may depend on the prospects and behavior of each expert e_i during the CRP. This aspect is partially considered in the agent semi-supervised autonomy approach presented in the following subsection.

B. Agent Semi-Supervised Autonomy Approach

The constant human supervision required by decision makers to revise and modify their preferences throughout the CRP can lead to several problems, including the excessively high amount of time invested, and the possibility that some experts might abandon the CRP, because of their lack of interest and motivation to continue with the tedious supervision process. Therefore, the most important novelty in our system is the inclusion of a agent semi-supervised approach aimed to eliminate such constant supervision, by increasing the system's autonomy during the overall CRP, thus achieving the main goal stated in the introduction.

Our system's underlying consensus model is managed by agents that operate cooperatively to reach an agreement. Such agents should be as much autonomous as possible, therefore they implement a semi-supervised approach that allows experts to modify and provide their opinions in a semi-supervised way, by delegating these tasks to agents, in order to minimize the need for human expert supervision during the process. It is remarkable that the semi-supervised approach presented here is not necessarily dependent on the theoretical consensus model proposed in the previous subsection, but it can be rather viewed as an additional module of our CSS which might be adapted and applied in combination with any other consensus models proposed in the literature by different authors [12], [13].

Two questions arise in the definition of the agent semi-supervised autonomy mechanism:

1) *Establishing a degree of change, i.e. the level of increase/decrease that an expert applies when he/she has received some recommendations to modify his/her preferences* (see *Generate Recommendations* phase in the consensus model, Sect. III-A).
   Regarding this question, it is usual in any real CRP that experts follow different strategies to reach the agreement, such as modifying their opinions significantly from the beginning of the process to achieve an agreement quickly or acting more conservatively to keep

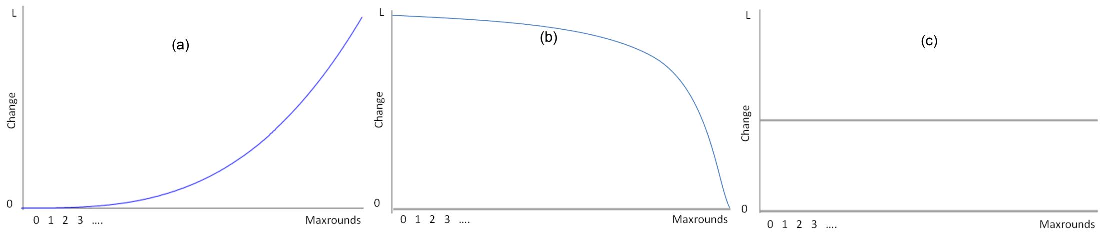


Fig. 2: Example of change functions for: (a) *sure* profile, (b) *unsure* profile and (c) *neutral* profile

their initial opinions as intact as possible. Based on these models of behavior, and inspired by [55], three different user *change profiles* are defined, as follows:

- *Sure profile*, representing experts who are quite sure about their initial opinions, so that they consider such opinions more important than achieving a consensus. Therefore, they are reluctant to apply changes on their preferences at the beginning of the process, but they become more concerned about achieving an agreement as the number of discussion rounds increases.
- *Unsure profile*, representing experts who want to achieve a consensus but are rather unsure about their initial opinions, therefore they are more determined to apply substantial changes on them, although the degree of change decreases as the discussion process develops.
- *Neutral profile*, representing experts who are moderately sure about their initial opinions, but are also convinced about the need for achieving an agreement, therefore they are determined to apply changes uniformly during all the process.

The multi-agent system allows each human expert to choose the profile that best reflects his/her individual concerns. An increasing, decreasing or constant mathematical function, so-called *change function*, can be used to model a sure, unsure or neutral profile, respectively, as shown in Figure 2 (similarly as other agent negotiation functions, such as *Kasbah* [55]). These change functions determine the degree of increase/decrease to be applied by experts on their assessments, depending on the profile chosen and the round of discussion where such changes have been suggested. From now on, an expert's assessment value at the beginning of a consensus round $r \in \{1, \ldots, Maxrounds\}$ will be denoted as p_{ir}^{lk}. Given an expert e_i, the change function associated to his/her chosen change profile is formally defined as follows:

$$\Delta_i : [0, Maxrounds] \rightarrow [0, L] \tag{9}$$

being $\Delta_i(r) \in [0, L]$ the variation (increase or decrease) applied to the assessment elicited in the previous round $p_{i(r-1)}^{lk}$. L is a parameter used to set an upper bound of the degree of change applied to an assessment in a given round r, according to the direction rules. Therefore, an expert's new assessment p_{ir}^{lk} on the pair (x_l, x_k) which has been given a recommendation at the end of round $r-1$ $(r \geq 2)$, can be computed for a given change profile as follows:

$$p_{ir}^{lk} = p_{i(r-1)}^{lk} \pm \Delta_i(r), \tag{10}$$

being the initial preference assessments denoted as p_{i1}^{lk}.

2) *Setting an appropriate degree of autonomy to let agents apply suggested changes by themselves, without requiring human supervision to do it.*
Regarding this issue, change profiles could initially eliminate the need for human expert supervision during all the process [15], as explained above. Notwithstanding, there are some situations where experts are quite sure about their preference towards a specific alternative and they would prefer to decide by themselves about changes suggested. Therefore, the proposed semi-supervised approach lets agents in the system apply changes suggested on experts' preferences p_{ir}^{lk} autonomously, unless such changes imply a substantial change in their preferences. Different rules can be proposed to decide whether a change on an assessment p_{ir}^{lk} should be supervised by a human expert e_i or not. These rules might consider different criteria to decide about the need for human supervision. It is noteworthy that such criteria and rules can be added and/or adjusted in our system based on each problem, and they can also be personalized by each expert according to his/her individual concerns. This aspect, together with the definition of multiple change profiles/functions, provides the agent semi-supervised approach with a high degree of flexibility. A possible example of criteria for supervision rules are:
   - Require human supervision on p_{ir}^{lk} when the preferred alternative varies respect to its corresponding initial assessment, p_{ir}^{lk}, i.e. $p_{ir}^{lk} > 0.5$ and $p_{i1}^{lk} \leq 0.5$, or vice versa.
   - Require human supervision when the degree of change respect to p_{ir}^{lk} exceeds a threshold.
   - Apply changes autonomously when the degree of change respect to p_{ir}^{lk} is too low to consider human supervision necessary.

Regarding the second criterion, we introduce a parameter so-called maximum change threshold $\kappa_i \in [0, 1]$, which must be defined by each expert e_i at the beginning of the CRP, and indicates how much increase/decrease does e_i accept on his/her assessments out without requiring supervision. Additionally, the acceptability threshold ε introduced in the advice generation phase in Sect. III-A, can be reused here to control situations where the variation in an expert's assessment is too small to consider the need for human supervision. Notice that the value of ε is fixed for the whole group.

Based on the above proposed criteria, some supervision rules can be formulated as follows:

R.1: If $|p_{i1}^{lk}-p_{ir}^{lk}| > \kappa_i$, i.e. the degree of change respect to the initial assessment is higher than κ_i, then request human supervision. Otherwise, check the following rule (R.2).

R.2: If either one of these conditions holds:
(a) $p_{ir}^{lk} > 0.5$ AND $p_{i1}^{lk} \leq 0.5$ AND $|p_{i1}^{lk}-p_{ir}^{lk}| \geq \varepsilon$.
(b) $p_{ir}^{lk} < 0.5$ AND $p_{i1}^{lk} \geq 0.5$ AND $|p_{i1}^{lk}-p_{ir}^{lk}| \geq \varepsilon$.
i.e., if the preferred alternative (either x_l or x_k) varies respect to the initial assessment and the degree of change is not lower than ε, then request human supervision. Otherwise, apply changes autonomously.

In the cases that the system requests supervision to the corresponding human expert before applying the changes, he/she is in charge of deciding whether accepting or not the proposed recommendation to modify preferences.

In order to give a better understanding of these rules, in the following we show some brief examples.

Example 1. *Suppose* $p_{i1}^{lk} = 0.9$*,* $p_{ir}^{lk} = 0.55$ *and* $\kappa_i = 0.3$*. By checking R.1, we can see that* $|p_{i1}^{lk} - p_{ir}^{lk}| = 0.35 > \kappa_i$*, therefore* e_i*'s supervision is required to set his/her assessment on* (x_l, x_k) *as* $p_{ir}^{lk} = 0.55$*.*

Example 2. *Suppose* $p_{i1}^{lk} = 0.6$*,* $p_{ir}^{lk} = 0.45$*,* $\kappa_i = 0.3$ *and* $\varepsilon = 0.05$*. By checking R.1, we can see that* $|p_{i1}^{lk} - p_{ir}^{lk}| = 0.15 < \kappa_i$*, therefore R.2. must be checked. Condition (b) in R.2 holds, therefore human supervision is required.*

Example 3. *Suppose* $p_{i1}^{lk} = 0.52$*,* $p_{ir}^{lk} = 0.48$*,* $\kappa_i = 0.3$ *and* $\varepsilon = 0.05$*. By checking R.1, we can see that* $|p_{i1}^{lk} - p_{ir}^{lk}| = 0.04 < \kappa_i$*, therefore R.2. must be checked. In this case, neither one of the two conditions in R.2 holds (despite* $p_{ir}^{lk} < 0.5$ *AND* $p_{i1}^{lk} > 0.5$*), because the degree of change is lower than* ε*, therefore agents in the CSS apply the change suggested autonomously.*

To sum up, the semi-supervised mechanism described preserves a full automation of moderator's tasks, and it introduces a high degree of autonomy for experts, who are only responsible for providing their initial preferences and accepting/rejecting suggested recommendations manually when they imply a substantial change on the assessment value and/or the alternative they prefer, for a given assessment p_{ir}^{lk}.

C. Multi-Agent System Architecture

In this subsection, the key components of our multi-agent based CSS are described. Such components are, namely: the software agents implemented to support CRPs, the communication mechanisms and protocols considered to allow agents communicate with each other, and the double ontology used by them to exchange information.

The system has been developed based on JADE, which complies with FIPA standards (as mentioned in Sec. II-B), and its main components are depicted in Figure 3. Several types of agents, each one with a specific role, have been designed and implemented with the purpose of supporting CRPs:

- *Expert Agent*: An expert agent represents a human expert in the system, acting autonomously. Expert agents implement the change profiles and rules defined by the semi-supervised autonomy approach (see Sect. III-B).
- *Moderator Agent*: This agent assumes the human moderator role, automating his/her responsibilities. Due to the complexity found to implement moderator's tasks into a single agent, several agents were introduced to support it:
  - *Consensus Evaluator Agent*: This agent is in charge of computing consensus degree, as well as informing moderator agent about it.
  - *Change Detector Agent*: Its responsibility is focused on carrying out the phase of generating recommendations.
  - *Analyst Agent*: It provides functions to store and recover information about past problems persistently.

Other essential components in the system architecture are:

- A set of *Interface Agents* that let human experts provide their preferences and communicate with their respective expert agents when human supervision is required.
- A double ontology [35], based on the ideas presented in [20], to facilitate communication between agents.
- The implementation of the underlying consensus model, as described in Section III-A.
- A database to store data about previous consensus rounds in the GDM problem.

Given the importance of the agents specifically developed for our CSS, following we describe in further detail their main responsibilities in the CRP, as well as the communication flow between them and the ontologies designed.

1) CSS Agents and their Responsibilites: Some of the most important tasks carried out by the implemented agents in the overall CRP are described below:

- **Moderator Agent**: Besides replacing the human moderator, this agent is responsible for mediating all communicative acts between agents, therefore it is a core element of the system. As occurs in real CRPs, only one moderator agent takes part in the solution of a problem. Its main functionalities are:
  i) *Call to participate in a problem*: The moderator agent sends a proposal to the rest of agents, inviting them to take part in a GDM problem.
  ii) *Assign importance weights to experts*: In the case that experts with different degrees of knowledge and/or expertise take part in the GDM problem, the moderator agent may assign each of them an importance weight λ_i (see Sect. III-A). The moderator agent can conduct this task automatically, based on each expert agent profile, which might

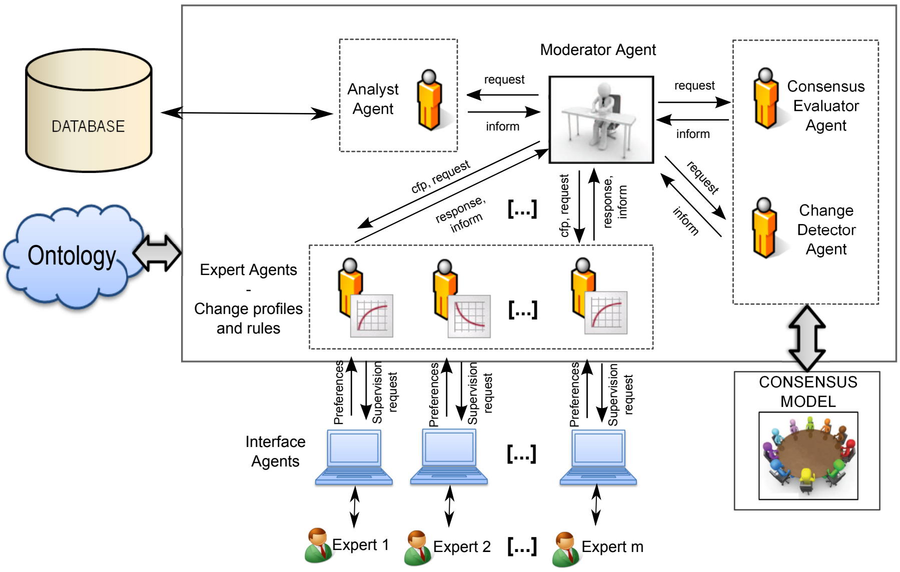


Fig. 3: Architecture of the system

include information about the role/status of the corresponding human expert.

iii) *Request Preferences*: At the beginning of each round, the moderator agent requests expert agents their preferences. If one or more discussion rounds have already taken place, the request includes the set of recommendations to be applied by each expert.

iv) *Request computing consensus degree* to consensus evaluator agent.

v) *Consensus Control*: If consensus degree is enough, moderator agent informs expert agents about it. Otherwise, it requests change detector agent to compute change recommendations.

- **Expert Agent**: Its goal is to automate in a semi-supervised way, as much as as possible, the tasks carried out by human experts in real CRPs. Therefore, the semi-supervised approach presented in Sect. III-B is implemented as part of the expert agents' behavior. Each human expert has associated an expert agent during a GDM problem in which he participates. The main expert agents' functionalities are described below:

i) *Send decision about participating in a problem*: The expert agent gathers and sends to the moderator agent an experts' decision about taking part in a proposed GDM problem. Before agreeing to participate, an expert may choose a change profile and/or personalize the supervision rules (see Sect. III-B), according to his/her requirements.

ii) *Elicitation of Preferences*: Expert agent provides to the moderator agent a preference relation on the set of alternatives considered. In the first discussion round, human experts are responsible for introducing such preferences.

iii) *Apply Changes on Preferences*: When a change recommendation on preferences is received, the expert agent checks it before giving preferences back to moderator agent. Here, the semi-supervised approach facilitates a high degree of autonomy to let agents carry out this task without human supervision in most cases.

- **Consensus Evaluator Agent**: Some of the human moderator's tasks are delegated to specific agents in our system, being the consensus evaluator agent one of them. This agent accesses the consensus model to perform the necessary operations to obtain a consensus degree at each round, which is sent to the moderator agent.
- **Change detector Agent**: This agent is invoked by moderator agent when consensus degree is not enough, to identify furthest preferences from the agreement and determine which expert agents must be given recommendations to modify such preferences.
- **Analyst Agent**: It is only responsible for storing in a database information related to each CRP carried out.

2) Agent Communication: Agents communicate each other by exchanging FIPA-ACL messages according to two FIPA communications protocols based on communicative acts [56]: *Call for Propose* and *Request* (see Fig. 3).

- *Call for Propose (cfp)* is part of a more complex protocol so-called *contract-net*. An initiator agent proposes one or more receivers to participate in an action. Each receiver may accept or reject the proposal. This protocol is used by the moderator agent to invite the rest of agents to participate in a problem.
- *Request* consists in the request of a resource (normally information) to one or several receiver agents, who decide whether agreeing or refusing it. If a receiver agent agrees

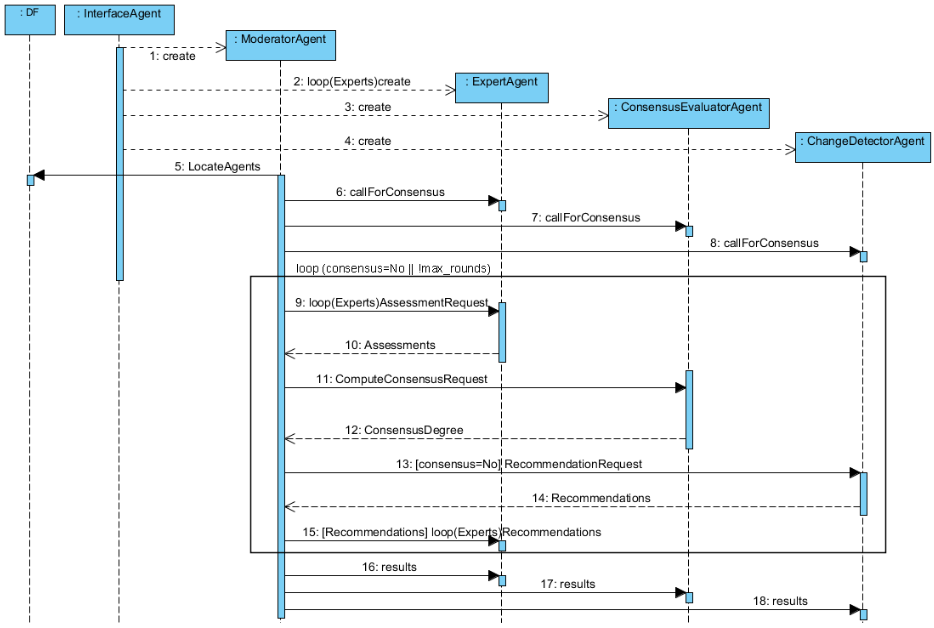


Fig. 4: Overall communication between agents

with the request, it must immediately return an inform message containing the requested resource. This protocol is used by the moderator agent during the CRP repeatedly, to request information such as consensus degree, recommendations, preferences, etc.

Since many communication flows between our agents have been defined and implemented based on these protocols, a simplified overall sequence diagram representing the main communicative acts between agents during a complete CRP is shown in Figure 4. Notice that before beginning a CRP, different *create* messages from interface agents are generated, since users of the system first communicate with such agents to instantiate the rest of agents in the platform. Dashed lines represent responses to request or proposal messages, brackets represent conditions, and the loop spanning the necessary messages to carry out a CRP round is represented as a rectangle.

Regarding agent communication from the viewpoint of the agent semi-supervised approach (see Sect. III-B), it is worth noting that the interaction between the moderator agent and an expert agent is not affected by the use of such an approach. The reason for this, is that once the moderator agent sends each expert agent the FIPA-ACL message containing its recommendations in a given discussion round, the latter is responsible for deciding whether applying each recommendation autonomously or asking its corresponding human expert for supervision to accept or not the change suggested. This process is inherent to the expert agent, which implements the change profile and supervision rules previously chosen by the human expert, hence the fact that the semi-supervised approach's operation does not affect agent communication flow.

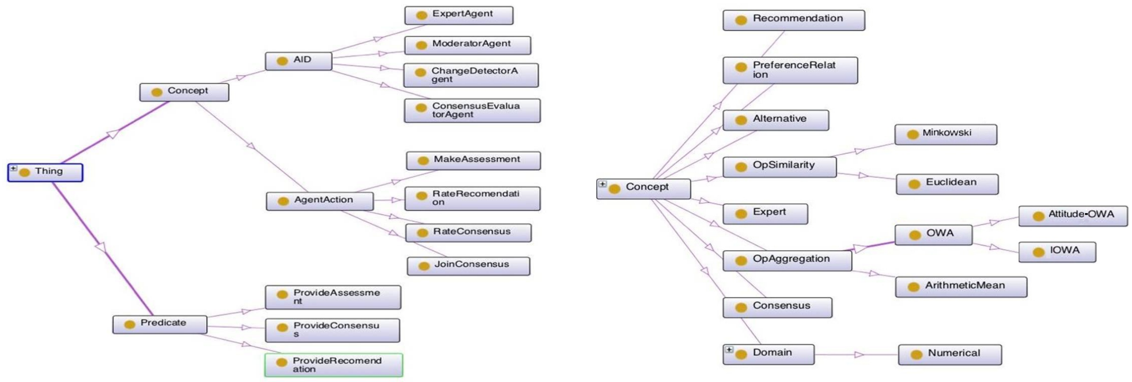
3) Ontology Design: A key aspect in the MAS design was the definition of an appropriate ontology to represent knowledge about the problem addressed, and facilitate an effective and comprehensive communication between agents in a common language and semantics [35].

It is necessary to design an ontology that defines all communicative acts carried out by agents throughout the overall CRP. To do so, we consider the approach proposed by Kacprzyk and Zadrozny in [20], where two ontologies to carry out CRPs were defined: (i) an ontology to represent general knowledge about CRPs, and (ii) and ontology to represent knowledge related to each particular GDM problem to be solved. Based on this idea, we define two ontologies:

i) An *application domain* ontology, including necessary elements to represent knowledge related to CRPs as conducted in the system, such as the agents' roles and the actions performed by the moderator agent, for instance (Fig. 5a).
ii) A *problem domain* ontology, used to represent knowledge about each particular GDM problem addressed in a given moment. An example of knowledge represented with this ontology is the set of alternatives defining the problem, or the consensus degree achieved in each round (Fig. 5b).

These ontologies are based on JADE content model [39], where elements are classified into three categories:

(i) *Concepts*, i.e. expressions representing objects and characterized by attributes, which are included in FIPA-ACL messages as part of a predicate or agent action. Concepts defined here include all elements defined in the problem domain ontology (see Fig. 5b), as well as agent



(a) Application Domain Ontology (b) Problem Domain Ontology

Fig. 5: System ontologies.

identifiers, defined in the application domain ontology.

(ii) *Predicates*, i.e. expressions about the state of the world, which can be either true or false and are normally used as a response to requests and proposals. The following predicates have been defined: *ProvideAssessment*, used by expert agents to provide their preferences; *ProvideConsensus*, used by consensus evaluator agent to inform about the consensus degrees achieved; and *ProvideRecommendation*, used by change detector agent to send each piece of advice generated.

(iii) *Agent actions*, i.e. expressions indicating actions to be conducted by agents, normally used as the content of FIPA-ACL request messages. Agent actions defined in our system are used by the moderator agent to request information to the rest of agents, and include: *JoinConsensus*, to invite agents to participate in a problem; *MakeAssessment*, to request experts' preferences; *RateConsensus*, to request computing consensus degrees; and *RateRecommendation*, to request carrying out the advice generation phase.

The only components that agents in our system manage directly to communicate with each other are predicates and agent actions. Such predicates and agent actions are usually composed by one or several terms, which could be either concepts, primitives (i.e. attributes belonging to a simple data type, e.g. numerical, string, etc.) or aggregates (e.g. collections of primitives or concepts). For example, the agent action *JoinConsensus*, which is used by the moderator agent as the content of a FIPA-ACL propose message when inviting the rest of agents to participate in a GDM problem, is formed by the following terms:

- *maxRounds*: A integer-type primitive indicating the value of parameter *Maxrounds*.
- *setOfAlternatives*: Aggregate of instances of the concept *Alternative*, containing the set of existing alternatives in the problem.
- *problemDescription*: String-type primitive that describes the problem to solve.

IV. Case Study

Once presented and described the operation and main features of the proposed semi-supervised MAS to support large-scale CRPs, this section shows a case study in which the system is used to solve a real-life large-scale GDM problem. Such a problem is solved twice, by using the semi-supervised CSS proposed in this paper and another version of the system that includes a full-supervised approach of experts' preferences, with the aim of providing a comparison between results and findings obtained from each system and showing the improvements achieved by using the semi-supervised approach.

This case study considers a large-scale GDM problem in a real-life environment, in which experts who are highly motivated and interested in such a problem take part. The problem is formulated as follows: the 2013 graduating class of Computer Science M.Sc. Degree, compound by 46 students, $E = \{e_1, \ldots, e_{46}\}$, needs to achieve an agreement before deciding the destination for their final year trip, amongst four possible choices, $X = \{x_1$: Mediterranean cruise, x_2: Tunisia tour, x_3: Canary Islands, x_4: Prague, Vienna and Budapest$\}$. All students' preferences are regarded as equally important. The students have to reach a high level of agreement ($\mu = 0.85$) before making the decision. The maximum number of discussion rounds allowed is $Maxrounds = 10$, the acceptability threshold for advice generation is set as $\varepsilon = 0.02$ and, without loss of generality, the *arithmetic mean* is chosen as the aggregation operator used throughout the process.

Before carrying out this case study, students had attempted to reach an agreement on the trip destination by themselves, without the aid of any CSS, but they found some difficulties, mainly due to the high amount of time invested in discussing about their opinions without reaching an agreement. Consequently, we invited them to solve the problem with the aid of a CSS, and organized a lab session to which all 46 students attended. In order to carry out the comparative study, we randomly separated them into two subgroups of 23 students, and each subgroup was allocated in a different computer lab

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in which they used a different version of the CSS (semi-supervised and full-supervised).

At the beginning of both CRPs, each student provided an initial fuzzy preference relation over the four alternatives. In this case study, a large amount of information about students' preferences is managed. Due to this fact, the assessments provided and modified by them across the CRP are omitted in this paper for the sake of space, but they can be consulted in a separate document[3].

In the following, it is shown the specific settings and results of applying each CRP.

A. Resolution of a semi-supervised CRP

Before beginning the CRP, students who used the semi-supervised CSS chose their preferred change profile to let agents apply some of the changes suggested on their preferences autonomously. For each change profile defined in Sec. III-B, the following change functions have been defined upon an upper limit of change set as $L = 0.2$, *Maxrounds* and the current discussion round r, as follows:

- Change function for Neutral Profile:

$$\Delta_i(r) = \frac{L}{2} = 0.1 \tag{11}$$

- Change function for Sure Profile:

$$\Delta_i(r) = L\left(\frac{r}{Maxrounds}\right)^3 = 0.2\left(\frac{r}{10}\right)^3 \tag{12}$$

- Change function for Unsure Profile:

$$\Delta_i(r) = L\left(1 - \left(\frac{r}{Maxrounds}\right)^3\right) = 0.2\left(1 - \left(\frac{r}{10}\right)^3\right) \tag{13}$$

A total of 12 students chose a *neutral* profile, five students chose an *unsure* profile and the remaining six students chose a *sure* profile. Regardless of the change profile chosen, all students assumed a maximum degree of autonomous change on their preferences of $\kappa_i = 0.35$, $i = 1, \ldots, 4$ (see Sect. III-B).

Results of the discussion process are summarized in Table I, and they consist of the following features gathered at each discussion round r:

- *cr*: overall consensus degree achieved.
- *# changes*: Total number of recommendations suggested on a single expert's assessment $p_{i,r}^{lk}$ at a given round.
- *#ch_applied*: Number of changes applied in the current round, either autonomously by an expert agent, or supervised by the corresponding human expert.
- *#sup*: Number of recommendations on assessments that require human supervision.
- *accepted*: Number of supervised recommendations which are accepted by the human expert.
- *rejected*: Number of supervised recommendations which are rejected by the human expert and, therefore, they are not applied in the current round considered.
- *# exp. involved*: Number of experts involved in one or more assessments' supervision in the current round.
- *resp. time (min.)*: Response time, in minutes, required by the group to revise and accept/reject all supervisions they received at a given round. The total time invested in human supervision throughout the CRP is the sum of these response times.

  Remark 2. *Students carried out the CRP in a computer lab in which they could do other tasks simultaneously (e.g. chatting or browsing in the Internet), therefore response times were strongly dependent on their availability and degree of occupation at each moment.*

Results show that consensus is achieved in the sixth round. The number of recommendations generated, *#changes*, tends to decrease as the CRP develops and experts' opinions become closer to each other. Most recommendations do not imply a substantial change in an assessment's value, therefore a low number of experts' human supervisions are necessary, which contributes to save much time and cost. As shown in the *# exp. involved* column, very few experts are required to supervise their preferences at each round of the CRP. Such supervisions are quite scarce at the beginning, and become slightly more frequent as the process develops, due to the fact that initial assessments begin to experience more noticeable changes with respect to their initial values. The total time required to supervise changes in preferences during the whole CRP was 12 minutes.

B. Comparison with the resolution of a full-supervised CRP

Once shown the results obtained by using the proposed semi-supervised CSS to conduct the CRP for the large-scale GDM problem considered, we compare them with results obtained from the second subgroup of students, who solved the same problem by using a version of the CSS that includes the full-supervised approach, in order to show the advantages of using the former one. In this case, students receive notification about *all* changes suggested on their assessments and, due to the fact that they can not adopt a specific change profile, they may either accept or reject them, as well as increase/decrease such assessments in the degree they wish to consider.

Table II shows the results obtained (notice that, in this case, *all* change recommendations are regarded as supervisions). Figure 6 shows graphically the convergence towards consensus, i.e. the evolution of consensus degree, cr, achieved by each CSS throughout the discussion process.

By comparing results shown in Tables I, II and Figure 6, it can be seen that, despite the subgroup of students who used the full-supervised CSS presented a slightly higher level of agreement on their initial preferences (recall that they were separated into two subgroups randomly), they experienced a lower convergence towards consensus, due to the fact that they applied little changes on their assessments in most cases. The number of supervisions and the number of experts involved in such supervisions is significantly lower when using the semi-supervised CSS. Finally, although in both cases students were continuously connected to the system during the CRP, the second group required more response time (35 minutes),

[3]The data associated to this problem consists of experts' preferences across the CRP, change profiles chosen by experts who used the semi-supervised CSS, and more detailed results. They can be consulted at: http://sinbad2.ujaen.es/cod/consensus_mas.

TABLE I: Results of the GDM problem resolution with the proposed semi-supervised CSS.

r	cr	# changes	#ch_applied	#sup.	accepted	rejected	# exp. involved	resp. time (min.)
1	0.6278	146	146	**0**/146	0	0	**0**/23	**0**
2	0.7282	100	99	**7**/100	6	1	**5**/23	**3**
3	0.7781	63	61	**6**/63	4	2	**4**/23	**2**
4	0.7990	45	44	**5**/45	4	1	**5**/23	**3**
5	0.8294	36	32	**10**/36	6	4	**8**/23	**4**
6	**0.8547**							
			Total supervisions:	**28**			Total resp. time:	**12 min.**

TABLE II: Results of the GDM problem resolution with a full-supervised CSS.

r	cr	# changes (= # sup.)	#ch_applied	#ch_rejected	# exp. involved	resp. time (min.)
1	0.6645	**84**	75	9	**15**/23	**4**
2	0.7080	**65**	54	11	**12**/23	**6**
3	0.7395	**54**	45	9	**10**/23	**5**
4	0.7653	**76**	69	7	**17**/23	**6**
5	0.8002	**61**	57	4	**19**/23	**5**
6	0.8269	**33**	30	3	**8**/23	**5**
7	0.8418	**20**	18	2	**10**/23	**4**
8	**0.8509**					
Total supervisions:		**393**			Total resp. time:	**35 min.**

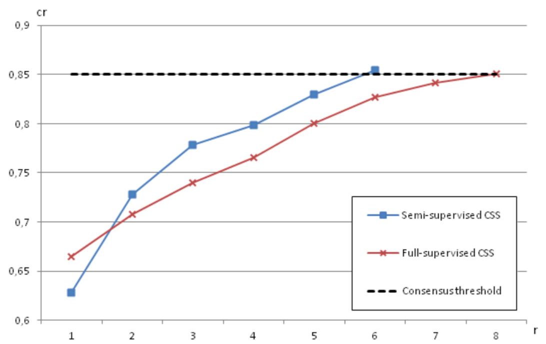

Fig. 6: Evolution of consensus degree, cr, during the CRP.

because they had to supervise a higher number of their assessments at each round, and they needed to think about the acceptance or rejection of each proposed change, as well as the degree of increase/decrease to which they applied an accepted change.

Based on results obtained, we conclude that our proposed semi-supervised CSS provided some remarkable advantages:

1) The problem of constant human supervision by experts was addressed by minimizing the number of suggested changes on preferences that they needed to revise.
2) The number of experts who had to revise their assessments at each discussion round was significantly reduced. As a result of this, the cost and time invested in conducting the whole CRP were significantly reduced, in comparison with using a full-supervised CSS.
3) The semi-supervised CSS contributed to achieve a higher convergence towards consensus, thus having been necessary a lower number of discussion rounds than those required by the full-supervised CSS.

V. Concluding Remarks

The necessity of automating consensus reaching processes to solve group decision making problems is leading to the design and implementation of different consensus support systems. Some of the challenges which still remain unsolved in these systems are the need for constant supervision of preferences by decision makers during the overall discussion process, and the increasing need for an approach to manage large-scale group decision making problems effectively. In this paper, we have presented a semi-supervised consensus support system based on a multi-agent architecture. Such a system is aimed to support consensus reaching processes in real-life group decision making problems where a high number of decision makers participate. Besides the full autonomy of the human moderator tasks, achieved thanks to the consensus model considered, an agent semi-supervised autonomy mechanism which means the main novelty in the proposed system, provides a high degree of autonomy for human experts, who only are requested supervision on their preferences in the cases they have to apply critical changes on them. Agents communicate each other by means of two ontologies that let them use a common language and semantics. Finally, even though the presented system is based on a specific consensus model described, its architecture lets implementing and using different models on it, therefore the system is also appropriate to study, simulate, evaluate and solve problems with different consensus models and approaches proposed in the literature.

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4.3. A consensus model to detect and manage non-cooperative behaviors in large-scale group decision making

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A Consensus Model to Detect and Manage Non-Cooperative Behaviors in Large Scale Group Decision Making

Iván Palomares, Luis Martínez, *Member, IEEE*, and Francisco Herrera, *Member, IEEE*

***Abstract*—Consensus reaching processes in group decision making attempt to reach a mutual agreement among a group of decision makers before making a common decision. Different consensus models have been proposed by different authors in the literature to facilitate consensus reaching processes. Classical models focus on solving group decision making problems where few decision makers participate. However, nowadays societal and technological trends that demand the management of larger scales of decision makers, such as e-democracy and social networks, add a new requirement to the solution of consensus-based group decision making problems. Dealing with such large groups implies the need for mechanisms to detect decision makers' non-cooperative behaviors in consensus, which might bias the consensus reaching process. This paper presents a consensus model suitable to manage large scales of decision makers, that incorporates a fuzzy clustering-based scheme to detect and manage individual and subgroup non-cooperative behaviors. The model is complemented with a visual analysis tool of the overall consensus reaching process based on Self-Organizing Maps, that facilitates the monitoring of the process performance across the time. The consensus model presented is aimed to the solution of consensus processes involving large groups.**

***Index Terms*—Group Decision Making, Consensus, Preference Relation, Fuzzy Clustering, E-democracy, Social Networks, Self-Organizing Maps.**

I. INTRODUCTION

Decision making processes are one of the most frequent mankind activities in daily life. The need for multiple views in decision making makes Group Decision Making (GDM) increasingly necessary in many societies and organizations nowadays. GDM problems can be defined as decision situations where a group of decision makers or experts try to achieve a common solution to a problem consisting of two or more possible solutions or alternatives [1]. In real world GDM problems, different situations might usually occur, such as collaboration and competitiveness among individuals, compatible or incompatible proposals, etc. Some guiding rules, including the majority rule, minority rule and unanimity [2], have been proposed to support decision making in such situations. For instance, the majority rule is classically the most usual rule for dealing with GDM problems in democratic systems [3].

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Traditionally, GDM problems have been solved by applying a selection process to choose the best alternative or subset of alternatives, paying no attention to the level of agreement achieved among experts [4]. However, many real world problems that affect entire groups or societies (civil rights, tax raising, political and religious issues, etc.) may require highly agreed decisions. Therefore, the need for making consensus-based decisions is becoming increasingly apparent in these contexts. Consensus Reaching Processes (CRPs) [2], [5] attempt to reach an experts' agreement before making a decision, thus yielding a more accepted solution by the whole group. In a CRP, experts discuss and modify their preferences, guided and supervised by a human figure known as *moderator* [6].

GDM and consensus models have been normally focused on dealing with a few number of decision makers [7]–[11], because classically in companies and administrations, important decisions have been made by one or a few number of them. However, current technological and societal demands have given birth to new paradigms in which decisions can be made taking into account a large number of decision makers (such as e-democracy [12], [13] and social networks [14]–[16]). Most current models are not appropriate to manage large groups, due to the high cost, complexity and human supervision required. Additionally, a noticeable drawback usually found in such large groups, is the presence of experts and subgroups of experts who present a behavior that does not contribute to achieve consensus [17], because they do not want to modify their initial position in order to achieve an agreement. In large groups, it is common that there exist several subgroups or *coalitions* of experts with similar interests. Some of these subgroups are prone to modify their preferences to achieve an agreement (they can be referred to as *pro-coalitions*), while some others do not modify their preferences or even do it on the contrary way to the remaining experts (they can be referred to as *con-coalitions*). Con-coalitions of experts introduce a bias in the collective opinion, since they move their preferences against consensus coordinately. Therefore, it would be advisable to detect and manage non cooperating individuals and subgroups [5], [17], with the aim of improving the CRP performance.

A visual analysis of the consensus evolution among decision makers' preferences throughout the discussion process, by means of a CRP monitoring tool to distinguish between those decision makers who move their preferences towards consensus and those ones who do not cooperate to achieve it, would also be very convenient to analyze consensus models. Self-

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Organizing Maps (SOMs) are a widely used tool capable of projecting high dimensional data (such as experts' preferences, for instance) into a low-dimensional space, maintaining the main topological properties of data to facilitate its visual analysis and interpretation [18]–[20].

In order to address the multiple challenges stated above, in this paper a consensus model capable of managing large groups of decision makers is proposed. Such a model incorporates an approach that classifies decision makers (based on their fuzzy preference relations) to detect non cooperative behaviors in CRPs and manage them. In order to achieve these objectives, fuzzy clustering techniques are used to facilitate the detection of non cooperating individuals or subgroups and deal with them accordingly. In line with the presented consensus model, we propose the use of a monitoring tool based on SOMs, which facilitates a visual analysis of experts' agreement evolution across the consensus process and their behavior.

This paper is organized as follows: in Section 2 some preliminaries related to consensus processes in GDM, CRPs, fuzzy clustering techniques and SOMs are reviewed. In Section 3, the consensus model that deals with large scales of decision makers is presented, describing in detail the mechanisms to detect and manage experts' non-cooperative behaviors. Section 4 describes the use of SOM-based techniques to develop a monitoring tool to visualize the CRP performance. An illustrative example of the model's utility and applicability, including a visual analysis of the CRP, is shown in Section 5. Finally, in Section 6, some concluding remarks are drawn.

II. Preliminaries

In this section, we revise GDM problems, CRPs and consensus models. We then briefly review fuzzy clustering techniques, which are the basis for the behavior detection scheme implemented in the proposed consensus model; and Self-Organizing Maps (SOMs), which will be considered to propose a visual monitoring tool of the CRP performance.

A. Group Decision Making

GDM problems are characterized by the participation of two or more experts in a decision problem, where a set of alternatives or possible solutions to the problem are presented [1], [2]. Formally, the main elements found in any GDM problem are:

- A set $X = \{x_1, \ldots, x_n\}$, $(n \geq 2)$ of *alternatives* to be chosen as possible solutions to the problem.
- A set $E = \{e_1, \ldots, e_m\}$, $(m \geq 2)$ of *decision makers* or *experts*, who express their judgements on the alternatives in X.

Each expert e_i, $i \in \{1, \ldots, m\}$, provides his/her opinions over alternatives in X by means of a preference structure. One of the most usual preference structures in GDM problems under uncertainty are the so-called preference relations [21], [22]. More specifically, *fuzzy preference relations* have proved to be especially effective to deal with uncertain information. They are defined as follows:

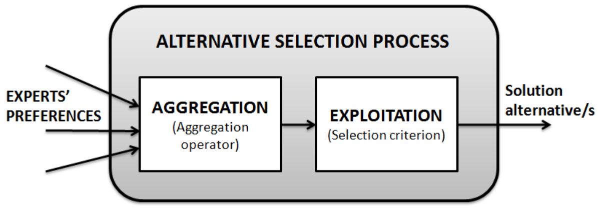

Fig. 1: Selection process in GDM problems.

Definition 1. *[23] Given an expert $e_i \in E$, $i \in \{1, \ldots, m\}$ and two different alternatives $x_l, x_k \in X$; $l, k \in \{1, \ldots, n\}$ $(l \neq k)$, a fuzzy preference relation's* assessment *on the pair (x_l, x_k), denoted as $p_i^{lk} \in [0, 1]$, represents the degree of preference of alternative x_l with respect to alternative x_k assessed by expert e_i, so that $p_i^{lk} > 1/2$ indicates that x_l is preferred to x_k, $p_i^{lk} < 1/2$ indicates that x_k is preferred to x_l, and $p_i^{lk} = 1/2$ indicates indifference between x_l and x_k.*

Definition 2. *[21], [24] A* fuzzy preference relation *P_i associated to expert e_i, $i \in \{1, \ldots, m\}$, on a set of alternatives X is a fuzzy set on $X \times X$, which is characterized by the membership function $\mu_{P_i} : X \times X \longrightarrow [0, 1]$. When the number of alternatives n is finite, P_i is represented by a $n \times n$ matrix of assessments $p_i^{lk} = \mu_{P_i}(x_l, x_k)$ as follows:*

$$P_i = \begin{pmatrix} - & \ldots & p_i^{1n} \\ \vdots & \ddots & \vdots \\ p_i^{n1} & \ldots & - \end{pmatrix}$$

Assessments p_i^{ll}, $l \in \{1, \ldots, n\}$, situated in the diagonal of the matrix, are not defined, since an alternative x_l is not assessed with respect to itself.

The solution to a GDM problem may be obtained either by a direct approach, where the solution is immediately obtained from experts' preferences; or by an indirect approach, where a social opinion is computed to determine the chosen alternative/s [4]. Regardless of the approach considered, it is necessary to apply a selection process to solve the GDM problem, which usually consists of two main phases (Fig. 1) [25]: (i) an Aggregation phase, where experts' preferences are combined, and (ii) an Exploitation phase, which consists in obtaining an alternative or subset of alternatives as the solution to the problem.

B. Consensus Reaching Processes (CRPs) and Consensus Models

The resolution of GDM problems by applying a selection process solely does not always guarantee that the decision would be accepted by all experts in the group, since some of them might consider that their opinions have not been sufficiently considered. In order to achieve a solution to the GDM problem which is accepted by the whole group, CRPs have attained a great attention as part of the decision process. *Consensus* can be understood as a state of mutual agreement among members of a group, in which the decision made

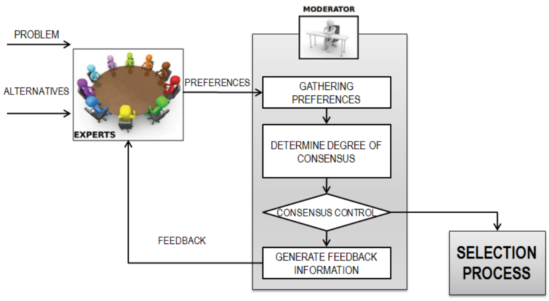

Fig. 2: General consensus process scheme in GDM problems.

satisfies all of them [2], [5]. Reaching a consensus usually requires that experts modify their initial opinions, making them closer to each other and towards a collective opinion which must be satisfactory for all of them.

The notion of consensus can be interpreted in different ways, ranging from consensus as total agreement to more flexible approaches [9], [26]. Consensus as a total agreement, where all experts achieve a mutual agreement in all their opinions, may be quite difficult to achieve in practice, and in cases that it could be achieved, the cost derived from the CRP would be unacceptable, and it might have been sometimes achieved under a normative point of view, through intimidation or other social strategies [17]. Subsequently, more flexible notions of consensus have been proposed to soften the strict view of consensus as a total agreement [26], [27], considering different degrees of partial agreement among experts to decide about the existence of consensus. One of the most widely accepted approaches for a flexible measurement of consensus is the so-called notion of *soft consensus*, proposed by Kacprzyk [1]. This approach introduces the concept of fuzzy linguistic majority, which establishes that consensus exists if *most experts, participating in a problem, agree on the most important alternatives*. *Soft consensus*-based approaches have been used in different GDM problems providing satisfactory results [28]–[30].

The process to reach a consensus in GDM problems is a dynamic and iterative discussion process [5], frequently coordinated by a human figure known as moderator, who is responsible for supervising and guiding experts in the overall process, as well as giving them advice to modify their opinions [6]. A general scheme of the phases required for conducting CRPs, depicted in Figure 2, is briefly described below:

- *Gather preferences*: Each expert provides moderator a preference structure with his/her opinion on the existing alternatives.
- *Determine degree of consensus*: The moderator computes the level of agreement in the group by means of a *consensus measure* [26], usually based on different *similarity measures* and *aggregation operators* [31].
- *Consensus control*: The consensus degree is compared with a threshold level of agreement desired by the group. If such degree is enough, the group moves on to the selection process, otherwise, more discussion rounds are required.
- *Generate feedback information*: The moderator identifies furthest preferences from consensus and gives experts some pieces of advice, suggesting them how to modify their opinions and make them closer. Afterwards, a new round of discussion begins with the gathering preferences phase.

In order to deal with CRPs, a large number of theoretical consensus models have been proposed in the literature by different authors [5], [8], [9], [11], [32]–[34]. These models have been designed to deal with GDM problems where small groups of decision makers participate, as traditionally occurred in most companies and organizations, where decisions were delegated to one or, at the most, a low number of them.

However, new trends stemming from current demands in societal and technological contexts, such as e-democracy [12], [13] and social networks [14]–[16], make necessary to cope with consensus challenges in order that CRPs would be suitable for dealing with larger scales of decision makers participating in the GDM problem [35], which implies a higher cost and complexity in such processes.

C. *Fuzzy Clustering*

Clustering is a widely-used methodology, categorized as an unsupervised machine learning technique, aimed to data analysis and interpretation [36]. The problem of clustering consists in separating a set of data objects into a number of groups so-called *clusters*, based on a measure of similarity, so that data objects within the same cluster are more similar to each other than data objects belonging to different clusters [37]. Usually, each cluster is represented by a prototype or *cluster centre* that characterizes all data objects belonging to such a cluster. Many clustering algorithms compute these cluster centres as the centroid of data belonging to the cluster considered.

Traditional or *crisp* clustering methods, such as k-means [38], are partitioning methods, i.e. each data object is assigned to one and only one cluster. Since this may not always provide a convincing representation of data, fuzzy clustering methods based on fuzzy set theory [39] have been later proposed, under the assumption that data objects may belong to multiple clusters with different degrees of membership [37]. Fuzzy clustering methods are objective function-based methods which seek to find cluster centres for a predefined number N of fuzzy clusters (for the sake of brevity, they will be referred to as clusters in the rest of the paper) and assign data objects a fuzzy membership degree to each cluster, during an iterative process aimed to minimize a predefined loss function [36], [40].

One of the most popular fuzzy clustering algorithms is the Fuzzy C-Means (FCM) algorithm [41], consisting in an optimization process where both cluster centres and data objects are iteratively updated until a locally optimal solution is found (which occurs when the variation between cluster membership degrees in two consecutive iterations of the algorithm approaches zero).

Algorithm 1 shows the basic steps in the standard FCM algorithm, defined according to our purpose of solving GDM

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Algorithm 1 Fuzzy C-Means (FCM) Algorithm applied to experts' fuzzy preference relations

1. Set the number of cluster centres N, ($N \geq 2$), and degree of fuzziness b.
2. Initialize N clusters C_h, $h \in \{1, \ldots, N\}$, by means of a cluster initialization technique.
3. **while** the stopping condition is not reached. **do**
4. Compute membership degrees of each preference relation P_i to each cluster centre C_h, $\mu_{C_h}(P_i) \in [0,1]$, as follows:

$$\mu_{C_h}(P_i) = \frac{(1/d(P_i, C_h))^{1/(b-1)}}{\sum_{u=1}^{N} (1/d(P_i, C_u))^{1/(b-1)}} \quad (1)$$

5. Update cluster centres C_h:

$$C_h = \frac{\sum_{i=1}^{m} \mu_{C_h}(P_i) P_i}{\sum_{i=1}^{m} \mu_{C_h}(P_i)} \quad (2)$$

6. **end while**.

problems with fuzzy preference relations, assuming the following:

- Considering the scope and purpose of this paper, the set of data objects is formed by all experts' preferences $P_1, \ldots, P_m$, therefore P_i is regarded as a data object. As a result, cluster centres C_h, $h = 1, \ldots, N$, also consist of fuzzy preference relations.
- Parameter b, ($b > 1$), indicates the degree of fuzziness of clusters. The larger b, the fuzzier the clusters are [41]. A common value for this parameter is $b = 2$.
- A cluster initialization technique is required to set initial values for cluster centres C_h. Different cluster initialization techniques to perform this task can be found in the literature [42], [43].
- Experts' fuzzy membership degrees to each cluster, $\mu_{C_h}(P_i)$, are computed by using similarity measures, which are based on distance metrics. The distance between preference P_i and cluster centre C_h is denoted as $d(P_i, C_h)$, and it will be introduced in Section III.
- Further detail about the specific stopping condition considered in our proposal will be given in Sect. III-B.

D. Self-Organizing Maps (SOMs)

Self-organizing Maps (SOMs) are a learning tool used in exploratory data mining, due to its prominent visualization properties [20], [44]. They were introduced by T. Kohonen [18] as a type of unsupervised learning algorithm based on neural networks [45], which is one of the most popular unsupervised learning methods for constructing topographic maps, i.e. low-dimensional (usually 2D or 3D) visualizations of high dimensional data.

In the SOM algorithm, a set of d-dimensional training data is used to iteratively modify connections between artificial neurons (with the same dimension d) situated in a rectangular or hexagonal-shaped grid, which is progressively adapted to such data. For each data object in the training set, the most similar neuron to such data object, so-called BMU (*Best Matching Unit*), must be found amongst all the artificial neurons in the grid. Connection weights in the BMU and its nearest neighboring neurons are updated upon the given data object. This process is iteratively conducted to progressively learn the structure of the whole SOM [36], [45]. The resulting SOM can be then used as a visualization surface to represent future sets of data objects in a low dimensional space, preserving its main topological properties [18], [45].

Once constructed, the SOM can visualize high-dimensional data sets. There are multiple methods based on SOMs to visualize data, such as distance matrices, similarity coloring, data histograms and PCA projections [19], the latter of which will be considered in this paper. Most of these methods can be used either for a two-dimensional or a three-dimensional visualization of data [20].

SOMs have proved themselves to be a useful tool in different data mining applications to obtain qualitative information, such as full-text and financial data analysis, cluster analysis, vector quantization and projection, etc., [19], [44].

III. Consensus Model to Detect and Manage Non-cooperative Behaviors

In this section, a consensus model suitable to deal with a large number of decision makers in the resolution of GDM problems is presented. The main novelty of such a model is the approach to classify decision makers according to their preferences and detect individual and subgroup non-cooperative behaviors in the CRP, based on fuzzy clustering techniques, as well as dealing with those experts who present such behaviors, with the aim of improving the overall CRP performance.

The consensus model description will be divided into three parts:

(a) A general scheme of the model, according to the main phases conducted in CRPs (see Sect. III-A).
(b) A fuzzy clustering-based method to classify experts' preferences and detect non-cooperative behaviors (see Sect. III-B).
(c) A scheme based on weights to manage non cooperating experts and subgroups of experts (see Sect. III-C).

Figure 3 shows a scheme of the consensus model, whose main phases and modules are developed in the following subsections.

A. Consensus Model Scheme

The proposed consensus model aims to serve as a guide to carry out the main tasks required to conduct CRPs, as stated in Sect. II-B. Such a model (see Fig. 3), extends the basic ideas of the ones previously proposed in [7], [10], and incorporates additional modules to achieve our goal of detecting experts' non-cooperative behaviors and dealing with them.

The consensus model design allows an easy automation of the human moderator tasks, thus removing his/her inherent subjective bias and facilitating the resolution of GDM problems with large groups of experts computationally. Let us remark that, regarding the scheme presented later to deal with non cooperating experts, which will be based on experts'

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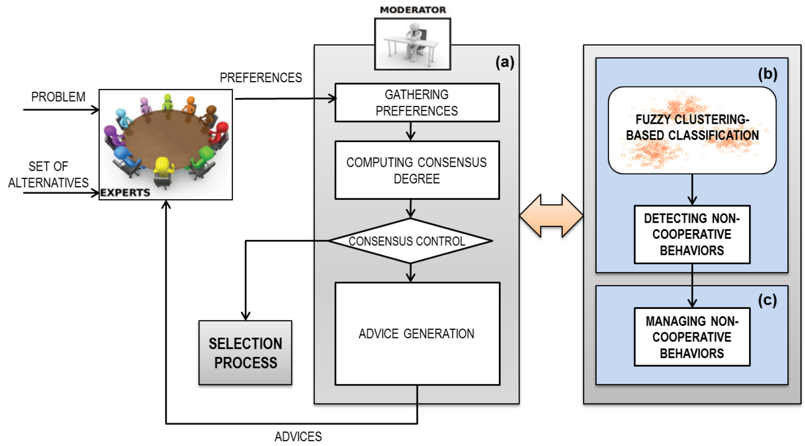

Fig. 3: Consensus model scheme.

importance weights, we propose that each expert $e_i \in E$ has an associated importance weight $w_i \in [0,1]$ which is initially $w_i = 1$, $\forall i \in \{1,\dots,m\}$, and may vary when the CRP goes on. Further detail on the meaning and use of such weights will be given in Sect. III-C.

Following, the four basic phases of the proposed consensus model are described in detail:

i) *Gathering Preferences*: Each expert $e_i \in E$ provides his/her preference on alternatives in X to the moderator, by means of a fuzzy preference relation $P_i = (p_i^{lk})^{n\times n}$, consisting of a matrix of assessments p_i^{lk} on each pair of alternatives (x_l, x_k), $l,k \in \{1,\dots,n\}$. Consistency in preferences can be improved if experts provide reciprocal assessments, i.e. if $p_i^{lk} = p, p \in [0,1]$, $l \neq k$, then $p_i^{kl} = 1 - p$.

ii) *Computing Consensus Degree*: The moderator computes the level of agreement between experts, by means of the following steps:

a) For each pair of experts e_i, e_j, $(i < j)$ a similarity matrix $SM_{ij} = (sm_{ij}^{lk})^{n\times n}$ defined by

$$SM_{ij} = \begin{pmatrix} - & \dots & sm_{ij}^{1n} \\ \vdots & \ddots & \vdots \\ sm_{ij}^{n1} & \dots & - \end{pmatrix}$$

is computed. $sm_{ij}^{lk} \in [0,1]$ is the similarity degree between experts e_i and e_j in their assessments p_i^{lk}, p_j^{lk}, obtained by means of a similarity function as follows [8]:

$$sm_{ij}^{lk} = 1 - |(p_i^{lk} - p_j^{lk})| \tag{3}$$

b) A consensus matrix $CM = (cm^{lk})^{n\times n}$, is computed by aggregating similarity matrices, taking into account the importance weights $w_{ij} \in [0,1]$ associated to each pair of experts (e_i, e_j), $i < j$. Each element $cm^{lk} \in [0,1]$, $l \neq k$, is computed as the weighted average of similarity degrees:

$$cm^{lk} = \frac{\sum_{i=1}^{m-1}\sum_{j=i+1}^{m} w_{ij} sm_{ij}^{lk}}{\sum_{i=1}^{m-1}\sum_{j=i+1}^{m} w_{ij}} \tag{4}$$

Further detail about weights w_{ij} and the way they are computed upon single experts' weights w_i, w_j, can be found in the scheme to manage non cooperating experts described in Sect. III-C. Notice here that if all experts are given equal importance weights, cm^{lk} can be computed as:

$$cm^{lk} = \frac{\sum_{i=1}^{m-1}\sum_{j=i+1}^{m} sm_{ij}^{lk}}{m(m-1)/2} \tag{5}$$

being $m(m-1)/2$ the number of different pairs of experts (e_i, e_j) in the group (in both Eq. (4) and Eq. (5)).

c) Consensus degree is computed at three different levels [8], [10]:

i) Level of pairs of alternatives (cp^{lk}): Obtained from CM as $cp^{lk} = cm^{lk}$, $l,k \in \{1,\dots,n\}, l \neq k$.

ii) Level of alternatives (ca^l): The level of agreement on each alternative $x_l \in X$ is computed as:

$$ca^l = \frac{\sum_{k=1,k\neq l}^{n} cp^{lk}}{n-1} \tag{6}$$

iii) Level of preference relation (overall consensus degree, cr):

$$cr = \frac{\sum_{l=1}^{n} ca^l}{n} \tag{7}$$

iii) *Consensus Control*: The overall consensus degree cr is compared with a consensus threshold $\mu \in [0,1]$ established a priori. If $cr \geq \mu$, then the CRP ends and the group moves on to the selection process; otherwise, more discussion rounds are required. A parameter *Maxround* can be used to limit the number of discussion rounds conducted in the cases that consensus can not be achieved.

iv) *Advice Generation*: If $cr < \mu$, the moderator advises experts to modify their preferences in order to increase the level of agreement in the following rounds. Since this is the last phase of each discussion round in the

CRP, the schemes to detect and manage non-cooperative behaviors must be applied in a parallel way (see Sections III-B and III-C), so that experts' importance weights w_i will be updated before initiating the following round of discussion. Three steps are considered in the advice generation phase:

(1) Compute a collective preference and proximity matrices for experts: A collective preference P_c is computed for each pair of alternatives by aggregating experts' preference relations:

$$p_c^{lk} = \frac{\sum_{i=1}^{m} w_i p_i^{lk}}{\sum_{i=1}^{m} w_i} \quad (8)$$

where $w_i \in [0,1]$ is the importance weight assigned to e_i (see Section III-C). If all experts have the same importance, then p_c^{lk} can be computed as:

$$p_c^{lk} = \frac{\sum_{i=1}^{m} p_i^{lk}}{m} \quad (9)$$

Once computed P_c, we have all the necessary data to initiate the fuzzy clustering-based algorithm to classify and group experts according with their preferences, as it will be shown in Section III-B.

(2) A proximity matrix $PP_i = (pp_i^{lk})^{n \times n}$ between each expert's preference relation and P_c, defined by

$$PP_i = \begin{pmatrix} - & \dots & pp_i^{1n} \\ \vdots & \ddots & \vdots \\ pp_i^{n1} & \dots & - \end{pmatrix}$$

is computed. Proximity values pp_i^{lk} are obtained for each pair (x_l, x_k) as follows:

$$pp_i^{lk} = 1 - |(p_i^{lk} - p_c^{lk})| \quad (10)$$

Proximity values are used to identify the furthest preferences from the collective opinion, which should be modified by some experts.

(3) Identify preferences to be changed (CC): Pairs of alternatives (x_l, x_k) whose consensus degrees ca^l and cp^{lk} are not enough, are identified:

$$CC = \{(x_l, x_k) | ca^l < cr \wedge cp^{lk} < cr\} \quad (11)$$

Afterwards, the model identifies experts who should change their opinion on each of these pairs, i.e. those experts e_i whose preference p_i^{lk} on the pair $(x_l, x_k) \in CC$ is furthest to p_c^{lk}. An average proximity $\overline{pp}^{lk}$ is calculated to identify them, as follows:

$$\overline{pp}^{lk} = \frac{\sum_{i=1}^{m} pp_i^{lk}}{m} \quad (12)$$

As a result, experts e_i whose $pp_i^{lk} < \overline{pp}^{lk}$ are advised to modify their assessment on pair (x_l, x_k).

(4) Establish change directions: Several direction rules are applied to suggest the direction of changes proposed to experts, in order to increase the level of agreement in the following rounds [10]. Here, an acceptability threshold $\varepsilon \geq 0$ which may take a positive value close to zero is introduced, to allow a margin of acceptability when p_i^{lk} and p_c^{lk} are close enough to each other.

- DIR.1: If $(p_i^{lk} - p_c^{lk}) < -\varepsilon$, then expert e_i should *increase* his/her assessment on the pair of alternatives (x_l, x_k).
- DIR.2: If $(p_i^{lk} - p_c^{lk}) > \varepsilon$, then expert e_i should *decrease* his/her assessment on the pair of alternatives (x_l, x_k).
- DIR.3: If $-\varepsilon \leq (p_i^{lk} - p_c^{lk}) \leq \varepsilon$, then expert e_i does not need to modify his/her assessment on the pair of alternatives (x_l, x_k).

B. Non-cooperative Behavior Detection

Once described the main phases of the proposed consensus model, here we define a method to identify those experts and subgroups of them who do not tend to modify their initial preferences to achieve a consensus, or might move such preferences against it, either individually or coordinately. We aim to develop such a method by applying the FCM algorithm for fuzzy clustering [41], in order to classify experts based on their fuzzy preference relations P_i. Once applied the FCM algorithm, the definition of several rules is proposed, based on cluster similarity, cluster distance metrics and fuzzy logic. These rules must be checked before deciding about the existence of the above mentioned behaviors.

The detection scheme is conducted once for each round in the discussion process, after the collective preference P_c for that round is obtained during the *Advice Generation* phase of the basic consensus model scheme (see Sect. III-A). Let $t \in \{1, \dots, Maxround - 1\}$ be the current discussion round of the CRP. From now onwards, experts' preference values in round t will be denoted as P_i^t, $i = 1, \dots, m$, and cluster centres in such a round will be denoted as C_h^t, $h = 1, \dots, N$.

The description of the proposed detection method is organized into three parts:

- Application and settings of the FCM algorithm to classify experts.
- Rules for the detection of subgroup behaviors contrary to consensus achievement (con-coalitions).
- Rules for the detection of individual behaviors contrary to consensus achievement (considered as outliers).

1) FCM algorithm settings: First, the FCM algorithm is applied on experts' preferences in the current CRP round t. Several specifications and variations respect to FCM will be considered here, and they are described below:

- FCM parameters: Without loss of generality, a fuzziness degree $b \approx 2$ is usually taken.
- Cluster initialization: As reviewed in Section II-C, the first phase in the FCM algorithm consists in initializing clusters, i.e. assigning each of them a cluster centre C_h^t, based on an initialization technique. We consider the method proposed by Katsavounidis et al. in [43] to define the initialization scheme described below for N clusters ($N > 2$):

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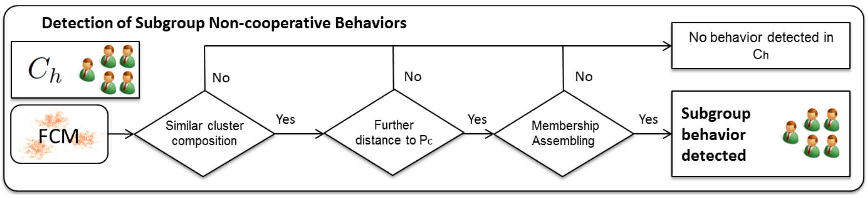

Fig. 4: Scheme of the method to detect subgroup non-cooperative behaviors.

a) The first cluster is initialized by assigning the collective preference in the current round, P_c^t, to cluster centre C_1^t.
b) Initialize the second cluster centre C_2^t as the expert preference P_i^t which is farthest from P_c^t.
c) For C_h^t ($h \geq 3$), compute the minimum distance between each of the remaining experts' preferences P_i^t and all current initial cluster centres, and find the expert preference whose minimum distance is the largest one, i.e. the one which accomplishes $\max_i \left(\min_{u<h} d(P_i^t, C_u^t)\right)$. Assign it to C_h^t.
d) Repeat step 3 until all N clusters are initialized.

- Update process: Cluster centres C_h^t ($h \geq 2$) and cluster membership degrees $\mu_{C_h^t}(P_i^t)$ are updated iteratively, as shown in Algorithm 1. Notice here that C_1^t is not updated, in order to preserve P_c^t as the centre of one of the clusters once applied the FCM algorithm, since it will play an essential role in the subsequent detection scheme.
- Distance metrics: In order to compute distances between preference relations (both experts' preferences and cluster centres indistinctly), the following normalized Minkowski-based distance measure [36] is considered:

$$d(P_i^t, C_h^t) = \sqrt[p]{\sum_{lk, l\neq k} (p_i^{lk,t} - c_h^{lk,t})^p} \qquad (13)$$

where $p > 0$ and $l, k \in \{1, \ldots, n\}$.
- Stopping criterion: The stopping condition considered is to finalize the update process when all clusters stabilize. This occurs when the variation in membership degrees between two consecutive iterations approaches zero. Formally, the iterative update process is stopped when:

$$\frac{\sum_{i=1}^{m}\sum_{h=1}^{N} |\mu_{C_h^t}^{y}(P_i^t) - \mu_{C_h^t}^{y-1}(P_i^t)|}{m \cdot N} \leq \epsilon \qquad (14)$$

where $y \in \mathbb{N}$ denotes the current iteration of the FCM algorithm and ϵ is a threshold value (which should be close to zero) used as a stopping condition. As an optimization algorithm where a locally optimal solution is always found, FCM guarantees the necessary convergence to achieve this condition.

2) Detection of Subgroup Non-cooperative Behaviors (Concoalitions): Once executed the FCM algorithm, we proceed to apply a method to detect individual and subgroup non-cooperative behaviors, which is aimed to facilitate the subsequent treatment of such experts, thus improving the performance of the CRP. Since a different rule-based scheme will be considered for each type of behavior (subgroup or individual), they will be explained separately. In this subsection we present the scheme corresponding to the detection of subgroup non-cooperative behaviors which, as stated in the Introduction, can be regarded in the scope of our paper as con-coalitions. This detection scheme is first applied in the second round of the CRP, because it requires comparisons between clusters obtained in the previous and current rounds of discussion, $t-1$ and t, and it is based on a set of three rules which must be checked for each cluster centre C_h^t, $h \geq 2$, to decide about the existence of a subgroup behavior on it (see Figure 4):

i) There exists a cluster with "similar" composition to C_h^t in round $t-1$.
ii) Distance between C_h^t and P_c^t increases.
iii) Membership of experts to C_h^t increases or membership to P_c^t decreases.

The accomplishment of *all* these rules by a cluster C_h^t can be assumed as a subgroup non-cooperative behavior performed by a con-coalition of experts belonging to it, whose preferences must be given some treatment, as will be explained in Section III-C.

Following, the rules are described in detail:

R1. *Similar Cluster Composition*: This rule is checked to determine whether a cluster is compound by the same experts across the time or not. To do this, the similarity between a given cluster C_h^t ($h \geq 2$) determined in the current CRP round t, $t \in \{2, \ldots, Maxrounds-1\}$, and each cluster C_u^{t-1} ($u \geq 2$) determined in the previous round, $t-1$, is computed. Two clusters C_h^t and C_u^{t-1} are considered to represent the same subgroup of experts, if experts' membership degrees to both of them, $\mu_{C_h}^t(P_i^t)$ and $\mu_{C_u}^{t-1}(P_i^{t-1})$ have close values to each other, for all $e_i \in E$.
In order to decide whether cluster similarity is enough to assume analogous cluster composition, a similarity threshold $\kappa \in [0,1]$ can be defined. A cluster similarity measure $sim(C_h^t, C_u^{t-1})$ is proposed as follows:

$$sim(C_h^t, C_u^{t-1}) = 1 - \frac{\sum_{i=1}^{m} \Delta_{hu}^t(P_i)}{m} \qquad (15)$$

where $\Delta_{hu}^t(P_i) \in [0,1]$ is the variation in P_i membership to both clusters, computed as:

$$\Delta_{hu}^t(P_i) = |\mu_{C_h}^t(P_i^t) - \mu_{C_u}^{t-1}(P_i^{t-1})| \qquad (16)$$

For a given cluster C_h^t, if $\exists C_u^{t-1} : sim(C_h^t, C_u^{t-1}) \geq \kappa$, then C_h^t and C_u^{t-1} are assumed to represent the same cluster across time, due to their similar composition.

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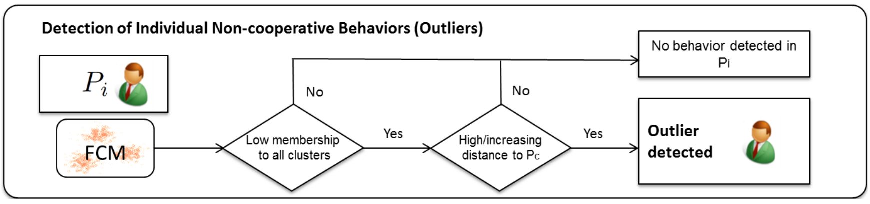

Fig. 5: Scheme of the method to detect individual non-cooperative behaviors.

Remark 1. *Since $sim(C_h^t, C_u^{t-1})$ takes values in the unit interval, the value fixed for similarity threshold, κ, should be close enough to 1, in order to guarantee an effective detection of similar clusters in consecutive rounds of the CRP.*

R2. *Further Distance to P_c*: Based on the previous rule and assuming that C_h^t and C_u^{t-1} are similar enough to be considered the same cluster, distances between a cluster centre and the collective preference (i.e. C_1^t) in rounds t and $t-1$, denoted as $d(C_1^t, C_h^t)$ and $d(C_1^{t-1}, C_u^{t-1})$ respectively, are computed by means of the distance measure shown in Eq. (13).
Let $\nu \in [0,1]$ be a parameter indicating a minimum distance between clusters, which should take a value close to zero, so that a distance value lower than ν means that cluster centres are close enough to each other and no further detection process is required. If $d(C_1^t, C_h^t) > \nu$ AND $d(C_1^t, C_h^t) \geq d(C_1^{t-1}, C_u^{t-1})$, i.e. cluster centres C_1^t and C_h^t are not close enough to each other and distance between them increases as the CRP progresses, then some experts in cluster C_h are presumably presenting a non-cooperative behavior.

R3. *Membership Assembling*: This rule is checked to decide whether one of the following conditions occurs: (i) a subgroup of experts become more assembled around a cluster C_h^t (i.e. their membership to the cluster increases), or (ii) there is a lower concentration of experts around the collective opinion P_c^t. Assuming again that C_h^t and C_u^{t-1} are considered to be same cluster, let $S_h^t = \sum_{i=1}^m \mu_{C_h^t}(P_i^t)$ and $S_u^{t-1} = \sum_{i=1}^m \mu_{C_u^{t-1}}(P_i^{t-1})$ be the sums of experts' membership degrees to cluster C_h, $(h \geq 2)$, in rounds t and $t-1$, respectively. Analogously, let $S_1^t = \sum_{i=1}^m \mu_{C_1^t}(P_i^t)$ and $S_1^{t-1} = \sum_{i=1}^m \mu_{C_1^{t-1}}(P_i^{t-1})$ be the sums of experts' membership degrees to the collective preference in the above mentioned rounds.
If $S_h^t > S_u^{t-1}$ then experts are becoming more assembled around C_h^t. On the other side, if $S_1^t < S_1^{t-1}$, then experts become less assembled around $C_1^t \equiv P_c^t$.

3) Detection of Individual Non-cooperative Behaviors (Outliers): Here, the scheme corresponding to the detection of individual behaviors is described. Such behaviors must also be managed later to optimize the performance of the consensus process, and they are determined by preference relations that present a low membership to all clusters in the group, therefore they can be viewed as *outliers* in the set of experts' preferences. This scheme is only applied towards the end of the CRP, i.e. when discussion between experts has been already developed and the consensus degree cr approaches the consensus threshold, μ. An additional consensus threshold $\gamma < \mu$, $\gamma \in [0,1]$, can be used to decide when the outlier detection mechanism is activated.

The following rules are checked to determine the existence of an individual non-cooperative behavior associated to a preference relation P_i^t (see Figure 5):

R1. *P_i does not present a high membership to any cluster*: A cluster membership threshold $\delta \in [0,1]$ is established. P_i^t does not present a high membership to any cluster iff $\mu_{C_h}^t(P_i^t) < \delta$, $\forall h \in \{1, \ldots, N\}$.

R2. *High/increasing distance to P_c*: Distance to the collective preference increases or it is higher than the average distance between all experts' preferences and the collective preference, i.e. either one of the following conditions holds,

a) $d(P_i^t, C_1^t) > \overline{d}$.
The average distance to the collective preference, denoted by $\overline{d}$, is computed as follows:

$$\overline{d} = \frac{\sum_{i=1}^m d(P_i^t, C_1^t)}{m} \quad (17)$$

b) $d(P_i^t, C_1^t) > d(P_i^{t-1}, C_1^{t-1})$.

Remark 2. *The rules described above have been proposed to detect the specific type of subgroup and individual non-cooperative behaviors this paper focuses on. However, the proposed model offers enough flexibility to introduce new rules and/or extend the current ones, if any new kind of behavior would be considered.*

C. Managing Non-cooperative Behaviors

Once individual and subgroup behavior detection mechanisms have been presented, it is necessary to define how to manage experts involved in such behaviors. There exist different proposals in the literature concerning this issue, for instance discarding preferences of experts who do not contribute to achieve consensus [5] or penalizing their importance weights, thus reducing their influence in the CRP [17], [26]. Here, a weight penalizing method is proposed, so that the weights of non cooperating experts' preferences are reduced accordingly throughout the discussion process.

As mentioned in the consensus model scheme in Section III-A, each expert $e_i \in E$ has an associated importance weight $w_i \in [0,1]$. At the beginning of the CRP, all experts have a

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maximum weight, $w_i = 1$, $\forall i$, and such a weight could be updated whenever a behavior detection occurs.

Given a cluster C_h^t which contains a con-coalition in round $t \geq 2$, the procedure shown in Algorithm 2 is applied to each expert preference relation $P_i^t \in C_h^t$ for updating its corresponding weight w_i. The procedure to manage individual behaviors (outliers) consists in applying steps 3 and 4 of Algorithm 2 for a detected P_i^t.

Algorithm 2 Procedure to update weights in a detected non cooperating subgroup, C_h^t

1. **for** each expert preference relation P_i^t, $i \in \{1, \ldots, m\}$ **do**
2. **if** C_h^t is the cluster to which P_i^t belongs the most, i.e. $\mu_{C_h^t}(P_i^t) = \max_u \mu_{C_u^t}(P_i^t)$, $u \in \{1, \ldots, N\}$. **then**
3. Compute $w_{i_{new}}$ upon current weight w_i, as follows:

$$w_{i_{new}} = w_i \left(1 - \frac{d(P_i^t, C_1^t)}{\max_j d(P_j^t, C_1^t)}\right) \tag{18}$$

4. Assign $w_i \leftarrow w_{i_{new}}$.
5. **end if**
6. **end for**

Eq. (18) is used to obtain the updated weight for e_i, $w_{i_{new}}$, based on its current weight w_i, the distance to the collective preference C_1^t and the maximum distance between an expert's preference and C_1^t, $\max_j d(P_j^t, C_1^t)$. This expression ensures that $w_{i_{new}}$ is bounded to the $[0, w_i] \subseteq [0, 1]$ interval and $w_{i_{new}} \leq w_i$. Notice here that if P_i^t is the furthest preference relation from P_c^t, then $d(P_i^t, C_1^t) = \max_j d(P_j^t, C_1^t)$, and consequently $w_{i_{new}} = 0$, therefore e_i's importance weight becomes null.

As previously shown in Section III-A, the reduction of experts' importance weights in round t affects two steps in the following round, $t+1$, of the CRP:

- The computation of the consensus matrix CM upon experts' similarities.
- The computation of P_c upon experts' preference relations.

Regarding the former step, since CM is obtained by aggregating similarity values sm_{ij}^{lk} for each pair of experts, it is necessary to combine w_i and w_j in order to obtain a weight w_{ij} associated to such a pair. It is assumed that, if at least one expert weight in the pair (e_i,e_j) has been penalized, then the importance weight w_{ij} assigned to their similarity degree sm_{ij}^{lk} should be decreased. Therefore, it is proposed computing the weight of the pair (e_i, e_j) as $w_{ij} = \min(w_i, w_j)$.

Finally, as it will be shown in the illustrative example in Section 5, two different weight penalizing schemes can be defined:

- *Partial weight penalizing*: Reduced weights are taken into account in the computation of P_c only (see Eq. (8)), with the aim of making P_c closer to the preferences of those experts who contribute to achieving a consensus.
- *Full weight penalizing*: An extended case of the partial weight penalizing where, besides considering reduced weights to compute P_c, the agreement positions of those experts who contribute to achieving a consensus are also taken into account, in order to improve the convergence in the consensus degree, cr. Therefore, reduced weights are also integrated in the computation of CM (see Eq. (4)).

The effect of using either one of these penalizing schemes in the CRP will be shown in Section 5. Figure 6 shows graphically the overall process to manage non cooperating experts.

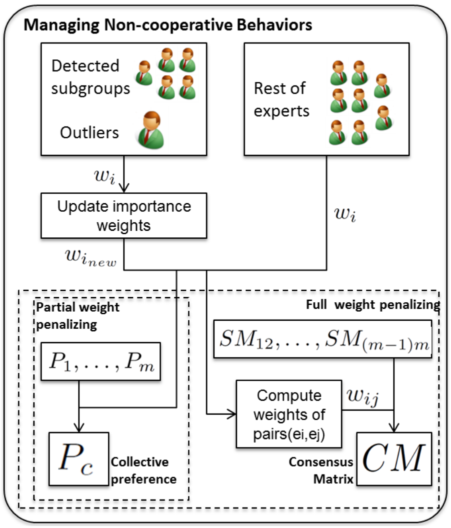

Fig. 6: Scheme of the method to manage non cooperating experts in CRPs.

IV. MONITORING TOOL BASED ON SOMS

Besides the proposed consensus model, and due to the necessity of having a visual insight on experts' preferences and their evolution across the CRP, in this section we propose a monitoring tool based on SOMs. Such a tool can be considered a complement to the consensus model presented in the previous section, that not only provides a clearer vision of the CRP performance, but also lets us find experts and subgroups of experts who may present different patterns of behavior against consensus, due to the fact that their preferences are moved against the collective opinion.

Different applications and tools have been implemented to support the SOM-based visualization of high dimensional data. One of them is SOM Toolbox[1], a powerful research-oriented plug-in for the widely-known MATLAB[2] software suite [20], which provides multiple ways of visualizing data, for instance by means of their two-dimensional PCA projection. SOM Toolbox can be used to process and visualize experts' preference relations and cluster centres managed by the consensus model proposed in this paper. To do so, we propose the following procedure, depicted in Figure 7, which is applied at the end of each CRP round t:

[1]http://www.cis.hut.fi/somtoolbox/
[2]http://www.mathworks.com

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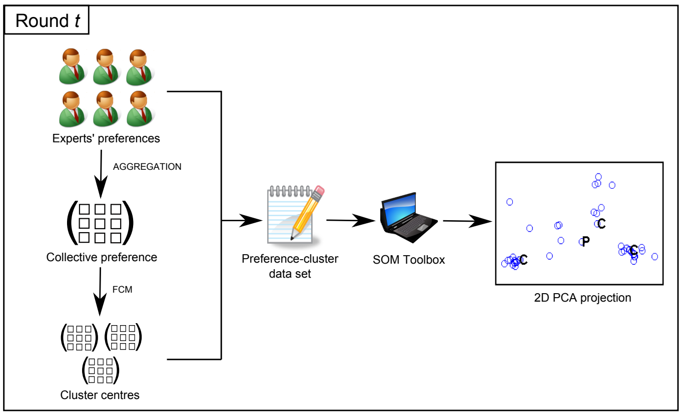

Fig. 7: Process to visualize experts' preferences and cluster centres in a CRP round t.

i) The collective preference P_c and cluster centres C_h in the current round are computed from experts' preferences P_i, as explained in Sect. III
ii) All preference relations, including P_c and cluster centres are gathered into a so-called preference-cluster data set file, where each data object is a preference relation, represented as a vector of dimension $n \times n$. The first line of the data set contains a number indicating the dimension of data. Data objects corresponding with cluster centres are given the label 'C', whereas the collective preference is given the label 'P', so that they can be easily localizable in the visual representation of preferences.
iii) The preference-cluster data set is processed by SOM Toolbox to generate a 2D PCA projection of experts' preferences and cluster centres in the current CRP round.

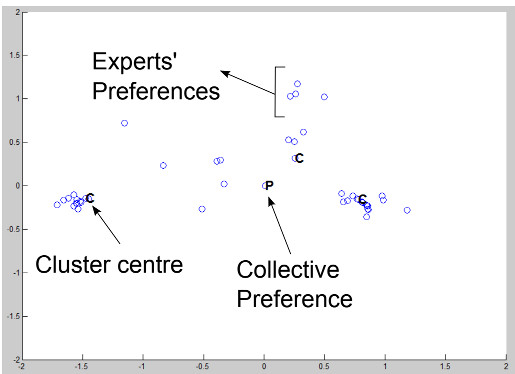

Fig. 8: 2-D visualization of experts' preferences and clusters with SOM Toolbox.

Figure 8 shows an example of 2-D visualization of experts' preferences and clusters, generated with SOM Toolbox.

V. Illustrative Example

In this section, an implemented version of the presented consensus model is used to solve a real-life GDM problem where a high number of decision makers participate. The main goal of such a simulation is to show the effectiveness and usefulness of our proposal when dealing with large groups of experts, some of which might present non-cooperative behaviors during the consensus process, thus hindering the achievement of an agreement.

The problem formulation is as follows: let us suppose that an expert commission compound by 50 members belonging to different areas, $E = \{e_1, \ldots, e_{50}\}$, must make an agreed decision regarding a recent discovery of fossil fuels in the province of Jaén, in Andalucía (Spain). The proposed alternatives $X = \{x_1, x_2, x_3, x_4\}$ are the following ones:

- x_1: Discard any exploitation actions, due to environmental factors.
- x_2: Authorize a national company to search for natural gas sources.
- x_3: Authorize a multi-national company to search for oil.
- x_4: Do a previous research on the area led by regional government.

The commission must achieve a minimum level of agreement of $\mu = 0.85$ before making a decision, the maximum number of rounds of discussion allowed is $Maxround = 10$ and the acceptability threshold is set as $\varepsilon = 0.02$. Some experts in the group may present individual behaviors, or they may form coalitions with a non-cooperative behavior, as it will be shown in the example.

Common parameters for the clustering, detection and management of behaviors are set below:

- Fuzziness coefficient: $b = 2$.
- Threshold for stopping condition in FCM: $\epsilon = 0.001$.
- Distance measure: Minkowski distance with $p = 1$.
- Cluster similarity threshold: $\kappa = 0.9$.
- Minimum detectable distance amongst clusters: $\nu = 0.01$.
- Consensus threshold to activate outlier detection, $\gamma = 0.75$.
- Membership threshold for outliers, $\delta = 0.4$.

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TABLE I: Consensus degree cr achieved, and detection of subgroup and individual non-cooperative behaviors in each round t ($N = 4$).

t	Without weight Penalizing			Partial weight Penalizing			Full weight Penalizing		
	cr	Subgroup detected	Outl. detected	cr	Subgroup detected	Outl. detected	cr	Subgroup detected	Outl. detected
1	0.63060	-	-	0.63060	-	-	0.63060	-	-
2	0.66655	-	-	0.66648	-	-	0.66614	-	-
3	0.69543	$\surd$	-	0.69573	$\surd$	-	0.69473	$\surd$	-
4	0.71914	$\surd$	-	0.71953	$\surd$	-	0.74663	$\surd$	-
5	0.72266	$\surd$	-	0.72605	$\surd$	-	0.84651	$\surd$	3
6	0.73317	-	-	0.75277	$\surd$	3	**0.85743**		
7	0.73525	-	-	0.78426	$\surd$	1			
8	0.74312	-	-	0.80119	$\surd$	1			
9	0.74468	$\surd$	-	0.80174	$\surd$	1			
10	0.74427			0.80160					

Two experimental studies have been conducted. In the first one, the effects of applying the different penalizing schemes to manage behaviors is shown; whereas the second one focuses on analyzing the effects in the CRP of using different values for the number of clusters, N, in the FCM algorithm.

Remark 3. *No comparison to other techniques is shown in the paper because, as far as we know, this is the first time a methodology based on fuzzy clustering is implemented and applied to support consensus reaching processes, and most current proposals of consensus models are for small groups and they do not focus on large-scale group decision making.*

A. *Experiments with Different Penalizing Schemes*

Firstly, the model is used to solve the GDM three times, applying the behavior detection scheme in all of them (with $N = n = 4$ clusters, see Section III-B), and different variations in the behavior management scheme for each one (see Section III-C):

1) *Without weight penalizing*: No penalizing is conducted upon detection.
2) *Partial weight penalizing*: A penalization on experts' weights is conducted only when computing the collective preference, P_c.
3) *Full weight penalizing*: A penalization is on experts' weights conducted when computing P_c and the consensus matrix, CM, from experts' similarity values.

Our hypothesis states that the application of behavior detection and management schemes on experts' preferences might improve the CRP performance, by increasing the convergence of cr towards the desired level of agreement, μ:

a) A partial weight penalizing may cause P_c to become closer to those experts who behave in favor of consensus, thus increasing slightly the convergence of cr towards μ.
b) A full weight penalizing may also take into account rather those experts who contribute to achieve an agreement in the computation of CM, which might imply a more substantial increase in the convergence of cr.

Once conducted the CRP, results are shown and analyzed. Table I shows the evolution of the consensus degree cr in each round, as well as the detection of subgroup and individual (outlier) behaviors, for each resolution of the GDM problem. The convergence of cr during the CRP is also graphically shown in Figure 9. In the cases of applying null or partial penalizing, consensus is not achieved, therefore it is necessary to apply a full penalizing to achieve it, by assigning low importance weights to non cooperating individuals and subgroups not only when computing P_c, but also when obtaining consensus degrees.

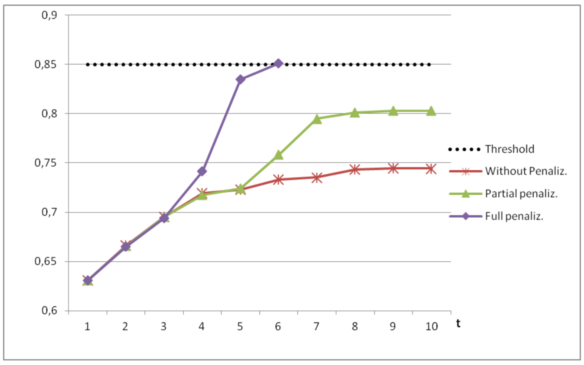
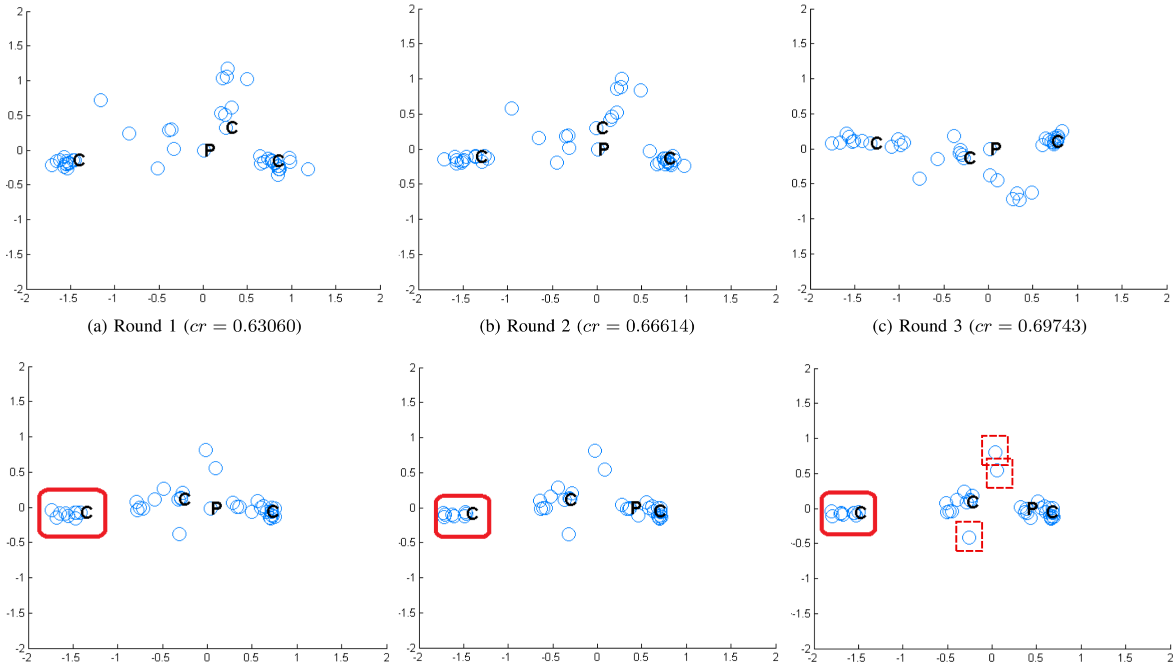


Fig. 9: Evolution of consensus degree cr in each round.

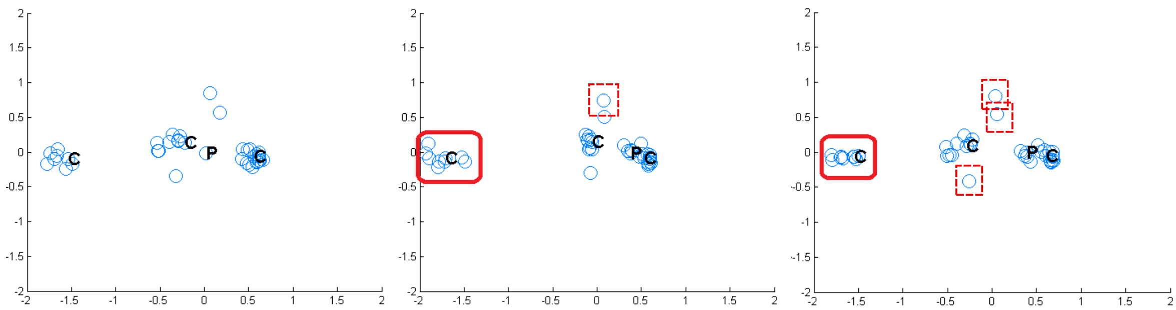
In order to provide a visual monitoring of the overall CRP performance, the SOM-based visualization tool SOM Toolbox is used to show experts' preferences in a 2D plot, as explained in Section IV. For the sake of space, we show the monitoring of the whole CRP for the case of applying full penalization. Figure 10 shows the visual representation of experts' preferences, the collective preference P_c (in the figure, denoted by 'P') and cluster centres C_h (in the figure, denoted by 'C') for each round of the CRP. As can be seen, a con-coalition of non cooperating experts is first detected at the end of the third round and consequently penalized from the fourth round onwards (solid rectangles represent penalized subgroups). When γ is exceeded, outliers (i.e. individual non-cooperative behaviors) are also detected and their weight is reduced (in the figure, they are surrounded by dashed rectangles). Additionally, from the fourth discussion round onwards, the position of P_c in the SOM shifts from the center of the SOM, which means that the weights of experts' preferences, which are used in the computation of P_c, have



(a) Round 1 ($cr = 0.63060$) (b) Round 2 ($cr = 0.66614$) (c) Round 3 ($cr = 0.69743$)

(d) Round 4 ($cr = 0.74663$) (e) Round 5 ($cr = 0.84651$) (f) Round 6 ($cr = 0.85743$)

Fig. 10: Experts' preferences visualization during the CRP, with full weight penalizing.



(a) No penalizing ($t = 10$, $cr \simeq 0.744$) (b) Partial penalizing ($t = 10$, $cr \simeq 0.802$) (c) Full penalizing ($t = 6$, $cr \simeq 0.857$)

Fig. 11: Visualization of experts' preferences in the last round of the CRP.

been updated due to penalizing, favoring those experts who contribute positively to achieve a consensus.

Remark 4. *The SOMs do not represent the absolute position of preference values, but rather the relative closeness of preferences amongst each other. Therefore, a P_c in the centre of several plots (e.g. Figures 10a)-c)) does not indicate equal values of P_c in them, but rather a collective preference obtained by using Eq. (9) (before penalizing weights).*

Finally, Figure 11 shows the visual representation of experts in the final round for each one of the three cases studied. It is remarkable here how the application of *any* of the two proposed weight penalizing schemes affects the value of P_c, which is moved with respect to the case of no penalizing, becoming closer to the opinions of those experts who contribute to achieving an agreement and further from the opinions of non cooperating experts. This may affect the subsequent alternative selection process and the final decision made. A similar position of P_c is obtained for both types of penalizing, since the main effect of applying a full penalizing with respect to a partial one is a higher convergence of cr.

These results allow us to confirm the hypothesis formulated, thus showing the importance and effectiveness of our approach to deal with large groups of decision makers, some of which

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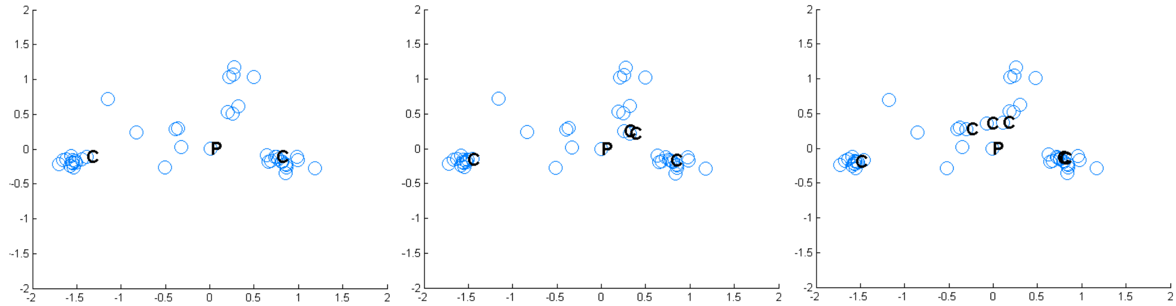
TABLE II: Consensus degrees cr and detection of behaviors for different numbers of clusters N (full penalizing).

t	N = 3			N = 5			N = 8		
	cr	Subgroup detected	Outl. detected	cr	Subgroup detected	Outl. detected	cr	Subgroup detected	Outl. detected
1	0.63060	-	-	0.63060	-	-	0.63060	-	-
2	0.66638	-	-	0.66853	-	-	0.66729	$\surd$	-
3	0.70582	-	-	0.69869	$\surd$	-	0.73263	$\surd$	-
4	0.72004	$\surd$	-	0.75141	$\surd$	6	0.75771	$\surd$	15
5	0.83488	$\surd$	3	0.83502	$\surd$	5	0.78596	$\surd$	15
6	**0.85374**			**0.85065**			0.81584	$\surd$	14
7							0.82774	$\surd$	21
8							0.82925	$\surd$	27
9							0.82981	$\surd$	15
10							0.83007	$\surd$	20



(a) N=3 (t=1) (b) N=5 (t=1) (c) N=8 (t=1)

Fig. 12: Visualization of cluster centres in the first round of the CRP.

might move their preferences against consensus and would prevent achieving the desired level of agreement if they are not detected and managed accordingly.

B. Experiments with Different Number of Clusters

Finally, some additional experiments are carried out by solving the consensus process with identical parameters, applying a full weight penalizing and using different values for the number of clusters considered in the FCM algorithm, N.

Table II shows the consensus degrees and detected behaviors for different values of N, and Figure 12 illustrates the position of cluster centres obtained in the first CRP round. From experiments conducted, it can be concluded that:

- $N = 2$ leads to undesired results, as C_2 always tends to approximate to P_c, which does not vary during the FCM algorithm, therefore its use has been discarded.
- If $N > n$ (being n the number of alternatives, in our case, $n = 4$), then some cluster centres are close to each other and they tend to overlap as N increases (see Fig. 12c, where $N = 8$ and some cluster centres overlap). Moreover, if a too high value of N is chosen, an excessive number of subgroup and individual misbehaviors are detected, which affects nearly all experts' weights during penalizing and, consequently, the convergence towards consensus is not improved with respect to applying no penalizing.
- Values of N which are close to the number of alternatives n provide good results in the behavior detection and an adequate convergence towards consensus.

It is concluded that an appropriate value for N is $n = 4$ (as it was considered in Sect. V-A), which makes sense if we assume that different experts' in the group might have a predilection for each one of the distinct alternatives $x_l \in X$, hence it is usual that at most n different subgroups with a clear preference over an specific alternative might appear during the CRP.

VI. Concluding Remarks

Current challenges for the improvement of consensus reaching processes in group decision making include the necessity of developing new consensus models capable of managing large scales of decision makers effectively, thus overcoming the difficulties derived from the high cost, complexity, the constant human supervision or even the possibility of dealing with subgroups of decision makers who present non-cooperative behaviors during the discussion process. Real-life decision making problems involving a large number of decision makers are becoming increasingly common, as occurs for instance with new trends such as e-democracy processes and social networks. In this paper, a consensus model capable of dealing with large groups of decision makers has been presented. Such a model utilizes an approach based on fuzzy clustering to detect and manage individuals or subgroups of decision makers who do not cooperate during the discussion process. Additionally, the model is complemented with a monitoring tool to visualize decision makers' preferences and their evolution during the consensus reaching process.

Despite the paper proposal has been presented under a methodological viewpoint, future works are mainly focused on developing a distributed software, which will be used to conduct a real large-scale experiment and prove the validity of the proposed model in a real-life problem. The proposed model is valid as such for its application in any business and organizational contexts, and it can be also easily extended by adapting it to more specific contexts, as mentioned above. Other future works are focused on the use of the proposed methodology in linguistic decision making [46], [47].

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4.4. MENTOR: A graphical monitoring tool of preferences evolution in large-scale group decision making

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MENTOR: A graphical monitoring tool of preferences evolution in large-scale group decision making

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ABSTRACT

Group decision making problems aim to manage situations in which two or more experts need to achieve a common solution to a decision problem. Different rules and processes can be applied to solve such problems (e.g. majority rule, consensus reaching, and so on), and several models have been proposed to deal with them. Some difficulties may arise in group decisions, being most of them caused by the presence of disagreement positions amongst experts. Given that group decision making problems have classically focused on a few number of experts, such difficulties have been relatively manageable by means of supporting tools based on textual or numerical information. However, such tools are not adequate when a large number of experts take part in the problem, therefore an alternate tool that provides decision makers with more easily interpretable information about the status of the problem becomes necessary. This paper proposes a graphical monitoring tool based on Self-Organizing Maps so-called MENTOR, that provides a 2-D graphical interface whose information is related to experts' preferences and their evolution during group decision making problems, and facilitates the analysis of information about large-scale problems.

1. Introduction

Decision Making is a common process in daily life. In Group Decision Making (GDM) problems, two or more individuals or *experts*, with their own attitudes and opinions, need to achieve a common solution to a decision problem consisting of several alternatives [1–3]. GDM problems are present in diverse application areas that require the participation of multiple experts, such as management and engineering and politics [4–6].

GDM problems can be solved by applying different processes, ranging from the use of classical decision rules (such as the majority or minority rule [7]), to the application of a consensus reaching process, which is a process of negotiation between experts, aimed to achieve a high level of agreement in the group before making a decision [8]. Consensus reaching processes are increasingly necessary in nowadays group decisions [9].

A large number of theoretical models and approaches to facilitate the resolution of GDM problems have been proposed in the literature [3,10–14]. Moreover, several authors have developed some computer-based Group Decision Support Systems (GDSS), to give groups further assistance in such problems [3,15]. Some of these GDSS make use of the Internet to allow groups to solve GDM problems ubiquitously [16,17].

Classically, GDM problems have been solved by a few number of experts. In these cases, when typical difficulties in group decisions arise (such as the presence of disagreement positions), they can be managed with the aid of GDSS and supporting tools that provide numerical or textual information about preferences of experts in the group [2,15,16]. Such tools could be often utilized with analytical purposes by a person who is responsible for making the final decision or *decision maker*. They can also be utilized by the *moderator* of a consensus reaching process [7,8].

However, new paradigms and ways of making group decisions, such as social networks [18] and e-democracy [5], have caused that decisions made by a larger number of experts become more frequent in recent years, therefore *large-scale GDM problems* are attaining greater importance. The resolution of large-scale GDM problems implies new challenges and requirements in terms of the higher cost and time invested to make the decision, and the increasing complexity of the problem. Additionally, in large-scale GDM problems, a considerable amount of information related to the preferences of experts must be managed, therefore a higher complexity appears in those analysis tasks that would be much more manageable in the case of dealing with small groups, for instance: (i) detecting conflicts amongst experts, (ii) determining the closeness between experts' opinions, (iii) identifying the

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number and identity of experts that agree/disagree with each other, and (iv) finding coalitions or subgroups of interest in the group, etc. Most existing GDSS focus on GDM problems with few experts, in which numerical information about the status of the problem can be easily analyzed by a decision maker interested in it. However, in large-scale GDM problems the amount of information available may become much larger and, consequently, much more complicated to understand.

Different solutions can be proposed to support the previous analysis tasks [16]. In large-scale GDM, it would be particularly interesting to increase knowledge about the problem and make it more accessible to the decision maker interested in it, by means of a graphical 2-D tool that visualizes information about the whole group. In this sense, Self-Organizing Maps (SOMs) [19,20] have previously proved to be an effective means to visualize high dimensional data in a low-dimensional space [21,22]. Therefore, a graphical tool based on two-dimensional SOMs would facilitate the analysis and interpretation of diverse aspects of interest in large-scale GDM problems.

In this paper, we present a SOM-based graphical monitoring tool so-called MENTOR, that supports decision makers in the analysis of information about the status of large-scale GDM problems during their resolution. Such a tool facilitates the obtaining of important information about diverse features in these problems, such as the detection of agreement/disagreement positions within the group, the evolution of experts' preferences, or the level of closeness between experts' opinions achieved during consensus reaching processes in the cases they are carried out. MENTOR is also presented as a tool that can be integrated with different GDSS proposed in the literature, therefore it implies a important step towards the design of new, highly-interpretable GDSS.

The paper is structured as follows: in Section 2, some preliminaries about GDM and SOMs are reviewed. Section 3 presents MENTOR, the graphical monitoring tool based on SOMs, by explaining how it works and describing its main features for analysis and interpretation of graphical information about the GDM problem. Section 4 shows an example of application of MENTOR in a large-scale GDM problem. Finally, some concluding remarks are exposed in Section 5.

2. Preliminaries

Given the paper proposal of a SOM-based graphical monitoring tool to support large-scale GDM problems, in this section we review GDM problems, paying special attention to consensus reaching processes as a means for smoothing group conflicts and finding agreed solutions. Eventually, it is revised some elementary concepts about SOMs, which are the basis for graphical representation of information in the proposed tool.

2.1. Group decision making problems

The need for making decisions in which multiple experts with different viewpoints are involved, is frequent in many complex real-life decision situations and organizational structures. GDM problems, where a group of experts must make a common decision together, are normally utilized in such situations [2,3]. Some examples of application of GDM problems are: political and democratical systems, engineering, management, etc. [4–6].

Formally, GDM problems can be defined as decision situations characterized by the participation of two or more experts, with their own knowledge and attitudes, in a decision problem consisting of a set of alternatives or possible solutions to such a problem [1,3]. The following elements are found in any GDM problem:

- A set $X = \{x_1, \ldots, x_n\}$, $(n \geqslant 2)$ of *alternatives*.
- A set $E = \{e_1, \ldots, e_m\}$, $(m \geqslant 2)$ of *experts*, who express their judgements on the alternatives in X.

Each expert e_i, $i \in \{1, \ldots, m\}$, provides his/her opinions over alternatives in X by means of a preference structure. Some types of preference structures commonly utilized in GDM are: preference relations [23], utility vectors [24] and preference orderings [25]. Preference relations have been specially utilized in many models of GDM under uncertainty. They are defined as follows:

Definition 1 (*[23,26]*). A *preference relation* P_i associated to expert e_i, $i \in \{1, \ldots, m\}$, on a set of alternatives X is a fuzzy set on $X \times X$, represented by a $n \times n$ matrix of assessments $p_i^{lk} = \mu_{P_i}(x_l, x_k)$ as follows:

$$P_i = \begin{pmatrix} - & \ldots & p_i^{1n} \\ \vdots & \ddots & \vdots \\ p_i^{n1} & \ldots & - \end{pmatrix}$$

where each *assessment*, $p_i^{lk} = \mu_{P_i}(x_l, x_k)$, represents the preference degree of alternative x_l over x_k according to e_i. Assessments $p_i^{ll}, l \in \{1, \ldots, n\}$, situated in the diagonal of the matrix, are not defined, since an alternative x_l is not assessed with respect to itself.

Experts' assessments are expressed in a specific information domain. Some information domains widely used in GDM are: numerical, interval-valued and linguistic [11].

The solution to a GDM problem is obtained by using either a *direct approach*, where the solution is directly obtained from experts' preferences, or an *indirect approach*, in which a collective opinion is computed before determining the chosen alternative/s [27]. In both approaches, the selection process to solve GDM problems consists of two phases [28]: (i) an *aggregation phase*, where individual preferences are combined and (ii) an *exploitation phase*, where an alternative or subset of alternatives are obtained as the solution to the problem.

Despite different classic guiding rules, such as the majority rule and minority rule, have been suggested to carry out the selection process in GDM [7], they do not guarantee a high level of agreement amongst experts regarding the decision made: it is possible that some of them may not accept the solution chosen, because they might consider that their opinions have not been considered sufficiently [8]. In such cases that a more agreed decision is necessary, a *negotiation phase* should be introduced as part of the GDM problem resolution process to achieve a high degree of agreement among experts before making a decision. A variety of formal negotiation models based on different theoretical backgrounds can be found in the literature [29,30]. Nevertheless, in the research field of GDM we move in, it is usually applied a *consensus reaching process* to achieve a collective agreement before making a group decision [8]. Consensus has attained a great importance to reach more appreciated solutions in GDM problems, and it has become a major research topic in the last decades [14–17,31,32].

The process to reach a consensus is a dynamic and iterative discussion process, frequently coordinated by a human figure known as moderator [7,8]. A general scheme of consensus reaching process is shown Fig. 1. Its phases are briefly described below:

1. *Gathering preferences*: Each expert provides his/her preferences to the moderator.
2. *Computing the level of agreement*: The moderator determines the level of agreement in the group.
3. *Consensus control*: If the level of agreement is enough, the group moves onto the selection process, otherwise more discussion is required.

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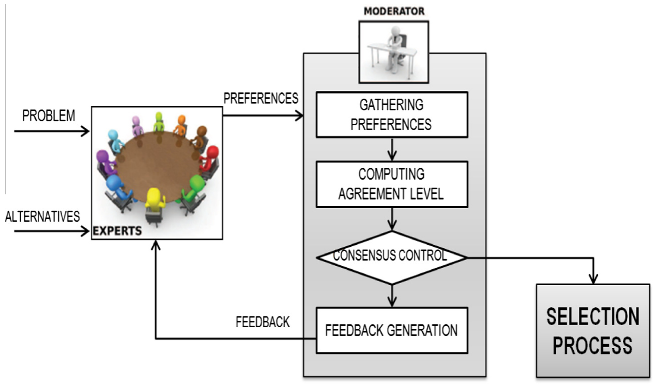

Fig. 1. General consensus reaching scheme in GDM.

4. *Feedback generation*: The moderator gives experts some feedback, suggesting them how to modify their preferences and make them closer to each other, to increase the level of agreement in the group.

In consensus-driven GDM problems, some crucial aspects that should be monitored during the consensus reaching process are the status of experts' preferences across the time, and the evolution of the level of agreement achieved in the group. Besides, in large-scale GDM problems, it is usual that some experts or subgroups of them disagree with each other on their opinions, they do not cooperate to reach a consensus or they try to deviate the collective opinion. A graphical tool that monitors these features both in GDM problems and in consensus reaching processes becomes then necessary, in order to analyze the positions of experts' preferences with respect to the group. The tool proposed in this paper is based on SOMs [19], therefore some basic concepts about this visualization technique will be reviewed in the following subsection.

2.2. Basic concepts on Self-Organizing Maps (SOMs)

Self-organizing Maps (SOMs) are a non-supervised learning technique used in exploratory data mining, introduced by Kohonen [19] and based on neural networks [33]. It is one of the best known methods for the construction of topographic maps, i.e. low-dimensional (usually 2D or 3D) visualizations of high dimensional data [21,22,34].

The SOM algorithm can be regarded as a "nonparametric regression" method, whose goal is fitting a number of discrete reference vectors to a distribution of vectorial input data samples [20]. The reference vectors define the nodes of a kind of *elastic* neural network, where a topologically ordered mapping is formed from the input space onto the neural network, thus obtaining a *feature map*. This adaptive process is biologically inspired by the organizations found in brain structures. If the network is a regular two-dimensional lattice, the feature map can be used to project and visualize high-dimensional data on it.

In the following, the basic SOM algorithm in the euclidean space is briefly reviewed [19,20]. Assume a two-dimensional regular (hexagonal or rectangular) lattice in which the array of nodes (*neurons*) are situated. Each node has associated a reference vector m_i of dimension n, which is defined by $m_i = [\mu_{i1} \dots \mu_{in}]^T \in \mathbb{R}^n$, being $i \in \mathbb{R}^2$ the position in the lattice of the node associated to m_i. Weights $\mu_{ij} \in \mathbb{R}$ are initialized either randomly or by means of an initializing technique. On the other hand, a training input vector x of dimension n is defined as $x = [\xi_1 \dots \xi_n]^T \in \mathbb{R}^n$.

At each iteration of the algorithm, an input data sample x is compared with all the m_i, and the location c of the *best matching unit* (BMU), i.e. the reference vector m_c whose weights are closest to values of x, is determined. x is then mapped onto this location. The BMU, denoted by m_c, accomplishes:

$$\|x - m_c\| = \min_i\{\|x - m_i\|\} \quad (1)$$

which is equivalent, in terms of the BMU location c, to:

$$c = \arg\min_i\{\|x - m_i\|\} \quad (2)$$

During learning, those nodes topographically close to the BMU (neighbor nodes), activate each other to learn something from input x. This process causes a smoothing effect on weights of nodes situated within this neighborhood. Fig. 2 illustrates this process [22]. Solid and dashed lines represent the situation before and after updating weights of nodes upon x, respectively.

Given an iteration t of the algorithm, $t = 0, 1, 2, \dots$, weights in reference vector m_i are updated as follows:

$$m_i(t+1) = m_i(t) + h_{ci}(t)[x(t) - m_i(t)] \quad (3)$$

where $h_{ci}(t) = h(\|r_c - r_i\|, t)$ is the so-called neighborhood function defined over the lattice nodes. $h_{ci}(t) \to 0$ when $t \to \infty$, thus ensuring convergence. $r_c, r_i \in \mathbb{R}^2$ are the locations of vectors m_c, m_i in the lattice. When $\|r_c - r_i\|$ increases, $h_{ci}(t) \to 0$. Let $N_c(t)$ be a

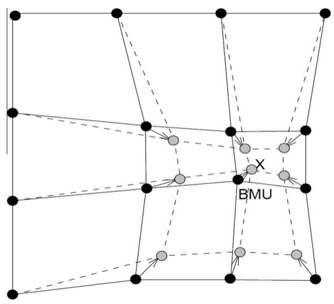

Fig. 2. Update of the BMU and its neighbors upon x (taken from [22]).

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neighborhood set of lattice nodes around c. Then, the neighborhood function can be defined as follows:

$$h_{ci}(t) = \alpha(t) \quad \text{if } i \in N_c, \qquad h_{ci}(t) = 0 \quad \text{otherwise} \tag{4}$$

being $\alpha(t) \in (0,1)$ a learning rate that decreases over time (a value commonly taken is $\alpha(t) = 0.9(1 - \frac{t}{100})$). The radius of $N_c(t)$ also decreases over time, thus reducing the neighborhood set of c progressively. Another possible, smoother neighborhood function in terms of the Gaussian function, is:

$$h_{ci} = \alpha(t) = \exp\left(-\frac{\|r_c - r_i\|^2}{2\sigma^2(t)}\right) \tag{5}$$

As a result of applying the above mentioned steps iteratively with a set of input data samples, reference vectors tend to approximate them in an orderly fashion, and the lattice becomes ordered, in the sense that reference vectors in neighboring nodes have similar weights. The training process ends when a sufficient number of input vectors have been processed and the iterative process given by Eq. (3) converges towards stationary values.

Variants of the basic SOM algorithm include the so-called ''Dot-Product'' SOM, which involves the use of a more *biological* matching criterion, based on dot product operations [19]. In this case, the BMU is determined by:

$$x^T(t) \cdot m_c(t) = \max_i \{x^T(t) \cdot m_i(t)\} \tag{6}$$

Once the SOM has been constructed, we can proceed to locate on the map projections of those data samples that must be interpreted and visually analyzed. There are multiple SOM-based methods to visualize data, such as distance matrices, similarity coloring, data histograms and PCA projections [21,22].

SOMs have been successfully utilized in different descriptive data mining applications, such as full-text and financial data analysis, cluster analysis, and vector quantization and projection [21,34].

3. MENTOR: SOM-based graphical monitoring tool of preferences to support group decision making

As stated in the introduction, large-scale GDM problems are increasingly common in multiple real-life contexts. In these problems, classical tools and GDSS based on numerical or textual information that have been proposed to support GDM problems with small groups, may not be appropriate for a decision maker, when he/she needs to analyze the large amount of information related to experts' preferences to have a deeper knowledge about the current status of the problem.

For these reasons, in this section we present a graphical monitoring tool based on SOMs, so-called MENTOR, that supports decision makers by providing them with easy interpretable information about the status of large-scale GDM problems during their resolution, thus facilitating the analysis of diverse crucial aspects that are common in these problems, such as:

- The closeness between experts' preferences.
- Detection of conflicts amongst experts.
- Identification of subgroups of experts that disagree with the rest of the group.

Firstly, we will show a detailed scheme of the tool operation during the resolution process of GDM problems. We will then describe some examples of GDM situations in which the tool can be utilized to overcome the difficulties stated above.

Fig. 3 shows the architecture of MENTOR. The tool has been conceived as a local application that receives a set of experts' preferences about a GDM problem and generates a 2-D graphical interface with their representation. Although the use of MENTOR is currently proposed as a self-contained tool that is directly used by decision groups, it is also suggested its integration with new or already existing GDSS, to make them more interpretable for decision makers and support them in the overall decision analysis process. Further detail on the use of the technologies used in MENTOR (Java, MATLAB and SOM Toolbox), is given in the following subsection.

3.1. Scheme of the monitoring tool

A scheme of operation of MENTOR is shown in Fig. 4. The procedure it follows to generate a graphical representation about the status of the GDM problem consists of three phases, which are described below:

(1) *Gathering Information about the GDM problem*: Information about the status of the GDM problem that will be graphically represented, is gathered in this phase. Such information usually consists in preferences of all experts in the group. Sometimes it would be also interesting to gather additional information, for example the collective preference of the group.
MENTOR deals with opinions expressed numerically. More specifically, we consider the use of fuzzy preference relations (in which assessments $p_i^{lk} \in [0,1]$), to generate graphical representation of them (as will be shown with more detail in the following phase). Nevertheless, the tool also allows the management of different preference structures [35]. To do so, it is proposed the use of existing approaches to unify them into fuzzy preference relations. For instance, in [35] it is shown the relationship between different representation formats (preference orderings, utility values, multiplicative and fuzzy preference relations), and a set of transformation functions are defined to obtain a fuzzy preference relation from preferences expressed under each of these representation formats.
Regarding preferences expressed under different information domains (such as intervals or linguistic values), some approaches to conduct them into a common information domain can be also found in the literature. For example, in [11] some transformation functions are proposed to unify numerical, interval-valued and linguistic assessments into fuzzy preference relations.
Taking into account the above mentioned approaches, it is shown that MENTOR can be utilized in large-scale GDM problems in which experts can use different preference structures or information domains to express their opinions. Consequently, its integration with existing GDSS that incorporate such approaches is also possible.

(2) *Transforming Information to SOM-based format*: Once information to be visualized has been gathered, it must be transformed into a suitable format for its treatment by MENTOR. Since the tool is based on SOMs, it is necessary to represent preferences as input data samples (vectors) that can be managed by SOM algorithms (see Section 2.2). To do so, a *preference data-set* is generated upon preferences.
The software that generates preference data-sets upon the set of experts' preferences has been implemented with Java[1]

[1] A sample version of the Java application to generate preference data-sets upon a set of preferences is available at our website: http://sinbad2.ujaen.es/cod/mentor.

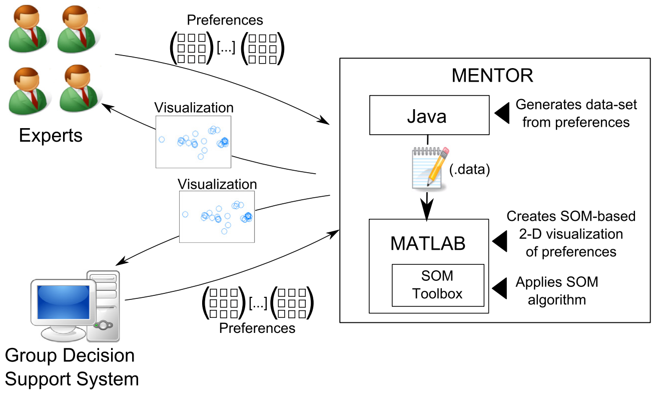

Fig. 3. Architecture of MENTOR.

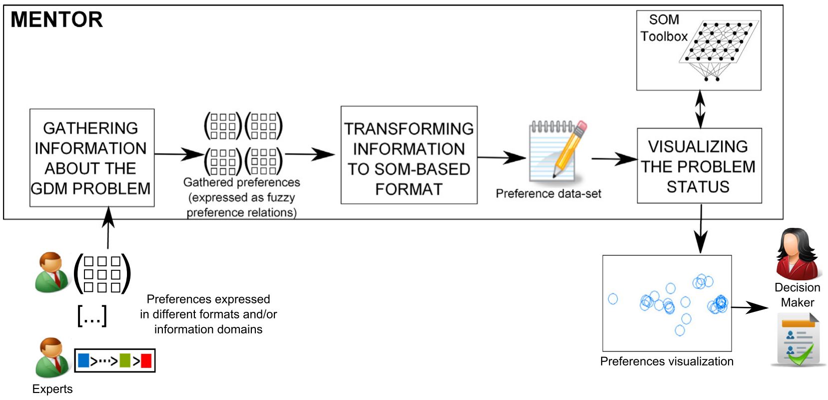

Fig. 4. General scheme of MENTOR.

(see Fig. 3). Preference data-sets are generated as files with extension *.data*. The structure of the preference data-set is as follows: the first row contains an integer value indicating the dimension of data samples, which is equal to the number of assessments each preference consists of. From the second row onwards, each row represents a data sample, corresponding to the preference of a single expert. The input preference format required to build the data-set is a numerical preference relation (e.g. a fuzzy preference relation, whose assessments are values in the unit interval [23]). Therefore, given a GDM problem with n alternatives, the dimension of data obtained from preferences must be equal to $n(n-1)$ (assessments of the type p_i^{ll}, $e_i \in E$, $x_l \in X$, are not considered, as stated in Section 2.1). Assessments are separated by blanks.

Data samples can be optionally tagged with informative purposes, by placing an alphanumerical tag at the end of the corresponding row. Tagging may provide additional information about a specific preference (for example, the name or role of its corresponding expert). Tags are not processed by the underlying SOM algorithm of MENTOR, but their content can be visualized together with the corresponding preference to provide additional knowledge about the problem.

Fig. 5 shows an extract of a preference data-set structure, in which two preferences have been tagged.

The following example illustrates the transformation of an expert's preference relation into an element of the preference data-set:

Example 1. Let P_i be the following fuzzy preference relation provided by an expert e_i, about a GDM problem consisting of $n = 4$ alternatives:

$$P_i = \begin{pmatrix} - & 1 & 0.5 & 0.9 \\ 0 & - & 0.15 & 0.4 \\ 0.5 & 0.85 & - & 1 \\ 0.1 & 0.6 & 0 & - \end{pmatrix}$$

Then, its corresponding data sample in the preference data-set obtained, is represented as follows:

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```
12
0.2 0.5 0.2 0.8 0.5 0.5 0.5 0.5 0.0 0.8 0.5 1.0
0.2 0.35 0.2 0.8 0.75 0.3 0.65 0.25 0.0 0.8 0.7 1.0
[...]
0.7 0.5 0.7 0.3 0.4 0.5 0.5 0.6 0.7 0.3 0.5 0.3 X
[...]
0.41 0.45 0.44 0.59 0.56 0.4 0.54 0.43 0.31 0.55 0.6 0.68 P
```

Fig. 5. Example of preference data-set with tags.

1 0.5 0.9 0 0.15 0.4 0.5 0.85 1 0.1 0.6 0

Although data samples in the data-set must be built upon numerical preference relations, some existing GDM models and approaches allow the unification into such format from experts' preferences expressed by means of different structures [35] or heterogeneous information [11], as aforementioned in the previous phase. Similarly, incomplete preferences [13] and preferences expressed in different scales [36] could be also considered, because the underlying SOM algorithm (which is applied in the following phase) can deal with incomplete data, and it also implicitly normalizes data values expressed in different numerical scales.

(3) *Visualizing the problem status*: The preference data-set is used as an input to apply a SOM-based technique that generates a 2-D graphical projection of data contained in it. Such a projection may be utilized by a group member (e.g. a decision maker who coordinates the whole group) for analyzing aspects of interest about the GDM problem.
The application to visualize preferences has been implemented by means of the software suite MATLAB[2] (see Fig. 3), which facilitates the management of data-sets and their graphical representation. MATLAB also offers possibilities to integrate its user-developed applications with a variety of widely used technologies, such as, Java, C++, and .NET, thus offering the possibility to communicate MENTOR with other systems.
Preference data-sets with extension *.data* obtained previously, are directly read by MATLAB, without the need for any further processing. Then, a SOM algorithm must be invoked to create the map on which data will be visualized. To do so, we have utilized the implemented SOM algorithms provided by a third-party MATLAB library so-called SOM Toolbox,[3] which was developed by Vesanto et al. [22] and constitutes a powerful research-oriented library with numerous functions and possibilities for managing SOMs and analyzing/visualizing data with them. By using this library, MENTOR offers the flexibility to apply different SOM algorithms defined by several settings, including: (i) the choice of the map size and shape (rectangular or hexagonal lattice), (ii) a matching criterion (see Eqs. (1) and (6) for instance), (iii) the neighborhood function, $h_{ci}(t)$, or (iv) the learning rate, $\alpha(t)$, amongst others.
Once constructed the map, each preference in the data-set is projected into it. The visualization method considered to show this task is a two-dimensional PCA projection of preferences [21]. Functions to generate and plot a graphical interface tho show PCA projections are also provided by MATLAB and SOM Toolbox.
It is noteworthy that in this phase, instead of obtaining a single graphical projection of experts' preferences solely, it would be sometimes useful to provide further detailed graphical information. For example, visualizing preferences at different levels of detail can be particularly interesting in GDM problems based on preference relations [26], because it would be sometimes convenient to view experts' opinions on each specific alternative (for purposes of disagreement detection, for instance). Then, a visual projection can be generated for each alternative $x_l \in X$ separately.
Tagging data might also be useful for several visualization purposes, some of which are:

- Viewing the collective preference of the group, by including and tagging it in the preference data-set.
- In some cases, it can be interesting to provide each expert with a visual representation of his/her own position with respect to the group. This can be done by generating a personalized graphical projection for each expert, in which his/her own preference is tagged.

Fig. 6 shows the graphical visualization corresponding to the complete data-set whose extract was shown in Fig. 5, in which the expert's self preference and the collective preference have been tagged.
In group decisions under consensus, the graphical visualization of the GDM problem status across the discussion process would be particularly convenient. Given that such processes consist of several rounds in which experts modify their opinions to increase agreement in the group (see Section 2.1), MENTOR can be iteratively used in consensus-based GDM problems, so that graphical information of the problem status is generated at each consensus round (as will be shown in the application example in Section 4). Visualizing the evolution of experts' preferences across the time may provide a better insight on the overall performance of this kind of problems and even a foresight of the future status of such problems in upcoming consensus rounds.

3.2. On the use of MENTOR in large-scale GDM

In the following, we illustrate how the graphical information provided by MENTOR can be used to facilitate the analysis of some important aspects and difficulties found in GDM problems, which are especially frequent in large-scale GDM. Such aspects and difficulties, and the way in which MENTOR facilitates their detection and analysis, are enumerated below:

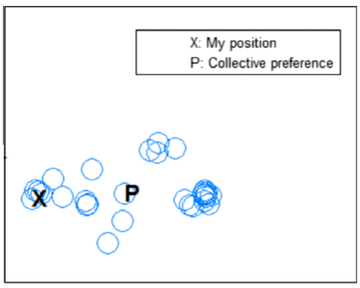

Fig. 6. Example of preferences visualization with tags.

[2] We are currently working on obtaining the necessary MATLAB license to release a sample version of the visualizing application in our website. Meanwhile, readers interested in obtaining a visualization of their preferences can follow the instructions found in: http://sinbad2.ujaen.es/cod/mentor.
[3] http://www.cis.hut.fi/somtoolbox/.

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- *Detecting conflicting opinions amongst experts*: Analyzing numerical or textual information about experts' preferences to identify conflicting opinions can be an affordable task if dealing with small groups, but not so adequate when the group size is large. The 2-D representation of preferences generated by MENTOR can provide a visual insight on conflicting opinions (if any) in these cases, because such preferences are visually represented as data points that are allocated far from each other.
- *Identifying disagreing experts*: When conflicting opinions are detected (see above), it would be interesting for the decision maker to view the identity of experts or subgroups of them who disagree with each other. This can be done by tagging the preferences of such experts, so that their names or identifiers can be also represented graphically.
- *Determining the closeness and agreement cardinality graphically*: Although most consensus models to support consensus reaching processes compute a global degree of agreement in the group analytically (usually as a numerical value in the unit interval) [1,14,15], such computations are frequently based on compensative consensus measures, in which case the collective agreement level computed might sometimes not reflect possible disagreement positions between some experts faithfully. In such cases, preferences visualization may help the decision maker to view the closeness between experts' opinions and decide whether the agreement cardinality (i.e. the number of experts who present a high agreement on the collective opinion with respect to the total group size) is enough or not to make a final decision, in situations of hesitancy.
- *Detecting non-cooperative behaviors in consensus reaching*: In GDM problems that require consensus, experts may adopt different types of behavior during the discussion process, regarding their predisposition to modify their initial opinions to make them closer to the collective opinion. Some experts or coalitions of experts with similar interests may not present a cooperative behavior in these problems, in the sense that they might move their preferences strategically trying to deviate the collective opinion [37]. If the necessary mechanisms to detect such behaviors analytically are utilized, then it is possible to tag experts involved in such behaviors and facilitate their graphical detection as well. Additional information about the relative size of the disagreeing subgroup with respect to the total group size would also be useful.

The illustrative example presented in the following section shows some of the above mentioned issues in practice.

4. Application example

In this section, an example of application of MENTOR to a real-life GDM problem is presented to show some of the possibilities such a tool offers, as well as its usefulness in practice. To do so, firstly an example of large-scale GDM problem is proposed. Then, the problem is solved by applying a simple GDM resolution scheme, and preferences in the group are visualized and analyzed by using MENTOR. Finally, a consensus reaching process is also applied to seek a higher degree of agreement, and MENTOR is used to visualize the evolution of experts' preferences across the process of negotiation.

4.1. Definition of the large-scale GDM problem

The GDM problem is formulated as follows: the 2013 graduating class of Computer Science M.Sc. Degree, compound by 46 students, $E = \{e_1, \ldots, e_{46}\}$, needs to decide the destination for their final year trip, amongst four possible choices, $X = \{x_1$: Mediterranean cruise, x_2: Tunisia tour, x_3: Canary Islands, x_4: Prague, Vienna and Budapest$\}$. During a lab session to which all 46 students attended, each one was requested to provide a fuzzy preference relation over the four alternatives.[4]

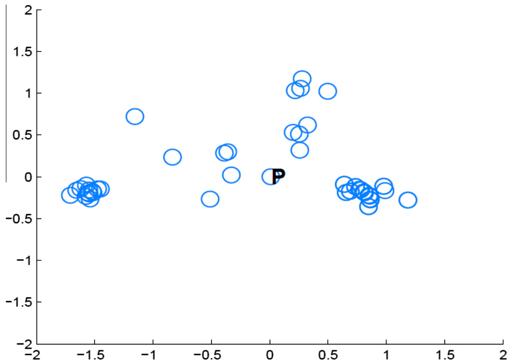

Fig. 7. Graphical representation of the group's preferences.

4.2. Visualization of a simple GDM resolution process

The large-scale GDM problem defined above was solved by applying a direct resolution scheme [27]. MENTOR was used to gather and visualize all experts' preferences and the collective preference obtained in the aggregation phase [28], having the latter been tagged to facilitate its detection.

Fig. 7 shows the graphical projection of preferences generated by MENTOR. The tag "P" indicates the position of the collective preference. As can be seen, some useful information can be easily obtained by analyzing the graphical representation generated: there exist two significant subgroups of students with very similar interests. However, such subgroups present a strong disagreement with each other and with the rest of students, who have diverse preferences that are situated far from the majority opinions.

The graphical representation of preferences provided by MENTOR let us conclude, without the need for analyzing the large amount of numerical information about experts' preferences, that the proposed solution to the GDM problem (given by the collective preference) is supported by a minor number of experts only, therefore it would not be a well-accepted solution by the group.

4.3. Visualization during a consensus reaching process

Given the low level of students' agreement on the initially obtained solution, it would be convenient to apply a consensus reaching process before carrying out the selection process. To do so, the consensus model proposed in [17] has been used, by considering the same initial preferences of students (see Fig. 7).

A total of five consensus rounds were carried out. At the end of each round, MENTOR generated a graphical projection of preferences to facilitate an analysis of their evolution, as well as the detection of possible disagreement positions and patterns of behavior adopted by some students. Fig. 8 shows the projections obtained from the second round onwards. Most students tended to move their preferences closer to the agreement position, which

[4] A large amount of information about preferences has been used in this example, therefore it was omitted for the sake of space. A supplementary material document that contains such preferences is also available at: http://sinbad2.ujaen.es/cod/mentor.

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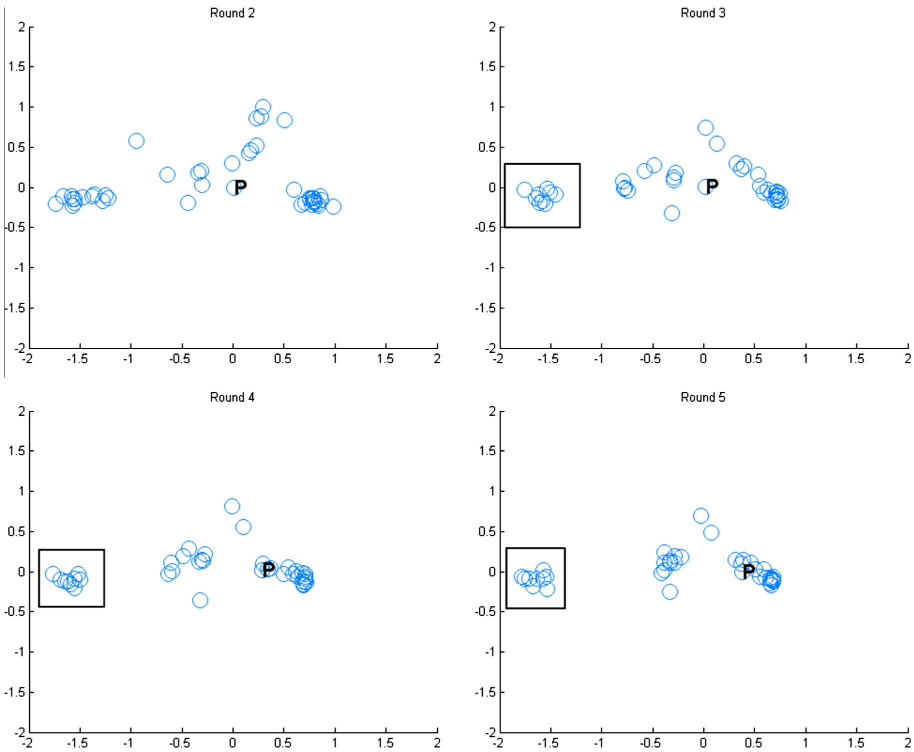

Fig. 8. Graphical representation of preferences during the consensus reaching process.

means they contributed positively to reach a consensus by applying changes suggested by the consensus model considered. However, MENTOR allowed us to notice that one of the two aforementioned subgroups of interest presented a different behavior, as students belonging to it did not move their preferences at all, showing that they were not interested in achieving an agreed decision, but rather in their own preferred options. This fact illustrates how MENTOR facilitates the detection of both disagreement positions and undesired behavioral patterns of experts or coalitions of them.

Based on subgroup behaviors detected, different alternate actions or decisions could be carried out by a decision maker depending of each particular problem circumstances, for example: informing experts involved that they are hindering the achievement of a consensus, moving onto the selection process to make the final decision before such experts can deviate the group opinion excessively, or penalizing experts who do not cooperate with the rest of the group [37].

5. Concluding remarks

This paper has presented MENTOR, a graphical monitoring tool based on Self-Organizing Maps to support large-scale Group Decision Making problems. The main goal of such a tool consists in helping decision makers to obtain and analyze easy interpretable information about the status of these problems during their resolution, as well as letting them analyze visually how different individuals or subgroups of them behave during the problem. MENTOR can also be used to detect and analyze visually a variety of aspects that are especially frequent in large-scale group decisions, such as the presence of subgroups of individuals with similar interests or the existence of agreement or disagreement positions. Additionally, it facilitates the monitoring of the problem status across the time in the cases that a consensus reaching processes is carried out. The visual analysis that MENTOR provides goes beyond the numerical information that Group Decision Support Systems or consensus models usually manage and provide: with MENTOR it is possible to find out, in a more understandable way, what does such numerical information mean, how do experts organize in subgroups, which experts do not contribute to achieve a consensus in the group, etc.

Although the tool is rather oriented towards giving support to a decision maker who is responsible for supervising the problem (e.g. a moderator in a consensus reaching process or a system administrator if the problem is solved with the aid of a Group Decision Support System), it has been shown that for some specific purposes (such as visualizing an expert's self position with respect of the rest of the group) it would be also interesting to provide experts with personalized visual information about the current problem status.

An example of application of the monitoring tool has been also presented, to solve a group decision making problem by applying both a direct selection process and a consensus reaching process. Such an example has illustrated how to analyze the behavior of experts through their preferences, as well as how to detect disagreement positions easily.

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4.5. Modelling experts' attitudes in group decision making

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ORIGINAL PAPER

Modelling experts' attitudes in group decision making

I. Palomares · J. Liu · Y. Xu · L. Martínez

Published online: 26 May 2012

Abstract Nowadays, important decisions that have a significant impact either in societies or in organizations are commonly made by a group rather than a single decision maker, which might require more than a majority rule to obtain a real acceptance. Consensus-reaching processes provide a way to drive group decisions which are more accepted and appreciated by people affected by such a decision. These processes care about different consensus measures to determine the agreement in the group. The correct choice of a consensus measure that reflects the attitude of decision makers is a key issue for improving and optimizing consensus-reaching processes, which still requires further research. This paper studies the concept of group's attitude towards consensus, and presents a consensus model that integrates it in the measurement of consensus, through an extension of OWA aggregation operators, the so-called Attitude-OWA. The approach is applied to the solution of a real-like group decision making problem with the definition of different attitudes, and the results are analysed.

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1 Introduction

Group decision making (GDM) problems are required throughout most companies and organizations nowadays, in order to guarantee a right development in them. They can be defined as decision situations where two or more decision makers or experts try to achieve a common solution to a decision problem, consisting of two or more possible solutions or alternatives (Kacprzyk 1986).

In real-world GDM problems, a range of situations including collaboration and competitiveness among individuals, compatible approaches or incompatible proposals might occur. Some guiding rules have been proposed to support decision making in such situations, for example the majority rule, minority rule and unanimity (Butler and Rothstein 2006). In democratic political systems, for instance, the majority rule is the most usual rule for dealing with GDM problems (Tocqueville 1840). However, in many real-world GDM problems that can affect groups or societies (civil rights, political or religious issues), the agreed solutions are highly appreciated. Therefore, the necessity of making decisions under consensus has become increasingly common in these contexts.

Consensus-reaching processes (CRPs) (Butler and Rothstein 2006; Saint and Lawson 1994) seek an experts' agreement about the problem before making the decision, thus yielding a more accepted solution by the whole group. CRPs are normally coordinated by a human figure, a so-called moderator, responsible for guiding experts throughout the overall discussion process. Different authors have proposed distinct approaches to handle CRPs, where Kacprzyk's *soft consensus* approach stands out (Kacprzyk 1986). In this approach, the concept of fuzzy linguistic majority is used to measure consensus between individuals in a flexible way. Later on, major achievements have been reached with the development of different consensus models, aimed to help decision makers to deal with CRPs. Some of these consensus models address aspects such as the use of different preference structures (Herrera-Viedma et al. 2002), management of incomplete preferences (Herrera-Viedma et al. 2007a, b), their extension to multi-criteria GDM problems (Parreiras et al. 2010; Pedrycz et al. 2011; Xu and Wu 2011) or even the introduction of adaptive consensus models based on the process performance (Mata et al. 2009).

However, some crucial aspects in CRPs still require a further study, for instance the idea of considering the group's attitude towards consensus, i.e. the experts' capacity to modify their own preferences during the CRP. Currently, consensus models found in the literature do not address the fact that if experts are reluctant to improve their attitude, the overall CRP might imply more time and cost. Additionally, it is important to consider the application of GDM problems with a large number of experts, because although real-world CRPs usually involve many experts, most developed models provide examples of performance with a small number of experts only (Herrera-Viedma et al. 2007a; Mata et al. 2009). This aspect may be addressed by designing consensus models with a high degree of automation, in which no human moderator is required to supervise experts' behaviour during the CRP.

In this paper, we develop the concept of group's attitude towards consensus and its application to CRPs, and present a consensus model that integrates it. Our goal consists in introducing such an attitude in the aggregation of information conducted during the CRP to measure the level of agreement in the group (Kuncheva and Krishnapuram 1995). To do so, we present the Attitude-OWA operator that extends the OWA operator (Yager 1988), so that it easily lets us reflect the group's attitude towards consensus. The model presented is applied to solve a real GDM problem where a large number of experts are involved, thus showing the importance and effects of integrating different attitudes.

This paper is organized as follows. In Sect. 2, some preliminaries related to consensus processes in GDM and OWA operators are reviewed. In Sect. 3, we develop in detail an approach to reflect the group's attitudes by means of the Attitude-OWA operator, and a consensus model based on such approach is defined and presented in Sect. 4. An application of the model to solve a real GDM problem by using different Attitude-OWA operators reflecting distinct attitudes is shown in Sect. 5. Finally, in Sect. 6, the main conclusions and some future works are drawn.

2 Preliminaries

In this section, we revise GDM problems and CRPs. We then briefly review OWA operators and linguistic quantifiers, which are the basis for our proposal.

2.1 Group decision making (GDM)

GDM problems are characterized by the participation of two or more experts in a decision problem, where a set of alternatives or possible solutions to the problem are presented (Butler and Rothstein 2006; Kacprzyk 1986). Formally, the main elements found in any GDM problem are:

- A set $X = \{x_1, \ldots, x_q\}, (q \geq 2)$ of possible *alternatives* to choose as possible solutions to the problem.
- A set $E = \{e_1, \ldots, e_m\}, (m \geq 2)$ of individuals or *experts*, who express their judgements or opinions on the alternatives in X.

Each expert $e_i, i \in \{1, \ldots, m\}$, provides his/her opinions over alternatives in X by means of a preference structure. One of the most usual preference structures, which also has been especially effective when dealing with uncertainty, is the so-called fuzzy preference relation.

Definition 1 (Bryson 1996; Herrera-Viedma et al. 2002) Given an expert $e_i \in E, i \in \{1, \ldots, m\}$ and two different alternatives $x_l, x_k \in X; l, k \in \{1, \ldots, q\} (l \neq k)$, a fuzzy preference relation's *assessment* on the pair (x_l, x_k), denoted as $p_i^{lk} \in [0, 1]$, represents the degree of preference of alternative x_l with respect to alternative x_k assessed by expert e_i, so that $p_i^{lk} > 1/2$ indicates that x_l is preferred to x_k, $p_i^{lk} < 1/2$ indicates that x_k is preferred to x_l, and $p_i^{lk} = 1/2$ indicates indifference between x_l and x_k.

Definition 2 (Herrera-Viedma et al. 2002) A *fuzzy preference relation* P_i associated with an expert $e_i, i \in \{1, \ldots, m\}$, on a set of alternatives X is a fuzzy set on $X \times X$, which is characterized by the membership function $\mu_{P_i} : X \times X \longrightarrow [0, 1]$. When the number of alternatives q is finite, P_i is represented by a $q \times q$ matrix of assessments $p_i^{lk} = \mu_{Pi}(x_l, x_k)$ as follows:

$$P_i = \begin{pmatrix} - & \cdots & p_i^{1q} \\ \vdots & \ddots & \vdots \\ p_i^{q1} & \cdots & - \end{pmatrix}$$

Notice here that assessments $p_i^{ll}, l \in \{1, \ldots, q\}$, situated in the diagonal of the matrix, are not defined, since an alternative x_l is not assessed with respect to itself.

In order to provide a better understanding of these definitions, a brief example is given below.

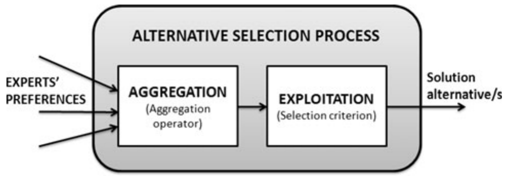


Fig. 1 Selection process in GDM problems

Example 1 Given $E = \{e_1, e_2, e_3\}$, $X = \{x_1, x_2, x_3, x_4\}$, let P_3 be the fuzzy preference relation on X expressed by e_3:

$$P_3 = \begin{pmatrix} - & 0.2 & 0.25 & 0 \\ 0.8 & - & 0.75 & 0.3 \\ 0.75 & 0.25 & - & 0 \\ 1 & 0.7 & 1 & - \end{pmatrix}$$

where we can see, for instance, that $p_3^{21} = 0.8$ indicates that x_2 is strongly preferred against x_1 by e_3, $p_3^{14} = 0$ indicates that x_1 is absolutely rejected with respect to x_4, and $p_3^{43} = 1$ indicates that x_4 is absolutely preferred against x_3.

The solution to a GDM problem may be obtained either by a direct approach, where the solution is immediately obtained from the experts' preferences, or by an indirect approach, where a social opinion is computed to determine the chosen alternative/s (Herrera et al. 1995). Regardless of the approach considered, it is necessary to apply a selection process to solve the GDM problem, which usually consists of two main phases (Fig. 1) (Roubens 1997): (1) an aggregation phase, where experts' preferences are combined and (2) an exploitation phase, which consists in obtaining an alternative or subset of alternatives as the solution to the problem.

2.2 Consensus-reaching processes (CRPs)

One of the main shortcomings found in classic GDM rules, such as the majority rule or minority rule, is the possible disagreement shown by one or more experts with the achieved solution, because they might consider that their opinions have not been taken into account sufficiently. Given the importance of obtaining an accepted solution by the whole group, CRPs as part of the decision process have attained great attention. *Consensus* can be understood as a state of mutual agreement among members of a group (Butler and Rothstein 2006; Saint and Lawson 1994), where the decision made satisfies all of them. Reaching a consensus usually requires that experts modify their initial opinions in a discussion process, making them closer to each other and towards a collective opinion which must be satisfactory for all of them.

The notion of consensus can be interpreted in different ways, ranging from consensus as total agreement to a more flexible approach (Herrera-Viedma et al. 2011). The strict notion of consensus assumes its existence only if all experts have achieved a mutual agreement in all their opinions (Tocqueville 1840). This may be quite difficult or even impossible to achieve in practice, and in the cases it could be achieved, the cost derived from the CRP would be unacceptable. Also, it might sometimes have been achieved through a normative point of view, through intimidation and other social strategies (Yager 2001). Subsequently, more flexible notions of consensus have been proposed to soften the strict view of consensus as unanimity (Elzinga et al. 2011; Herrera-Viedma et al. 2011; Kacprzyk and Fedrizzi 1988). These flexible approaches, more feasible in practice, consider different degrees of partial agreement to decide about the existence of consensus. Such degrees usually indicate how far a group of experts is from ideal consensus or unanimity.

One of the most widely accepted approaches for a flexible measurement of consensus is the so-called notion of *soft consensus*, proposed by Kacprzyk (1986). This approach introduces the concept of fuzzy linguistic majority, which establishes that there exists consensus if most experts participating in a problem agree with the most important alternatives. Soft consensus-based approaches have been used in different GDM problems, providing satisfactory results (Fedrizzi et al. 1999; Herrera et al. 1996; Kacprzyk and Zadrozny 2010; Zadrozny and Kacprzyk 2003). Consensus measures based on soft consensus are more human consistent and ideal for reflecting human perceptions of the meaning of consensus in practice (Kacprzyk and Fedrizzi 1989). The aforementioned concept of fuzzy linguistic majority has been captured by using linguistic quantifiers (Zadeh 1983).

The process to reach consensus in GDM problems is a dynamic and iterative discussion process (Saint and Lawson 1994), frequently coordinated by a human figure known as moderator, who plays a key role in CRPs (Martínez and Montero 2007). The moderator's main responsibilities are:

- Evaluate the degree of agreement achieved in each round of discussion, and decide whether it is enough to accept or not the existence of consensus.
- Identify those alternatives that hamper reaching a consensus.
- Give feedback to experts regarding changes they should make in their opinions on the previously identified alternatives, in order to increase the level of agreement in the next few rounds.

A general scheme of the phases required for conducting CRPs, depicted in Fig. 2, is briefly described below:

- *Gather preferences*: Each expert provides the moderator a preference structure with his/her opinion on the existing alternatives.

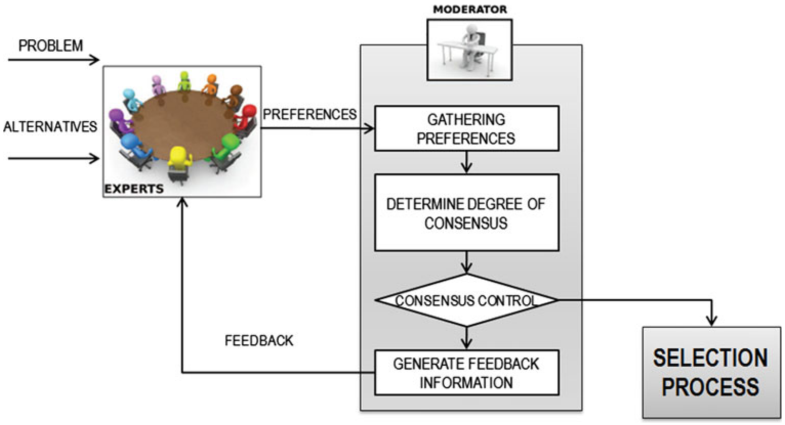


Fig. 2 General consensus process scheme in GDM problems

- *Determine degree of consensus*: The moderator computes the level of agreement in the group by means of a *consensus measure* (Herrera-Viedma et al. 2011), usually based on different *similarity measures* and *aggregation operators* (Beliakov et al. 2007).
- *Consensus control*: The consensus degree is compared with a threshold level of agreement desired by the group. If such degree is sufficient, the group moves on to the selection process; otherwise, more discussion rounds are required.
- *Generate feedback information*: The moderator identifies the farthest preferences from consensus and gives experts some pieces of advice, suggesting how to modify their opinions and make them closer to agreement. Afterwards, a new round of discussion begins with the gathering preferences phase.

2.3 OWA operators: weights computation

One of the most widely applied families of weighted aggregation operators (Beliakov et al. 2007) in different GDM approaches in the literature are the so-called ordered weighted averaging (OWA) operators, introduced by Yager:

Definition 3 (Yager 1988) Let $A = \{a_1, \ldots, a_n\}, a_i \in R$, be a set of n values to aggregate. An *OWA operator* is a mapping $F : R^n \to R$, with an associated weighting vector $W = [w_1 \ldots w_n]^\top$ $(w_i \in [0,1], \sum_i w_i = 1)$:

$$F(a_1, \ldots, a_n) = \sum_{j=1}^{n} w_j b_j \tag{2.1}$$

where b_j is the jth largest of a_i values.

Note that a weight w_i is associated with a particular ordered position instead of a particular element, i.e. w_i is associated with the ith largest element in $a_1, \ldots, a_n$. OWA operators are idempotent, continuous, monotone, neutral and compensative (Grabisch et al. 1998).

OWA operators are averaging aggregation functions, i.e. they lie between minimum and maximum functions, and therefore can be classified according to their optimism degree, by means of a measure, the so-called orness, associated with W. This measure provides the attitudinal character of aggregation, by determining how close the operator is to the maximum (OR) function, and is defined as (Beliakov et al. 2007):

$$\text{orness}(W) = \frac{1}{n-1} \sum_{i=1}^{n} (n-i) w_i \tag{2.2}$$

While optimistic or OR-LIKE OWA operators are those whose orness$(W) > 0.5$, in pessimistic or AND-LIKE operators we have orness$(W) < 0.5$ (Yager 1988, 1993).

Another measure, the *dispersion* (Shannon and Weaver 1949), can be used to let a further distinction amongst different OWA operators with an equal degree of optimism:

$$\text{Disp}(W) = -\sum_{i=1}^{n} w_i \ln w_i \tag{2.3}$$

This measure can be used as an indicator of the degree to which information contained in values $a_1, \ldots, a_n$ is really used in the aggregation process.

Several approaches have been proposed to compute OWA weights (Grabisch et al. 1998), for instance by using linguistic quantifiers (Yager 1996), as considered in this paper. Linguistic quantifiers were introduced by Zadeh (1983). They can be used to semantically express aggregation policies and actually capture Kacprzyk's notion of soft consensus in consensus models (Kacprzyk 1986; Kacprzyk and Fedrizzi 1989). This paper focuses on using a particular type of relative linguistic quantifiers, the so-called regular increasing monotone (RIM) quantifiers (Liu and Han 2008; Yager 1996), defined as a fuzzy subset Q of the unit interval (Klir and Yuan 1995; Yager and Filev 1994) where for a given proportion $r \in [0,1], Q(r)$ indicates the extent to which this proportion satisfies the semantics defined in Q. RIM quantifiers are characterized by the following properties: (1) $Q(0) = 0$, (2) $Q(1) = 1$ and (3) if $r_1 > r_2$ then $Q(r_1) \geq Q(r_2)$.

Yager (1988) proposed the following method to compute OWA weights with the use of RIM quantifiers:

$$w_i = Q\left(\frac{i}{n}\right) - Q\left(\frac{i-1}{n}\right), \quad i = 1, \ldots, n \tag{2.4}$$

where the linear membership function of a RIM quantifier $Q(r)$ is defined by the use of two parameters $\alpha, \beta \in [0,1]$ as

$$Q(r) = \begin{cases} 0 & \text{if } r \leq \alpha, \\ \frac{r-\alpha}{\beta-\alpha} & \text{if } \alpha < r \leq \beta, \\ 1 & \text{if } r > \beta. \end{cases} \tag{2.5}$$

OWA operators based on linguistic quantifiers have been widely applied in the literature, with multiple purposes (Reformat et al. 2011).

3 Integrating experts' attitude in consensus-reaching processes

The aim of this paper is to introduce and manage the concept of group's attitude towards consensus in CRPs, by means of a new aggregation operator based on the OWA operator that allows managing this concept in the measurement of consensus, and defines a consensus model upon it. In this section, we develop such a concept and show in detail how to implement attitude-based OWA operators. Furthermore, we will introduce in the coming sections a new attitude-based consensus model, as well as a complete study of its performance.

The concept of *attitude* refers to the importance that experts give to reach a consensus, compared to modifying their own preferences, and can be roughly classified into two types:

- *Optimistic attitude*: Achieving an agreement is more important than experts' own preferences; therefore, those positions in the group whose level of agreement is higher are given more importance in the aggregation process.
- *Pessimistic attitude*: Experts' own preferences are considered more important than achieving an agreement; therefore, those positions in the group where the level of agreement is lower attain more importance in aggregation.

The choice of an attitude depends on the prospects considered by experts in the group and the nature of the decision problem to be addressed.

Our proposal begins introducing the attitudinal parameters used by the group to reflect their attitude towards consensus, and then the Attitude-OWA operator is defined to capture such an attitude in the CRP. Attitude-OWA shall be applied to aggregate similarities between experts in the phase of computing consensus degree, as will be further detailed in Sect. 4.

3.1 Attitudinal parameters and Attitude-OWA operator

In Sect. 2.3, we reviewed RIM quantifiers and stated the membership function for a linear RIM quantifier upon two parameters α, β. Note that $[\alpha, \beta] \subseteq [0,1]$ ($\alpha < \beta$) defines the range of proportions r where the membership function $Q(r)$ increases, i.e. the slope of the RIM quantifier. Therefore, we have either $Q(r) = 0$ or $Q(r) = 1$ for any r situated to the left or to the right side of the slope, respectively. For a slope $[\alpha, \beta]$, its amplitude d is defined as $d = \beta - \alpha$.

When computing OWA weights from $Q(r)$ using Eq. (2.4), non-null weights w_i are assigned to elements b_i whose $r = i/n$ is situated inside the quantifier's slope, i.e. $r \in [\alpha, \beta]$. As we can see, d indicates the amount of values considered in the aggregation. In addition, orness(W) indicates how optimistic this aggregation is. These two elements let us define the *attitudinal parameters* used by the decision group to reflect an attitude towards consensus.

- $\vartheta = \text{orness}(W) \in [0,1]$ represents the group's attitude to be taken into account in the aggregation process (see Sect. 2.3). This attitude can be either optimistic if $\vartheta > 0.5$, pessimistic if $\vartheta < 0.5$ or neutral if $\vartheta = 0.5$.
- $\varphi = d \in [0,1]$ indicates the amount of values which are given non-null weight and therefore are considered in the aggregation. The higher the d, the wider is the range of ranked values given non-null weight and the higher is the dispersion in the corresponding Attitude-OWA operator.

We can now define an extension of OWA operators so-called Attitude-OWA for reflecting specific aggregation attitudes as follows:

Definition 4 An *Attitude-OWA operator* of dimension n on a set $A = \{a_1, \ldots, a_n\}$ of values to be aggregated, is an OWA operator based on two attitudinal parameters ϑ, φ given by a decision group to indicate their attitude towards consensus,

$$\text{Attitude-OWA}_W(A, \vartheta, \varphi) = \sum_{j=1}^{n} w_j b_j \tag{3.1}$$

where b_j is the jth largest of the a_i values, $\vartheta, \varphi \in [0, 1]$ are two input attitudinal parameters, and the set of weights, W, is computed by using a RIM quantifier, as shown in Eq. (2.4).

The attitude ϑ of an Attitude-OWA operator can be determined given the associated RIM quantifier Q, when the number of elements to aggregate n is sufficiently large, as follows:

Theorem 1 *Let ϑ be the attitude of an Attitude-OWA operator based on an RIM quantifier with a differentiable membership function $Q(r)$. Then for $n \to \infty, \vartheta \in [0, 1]$ is determined as follows*

$$\vartheta = \int_0^1 Q(r)\mathrm{d}r \tag{3.2}$$

The detailed analytical proof to obtain this expression is given as follows:

Proof Based on Eq. (2.2) and Eq. (2.4), we have

$$\vartheta(n) = \text{orness}(W)(n)$$
$$= \frac{1}{n-1}\sum_{i=1}^{n}(n-i)\left[Q\left(\frac{i}{n}\right) - Q\left(\frac{i-1}{n}\right)\right]$$

To calculate ϑ when n is sufficiently large, $n \to \infty$,

$$\vartheta = \lim_{n\to\infty} \vartheta(n)$$
$$= \lim_{n\to\infty} \frac{1}{n-1}\sum_{i=1}^{n}(n-i)\left[Q\left(\frac{i}{n}\right) - Q\left(\frac{i-1}{n}\right)\right]$$
$$= \lim_{n\to\infty} \frac{1}{n-1}\sum_{i=1}^{n-1}(n-i)\left[Q\left(\frac{i}{n}\right) - Q\left(\frac{i-1}{n}\right)\right]$$

If we consider $P = \sum_{i=1}^{n-1}(n-i)\left[Q\left(\frac{i}{n}\right) - Q\left(\frac{i-1}{n}\right)\right]$, then we have

$$P = \sum_{i=1}^{n-1}\left[n\left[Q\left(\frac{i}{n}\right) - Q\left(\frac{i-1}{n}\right)\right] - i\left[Q\left(\frac{i}{n}\right) - Q\left(\frac{i-1}{n}\right)\right]\right]$$
$$= \sum_{i=1}^{n-1} n\left[Q\left(\frac{i}{n}\right) - Q\left(\frac{i-1}{n}\right)\right] - \sum_{i=1}^{n-1} i\left[Q\left(\frac{i}{n}\right) - Q\left(\frac{i-1}{n}\right)\right]$$
$$= n\sum_{i=1}^{n-1}\left[Q\left(\frac{i}{n}\right) - Q\left(\frac{i-1}{n}\right)\right] - \sum_{i=1}^{n-1} i\left[Q\left(\frac{i}{n}\right) - Q\left(\frac{i-1}{n}\right)\right]$$

where, expanding it into the sum form, some terms are mutually deleted and finally we have

$$P = nQ\left(\frac{n-1}{n}\right) - \left[-\left[\sum_{i=1}^{n-2} Q\left(\frac{i}{n}\right)\right] + (n-1)Q\left(\frac{n-1}{n}\right)\right]$$
$$= nQ\left(\frac{n-1}{n}\right) - (n-1)Q\left(\frac{n-1}{n}\right) + \sum_{i=1}^{n-2} Q\left(\frac{i}{n}\right)$$
$$= Q\left(\frac{n-1}{n}\right) + \sum_{i=1}^{n-2} Q\left(\frac{i}{n}\right) = \sum_{i=1}^{n-1} Q\left(\frac{i}{n}\right)$$

Therefore,

$$\vartheta = \lim_{n\to\infty} \frac{1}{n-1}\sum_{i=1}^{n}(n-i)\left[Q\left(\frac{i}{n}\right) - Q\left(\frac{i-1}{n}\right)\right]$$
$$= \lim_{n\to\infty} \frac{1}{n-1}\sum_{i=1}^{n-1} Q\left(\frac{i}{n}\right)$$

When $n \to \infty$, it follows from the limit definition of definite integral that (Yager 1996)

$$\vartheta = \lim_{n\to\infty} \frac{1}{n-1}\sum_{i=1}^{n-1} Q\left(\frac{i}{n}\right) = \int_0^1 Q(r)\mathrm{d}r$$

where $r = i/n$. □

Since this statement is true for any function $Q(r)$ differentiable in [0,1], it can be easily extended to different types of RIM quantifiers, as shown below.

Corollary 1 *Given an RIM quantifier Q with a linear membership function $Q(r)$ as shown in Eq. (2.5), when the number of elements to aggregate n is sufficiently large, it is possible to compute the optimism degree ϑ of the Attitude-OWA operator based on Q as follows,*

$$\vartheta = 1 - \alpha - \frac{\varphi}{2} \tag{3.3}$$

Proof Based on the previous theorem and Eq. (2.5), we have

$$\vartheta = \int_0^1 Q(r)\mathrm{d}r = \int_\alpha^1 Q(r)\mathrm{d}r = \text{Area}(Q)$$
$$= \frac{1}{2}(\beta - \alpha) + [1 - \beta] = \frac{1}{2}\varphi + 1 - (\alpha + \varphi) = 1 - \alpha - \frac{\varphi}{2}$$

Notice that the interval [α,1] defines the *support* of the quantifier and $\beta - \alpha = \varphi$. The meaning of the integral states that ϑ is equal to the area under the membership function $Q(r)$ (Liu and Han 2008; Yager 1996), as shown in Fig. 3. □

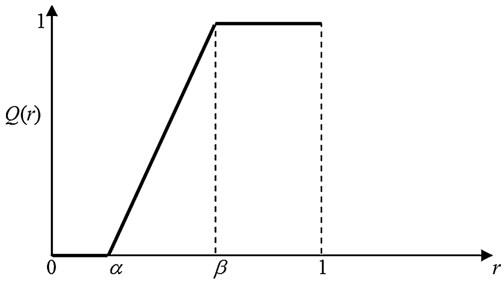


Fig. 3 Membership function in RIM quantifiers considered

Therefore, when using linear RIM quantifiers, ϑ may closely approximate to the result of Eq. (3.3) when measuring consensus in large groups where a high number of agreement values must be aggregated to measure consensus, i.e. when $n \rightarrow \infty$. As a result, since we are interested in integrating a group's attitude towards consensus by means of ϑ and φ, we use Eq. (3.3) to determine the value of α, necessary to define the RIM quantifier and compute Attitude-OWA weights, as follows:

$$\alpha = 1 - \vartheta - \frac{\varphi}{2} \tag{3.4}$$

3.2 Relations and restrictions between attitudinal parameters

Attitudinal parameters' values are related to each other, so it is convenient to clarify some existing relations and restrictions between them. As stated earlier, α and φ are used to univocally define a linear RIM quantifier Q, but the following condition must be fulfilled to define a valid RIM quantifier and therefore integrate a valid attitude in the process:

Theorem 2 *Given* $\alpha, \varphi \in [0, 1]$, *a valid attitude given by* ϑ *can be guaranteed only if* $\alpha + \varphi \leq 1$.

Proof Let us suppose $\alpha + \varphi > 1$. Considering that $\varphi = \beta - \alpha$, Eq. (3.3) leads to

$$\vartheta = 1 - \alpha - \frac{\varphi}{2} = 1 - \frac{\alpha + \beta}{2} \tag{3.5}$$

where $\frac{\alpha+\beta}{2}$ is the central value of the quantifier's slope, so that

$$\alpha \leq \frac{\alpha + \beta}{2} \leq \beta$$
$$1 - \alpha \geq 1 - \frac{\alpha + \beta}{2} \geq 1 - \beta$$
$$1 - \alpha \geq \vartheta \geq 1 - (\alpha + \varphi)$$

where $\beta = \alpha + \varphi$. Notice here that if $\alpha + \varphi > 1$ as we supposed, then ϑ can be negative; therefore, $\alpha + \varphi$ must be equal or less than one to ensure a valid attitude is defined. □

In order to avoid expressing invalid attitudinal parameters, we present the restrictions to be considered by the decision group when providing them.

Corollary 2 *The following condition must be fulfilled when the group provides a value of* ϑ:

$$\frac{\varphi}{2} \leq \vartheta \leq 1 - \frac{\varphi}{2} \tag{3.6}$$

Proof According to Eq. (3.4), α is negative if $(\vartheta + \varphi/2) > 1$. We need $\alpha \geq 0$, i.e.

$$1 - \vartheta - \frac{\varphi}{2} \geq 0$$
$$\vartheta + \frac{\varphi}{2} \leq 1$$
$$\vartheta \leq 1 - \frac{\varphi}{2}$$

However, according to Theorem 2, it is also necessary to guarantee $\alpha + \varphi \leq 1$. Based on Eq. (3.4) we have,

$$\alpha + \varphi = 1 - \vartheta - \frac{\varphi}{2} + \varphi \leq 1$$
$$1 - \vartheta + \frac{\varphi}{2} \leq 1$$
$$\vartheta \geq \frac{\varphi}{2}$$

The fulfillment of both inequalities leads to the aforementioned restriction. □

As a result, the higher the proportion of values to consider in aggregation (given by φ), the narrower range of possible attitudes or optimism degrees (given by ϑ) can be considered.

Corollary 3 *The following condition must be fulfilled when the group provides a value of* φ:

$$\varphi \leq 1 - |2\vartheta - 1| \tag{3.7}$$

Proof Based on the previous proof in Corollary 1, $\alpha \geq 0$ requires

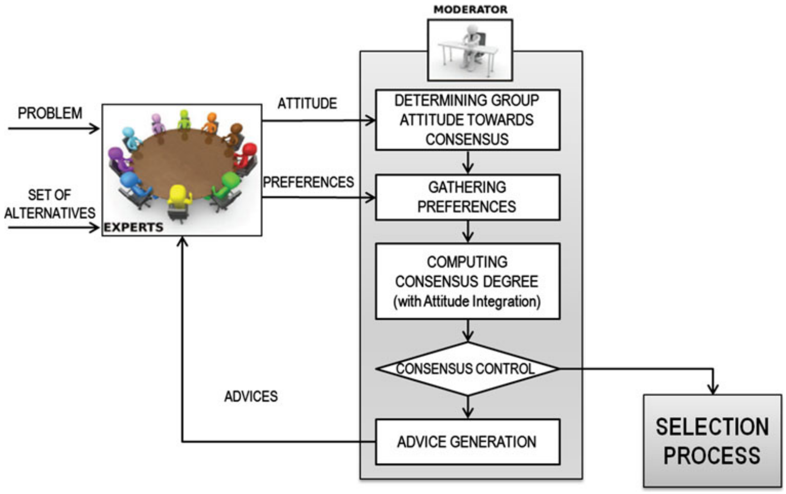
$$\vartheta + \frac{\varphi}{2} \leq 1 \quad \text{i.e.,}$$
$$\varphi \leq 2(1 - \vartheta)$$

which is valid for $\vartheta \in [0.5, 1]$, but may give rise to $\varphi > 1$ and fail to fulfill Theorem 2 when $\vartheta < 0.5$. Let us consider Theorem 2 and Eq. (3.4). We then have

$$\alpha + \varphi = 1 - \vartheta - \frac{\varphi}{2} + \varphi \leq 1$$
$$1 - \vartheta + \frac{\varphi}{2} \leq 1$$
$$\varphi \leq 2\vartheta$$

which is valid for $\vartheta \in [0, 0.5]$, but $\varphi > 1$ may still be possible when $\vartheta > 0.5$; hence, a valid quantifier can be defined only if these restrictions are satisfied,

Fig. 4 Attitude-based consensus model scheme



$\varphi \leq 2\vartheta \quad$ if $\vartheta \in [0, 0.5]$
$\varphi \leq 2(1-\vartheta) \quad$ if $\vartheta \in [0.5, 1]$

We finally proceed to find a single expression which considers both restrictions. On the one hand, we have

$2\vartheta = 1 - (-2\vartheta + 1)$

where, when $\vartheta \in [0, 0.5]$, the term $(-2\vartheta + 1) \geq 0$. On the other hand,

$2(1-\vartheta) = 1 - (2\vartheta - 1)$

where, when $\vartheta \in [0.5, 1]$, the term $(2\vartheta - 1) \geq 0$. This means we can consider the absolute value of the term $(2\vartheta - 1)$ to integrate both restrictions as

$\varphi \leq 1 - |2\vartheta - 1|$

This restriction can be interpreted as the fact that the closer ϑ is to a neutral attitude (0.5), the wider the range of possible degrees for φ that can be considered. □

If restrictions pointed out in Eqs. (3.6) and (3.7) are taken into account when expressing any two values for input attitudinal parameters (ϑ, φ), then a valid RIM quantifier is always defined, thus guaranteeing a valid Attitude-OWA operator.

4 Attitude-based consensus model

Once presented the concept of attitude towards consensus and the main features of the Attitude-OWA operator used to reflect it, in this section we present the consensus model designed to integrate such an attitude in CRPs. The model extends the main ideas of some models presented in (Herrera-Viedma et al. 2002; Mata et al. 2009) and its design allows to automate all the human moderator tasks, thus removing his/her inherent subjective biasness towards experts and facilitating the resolution of GDM problems with large groups of experts computationally.

Figure 4 shows the five phases conducted in the model, which are described in the following subsections:

4.1 Determining group attitude towards consensus

This phase is carried out at the beginning of the CRP, as part of a *pre-consensus process* (Saint and Lawson 1994). The moderator is responsible for reflecting the group's attitude towards consensus, by assigning a value to attitudinal parameters ϑ and φ, considering both the context and characteristics of the decision problem to solve, and the experts' individual concerns. Figure 5 shows the procedure to determine a group's attitude towards the achievement of consensus and integrate it in the CRP, defining the corresponding Attitude-OWA operator used in a later phase to measure consensus.

4.2 Gathering preferences

Each expert e_i provides his/her preference on alternatives in X to the moderator, by means of a fuzzy preference relation P_i, consisting of a $q \times q$ matrix of assessments p_i^{lk} on each pair of alternatives $(x_l, x_k), l, k \in \{1, \ldots, q\}$. It is advisable that experts provide consistent opinions that could be easier to achieve if they provide reciprocal assessments, i.e. if $p_i^{lk} = x, x \in [0, 1], l \neq k$, then $p_i^{kl} = 1 - x$.

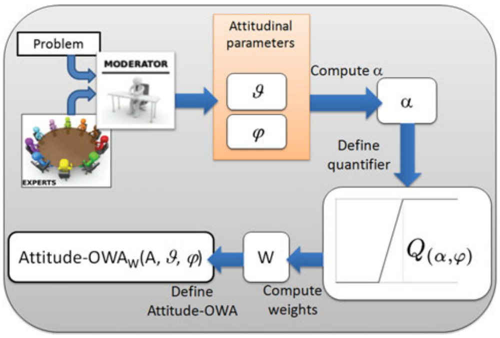


Fig. 5 Process to determine the Attitude-OWA operator used to measure consensus based on the group's attitudinal parameters ϑ and φ

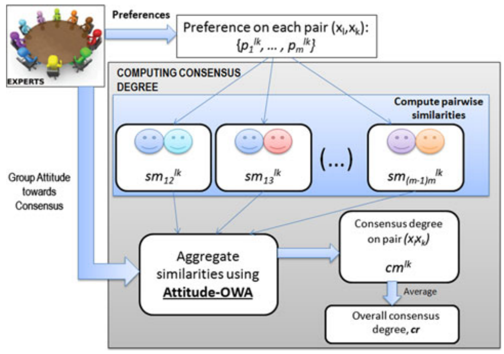


Fig. 6 Procedure to compute consensus degree based on the group's attitude

4.3 Computing consensus degree

The moderator computes the level of agreement between experts, by means of the following steps (see Fig. 6):

1. For each pair of experts e_i, e_j, $(i \neq j)$, a similarity matrix SM, defined by

$$SM_{ij} = \begin{pmatrix} - & \dots & sm_{ij}^{1q} \\ \vdots & \ddots & \vdots \\ sm_{ij}^{q1} & \dots & - \end{pmatrix},$$

is computed as follows (Herrera-Viedma et al. 2005):

$$sm_{ij}^{lk} = 1 - |(p_i^{lk} - p_j^{lk})| \tag{4.8}$$

where $sm_{ij}^{lk} \in [0,1]$ is the similarity degree between experts e_i and e_j in their assessments p_i^{lk}, p_j^{lk}.

2. A consensus matrix CM of dimension $q \times q$, defined by

$$CM = \begin{pmatrix} -\dots & cm^{1q} & \\ \vdots & \ddots & \vdots \\ cm^{q1} & \dots & - \end{pmatrix},$$

is computed, taking into account the group's attitude by aggregation of similarity matrices. Each element cm^{lk}, $l \neq k$, is computed as:

$$cm^{lk} = \text{Attitude-OWA}_W(SIM^{lk}, \vartheta, \varphi) \tag{4.9}$$

where $SIM^{lk} = \{sm_{12}^{lk}, \dots, sm_{1m}^{lk}, sm_{23}^{lk}, \dots, sm_{2m}^{lk}, \dots, sm_{(m-1)m}^{lk}\}$ is the set of all pairs of experts' similarities in their opinion on (x_l, x_k). Attitude-OWA operator is used here to integrate the group's attitude towards consensus, previously gathered by means of ϑ and φ.

3. Consensus degree is computed at three different levels:

   (a) Level of pairs of alternatives (cp^{lk}): obtained from CM as $cp^{lk} = cm^{lk}$, $l, k \in \{1, \dots, q\}$, $l \neq k$.

   (b) Level of alternatives (ca^l): the level of agreement on each alternative $x_l \in X$ is computed as:

$$ca^l = \frac{\sum_{k=1, k \neq l}^{q} cp^{lk}}{q-1} \tag{4.10}$$

   (c) Level of preference relation (overall consensus degree, cr): it is computed as:

$$cr = \frac{\sum_{l=1}^{q} ca^l}{q} \tag{4.11}$$

4.4 Consensus control

The overall consensus degree cr is compared with a consensus threshold $\mu \in [0,1]$ established a priori. If $cr \geq \mu$, then the CRP ends and the group moves on to the selection process; otherwise, more discussion rounds are required. A parameter *Maxrounds* can be used to limit the number of discussion rounds conducted in the cases that consensus cannot be achieved.

4.5 Advice generation

If $cr < \mu$, the moderator advises experts to modify their preferences in order to increase the level of agreement in the following rounds. Three steps are considered in this phase:

1. Compute a collective preference and proximity matrices for experts: A collective preference P_c is computed

for each pair of alternatives by aggregating experts' preference relations:

$$p_c^{lk} = \phi(p_1^{lk}, \ldots, p_m^{lk}) \quad (4.12)$$

where ϕ is the aggregation operator considered. Afterwards, a proximity matrix $PP_i = (pp_i^{lk})$ between each expert's preference relation and P_c is obtained:

$$PP_i = \begin{pmatrix} - & \ldots & pp_i^{1q} \\ \vdots & \ddots & \vdots \\ pp_i^{q1} & \ldots & - \end{pmatrix}$$

Proximity values pp_i^{lk} are obtained for each pair (x_l, x_k) as follows:

$$pp_i^{lk} = 1 - |(p_i^{lk} - p_c^{lk})| \quad (4.13)$$

Proximity values are used to identify the farthest preferences from the collective opinion, which should be modified by some experts.

2. Identify preferences to be changed (CC): pairs of alternatives (x_l, x_k), whose consensus degrees ca^l and cp^{lk} are not sufficient, are identified:

$$CC = \{(x_l, x_k) | ca^l < cr \wedge cp^{lk} < cr\} \quad (4.14)$$

Afterwards, the model identifies experts who should change their opinion on each of these pairs, i.e. those experts e_i whose preference p_i^{lk} on the pair $(x_l, x_k) \in CC$ is farthest to p_c^{lk}. An average proximity $\overline{pp}^{lk}$ is calculated to identify them, as follows:

$$\overline{pp}^{lk} = \phi(pp_1^{lk}, \ldots, pp_m^{lk}) \quad (4.15)$$

As a result, experts e_i whose $pp_i^{lk} < \overline{pp}^{lk}$ are advised to modify their assessment on the pair (x_l, x_k).

3. Establish change directions: several direction rules are applied to suggest the direction of changes proposed to experts, in order to increase the level of agreement in the following rounds (Mata et al, 2009).

- DIR.1: If $(p_i^{lk} - p_c^{lk}) < 0$, then expert e_i should *increase* his/her assessment on the pair of alternatives (x_l, x_k).
- DIR.2: If $(p_i^{lk} - p_c^{lk}) > 0$, then expert e_i should *decrease* his/her assessment on the pair of alternatives (x_l, x_k).
- DIR.3: If $(p_i^{lk} - p_c^{lk}) = 0$, then expert e_i should not modify his/her assessment on the pair of alternatives (x_l, x_k).

5 Experimental simulation

In this section, we use a multi-agent based consensus support system to simulate the resolution of a real GDM problem defined under uncertainty, with different instances of Attitude-OWA operator based on different group attitudes towards consensus, having a considerable number of experts in the group. Our main hypothesis focuses mainly on the effect of using different attitudes towards consensus in the process, and states that optimism, given by OR-LIKE operators, may favour a greater convergence towards consensus with a lower number of rounds; whereas pessimism, given by AND-LIKE operators, may favour a lower convergence towards consensus and, therefore, more rounds of discussion are required.

Table 1 Attitudinal parameters and RIM quantifiers used

Attitude	ϑ	φ	α	$Q_{(\alpha,\varphi)}$
Highly pessimistic	0.1	0.1	0.85	$Q_{(0.85,0.1)}$
	0.1	0.2	0.8	$Q_{(0.8,0.2)}$
Pessimistic	0.3	0.2	0.6	$Q_{(0.6,0.2)}$
	0.3	0.6	0.4	$Q_{(0.4,0.6)}$
Indifferent	0.5	0.6	0.2	$Q_{(0.2,0.6)}$
	0.5	1	0	$Q_{(0,1)}$
Optimistic	0.7	0.2	0.2	$Q_{(0.2,0.2)}$
	0.7	0.6	0	$Q_{(0,0.6)}$
Highly Optimistic	0.8	0.1	0.15	$Q_{(0.15,0.1)}$
	0.8	0.2	0.1	$Q_{(0.1,0.2)}$

The presented attitude-based consensus model has been applied to simulate a real-life problem, whose formulation is as follows: let us suppose that a conference scientific committee compound by 50 scientists, $E = \{e_1, \ldots, e_{50}\}$, must grant a best Ph.D. student paper award to one out of four possible candidate papers, $X = \{x_1 = \text{John's paper}, x_2 = \text{Wang's paper}, x_3 = \text{Sue's paper}, x_4 = \text{Michael's paper}\}$. The committee must achieve a minimum level of agreement of $\mu = 0.85$ before making a decision.

The experiments consisted in defining a total of five different attitudes towards consensus, ϑ, where both optimistic, indifferent and pessimistic attitudes are reflected, and applying a CRP based on the model presented in Sect. 4. For each attitude, two different degrees of the amount of information used, given by φ, have been considered (taking into account the restrictions pointed out in Sect. 3.2). Table 1 shows the different group attitudes used in simulations, the obtained value of α [as stated in Eq. (3.4)] and the subsequent definition of ten different RIM quantifiers (denoted as $Q_{(\alpha,\varphi)}$) used in experiments. For each instance of Attitude-OWA, 20 experiments have been run.

Results from experiments include the convergence to consensus achieved, i.e. the average number of rounds of discussion required to reach a consensus for each Attitude-OWA operator defined upon an RIM quantifier. These results, which are shown in Fig. 7, allow us to confirm our hypothesis that the use of Attitude-OWA operator based on

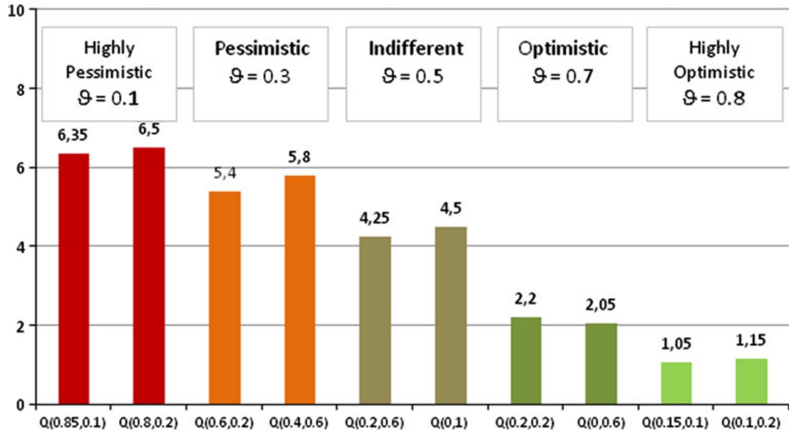


Fig. 7 The average number of required rounds of discussion for RIM quantifier-based Attitude-OWA operators with different attitudinal parameters given by ϑ and φ

an optimistic attitude favours a greater convergence towards consensus, whereas the use of Attitude-OWA operator based on a pessimistic attitude favours a lower convergence and a further discussion process, regardless of the proportion of values considered, φ.

It can be concluded that the main advantage of integrating the group's attitude in the CRP is the fact that it lets us adapt and optimize such a process, according to the specific needs of decision makers for each GDM problem to be addressed. For instance, if decision makers' priority is achieving a consensus in a fast discussion process and they do not care about considering the highest agreement positions, they would adopt an optimistic attitude. On the other hand, if they consider that the problem requires further discussion and they want to ensure that even the most discrepant experts finally reach an agreement, they would rather consider a pessimistic attitude.

6 Conclusions and future works

In this paper, we have studied the concept of group's attitude towards consensus by means of an extension of OWA operators, the so-called Attitude-OWA, and presented a consensus model which allows to integrate it in the consensus-reaching process. The attitudinal parameters involved in the defined operator have been thoroughly studied. In addition, the performance of the proposed approach has been analysed through a simulation to solve a real group decision making problem with many experts in an automatic consensus support system. Having shown the effect of using optimistic/pessimistic attitudes in the number of discussion rounds necessary to achieve an agreement (the more optimistic the attitude, the higher is the convergence towards consensus, and vice versa), we conclude that the integration of the group's attitude provides the advantage that the consensus-reaching process can be easily adapted and optimized according to the group's needs, by choosing the appropriate values for attitudinal parameters.

Our future works are currently focused on a further analysis of the proposed attitudinal parameters, as well as introduction of the possibility that experts can express their desired attitudes in a linguistic background, thus giving them an even more natural way to provide attitudinal information. We also aim to extend Attitude-OWA operator to apply it to consensus processes where different types of quantifiers with diverse membership functions can be used, and extend the consensus model to make it adaptive, under the assumption that the group's attitude might change during the discussion process

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4.6. An attitude-driven web consensus support system for heterogeneous group decision making

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An attitude-driven web consensus support system for heterogeneous group decision making

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A B S T R A C T

Consensus reaching processes are applied in group decision making problems to reach a mutual agreement among a group of decision makers before making a common decision. Different consensus models have been developed to facilitate consensus reaching processes. However, new trends bring diverse challenges in group decision making, such as the modelling of different types of information and of large groups of decision makers, together with their attitude to achieve agreements. These challenges require the capacity to deal with heterogenous frameworks, and the automation of consensus reaching processes by means of consensus support systems. In this paper, we propose a consensus model in which decision makers can express their opinions by using different types of information, capable of dealing with large groups of decision makers. The model incorporates the management of the group's attitude towards consensus by means of an extension of OWA aggregation operators aimed to optimize the overall consensus process. Eventually, a novel Web-based consensus support system that automates the proposed consensus model is presented.

1. Introduction

Decision making processes are one of the most frequent mankind activities in daily life. In group decision making (GDM) problems, a group of decision makers try to achieve a common solution to a problem consisting of two or more possible solutions or alternatives (Kacprzyk, 1986). Classically, GDM-based approaches are aimed to make decisions where few decision makers participate. However, nowadays technologies and societal models could imply the participation of large groups of decision makers in GDM problems.

A key aspect in GDM problems is to achieve a solution which is accepted by all decision makers in the group. Usually, GDM problems have been solved applying classic approaches, such as the majority rule, minority rule or total agreement (Butler & Rothstein, 2006; Kacprzyk, 1986; Martínez & Montero, 2007). However, these approaches do not guarantee achieving a solution accepted by all decision makers. Therefore, Consensus Reaching Processes (CRPs) are becoming increasingly necessary (Saint & Lawson, 1994) as part of GDM problems resolution. A number of theoretical consensus models have been proposed in the literature to conduct CRPs (Herrera-Viedma, Martínez, Mata, & Chiclana, 2005; Kacprzyk, Fedrizzi, & Nurmi, 1992; Parreiras, Ekel, Martini, & Palhares, 2010; Pedrycz, Ekel, & Parreiras, 2011; Saint & Lawson, 1994).

Some aspects in recent research for consensus have attained much attention, such as: (i) the need for ubiquitous CRPs, so that they can be conducted anywhere and anytime without physical meetings, which could be achieved by developing Consensus Support Systems (CSSs) that automate the CRP to a high extent; and (ii) the necessity of improving the static behavior present in most consensus models, irrespective of the changing complexity found in each particular problem, which may be addressed by developing adaptive consensus models (Mata, Martínez, & Herrera-Viedma, 2009).

Most classical consensus models and recent ones assumed that the group of decision makers were formed by a low number of decision makers. However, nowadays new trends like social networks (Sueur, Deneubourg, & Petit, 2012; Yager, 2008) and e-democracy (Kim, 2008), imply larger groups of decision makers in GDM problems, thus bringing new challenges to this research area:

(i) Dealing with heterogeneous information: A large number of decision makers implies many different profiles. Therefore, each decision maker may express his/her preferences in different information domains, depending on the level of knowledge, experience or the nature of alternatives. In such a case the GDM problem is defined in a heterogeneous framework, and an approach to deal with heterogeneous

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information is required (Herrera, Martínez, & Sánchez, 2005; Li, Huang, & Chen, 2010; Zhang & Lu, 2003).

(ii) Group's attitude towards consensus: The *attitude* of decision makers towards consensus is the capacity to modify preferences that they present during the CRP, which may affect to its performance significantly (Palomares, Liu, Xu, & Martínez, in press).

(iii) Automation: The management of large groups increases the complexity of physical meetings, therefore the development of a consensus model that allows a certain degree of automation on CRPs by implementing a CSS upon it becomes compulsory (Herrera, Herrera-Viedma, & Verdegay, 1996; Herrera-Viedma et al., 2005; Mata et al., 2009).

In this paper, we propose a novel large-scale oriented consensus model for GDM problems defined in heterogeneous contexts that is able to integrate decision makers' attitude regarding consensus. Once defined the model, and due to the necessity of automation, a Web-based CSS that integrates such a consensus model is presented.

It is remarkable that the proposed consensus model is able to integrate the group's attitude towards consensus in the measurement of the level of agreement, by means of an extension of OWA operators (Yager, 1988), so-called Attitude-OWA.

This paper is organized as follows: Section 2 reviews some preliminaries related to GDM problems in heterogeneous contexts, consensus processes and CSSs. Section 3 introduces the consensus model that integrates the group's attitude towards consensus and provides an approach to deal with heterogeneous information. Section 4 presents the Web-based CSS that uses the previous model and an illustrative example of its performance. Finally, some concluding remarks are pointed out in Section 5.

2. Preliminaries

This section reviews the formalization and management of GDM problems defined in heterogeneous contexts, and revises basic concepts about CRPs to understand the proposed consensus model. Because of the need for automating the proposed model, different CSSs are also revised.

2.1. GDM Problems with heterogeneous information

A GDM problem can be defined as a decision situation where a group of *decision makers* or *experts*, $E = \{e_1, \ldots, e_m\}$ ($m \geqslant 2$), express their preferences over a set of feasible *alternatives*, $X = \{x_1, \ldots, x_n\}$ ($n \geqslant 2$) (Kacprzyk, 1986). Each decision maker, e_i, provides his/her opinions on X by means of a preference relation P_i, $\mu_{P_i} : X \times X \to D$,

$$P_i = \begin{pmatrix} - & \ldots & p_i^{1n} \\ \vdots & \ddots & \vdots \\ p_i^{n1} & \ldots & - \end{pmatrix}$$

where each *assessment*, $p_i^{lk} = \mu_{P_i}(x_l, x_k)$, represents the preference degree of alternative x_l over x_k according to decision maker e_i, expressed in an information domain D, $i \in \{1, \ldots, m\}$ and $l,k \in \{1, \ldots, n\}$, $l \neq k$. In complex GDM problems usually defined with a high degree of uncertainty, decision makers might have different backgrounds and different levels of knowledge about a specific problem. Therefore, they could prefer to provide their preferences by using different domains according to their own characteristics. In such a case, the GDM problem is defined in an heterogeneous context. In this paper, we focus on this type of problems, so-called *heterogeneous GDM problems*, in which each decision maker e_i, may express his/her opinions on X by using different information domains $D_i \in \{numerical, interval-valued, linguistic\}$ (Herrera et al., 2005; Li et al., 2010; Zhang & Lu, 2003). Therefore, preferences could be assessed as:

- *Numerical*: Assessments p_i^{lk} are represented as values in $[0,1]$.
- *Interval-valued*: Assessments p_i^{lk} are represented as intervals, $I([0,1])$.
- *Linguistic*: Assessments p_i^{lk} are represented as linguistic labels $s_j \in S$, where $S = \{s_0, \ldots, s_g\}$ is a set of labels.

2.2. Consensus reaching processes (CRPs)

GDM problems have been usually solved by performing a *selection process* where the best alternative or subset of alternatives is obtained from decision makers' preferences (Roubens, 1997), which does not always guarantee that the decision would be accepted by all decision makers in the group, since some of them might consider that their opinions have not been sufficiently considered. In order to overcome this drawback and attempt to achieve a solution to the GDM problem which is accepted by the whole group, CRPs have attained a great attention as part of the decision process. *Consensus* can be understood as a state of mutual agreement among members of a group, where the decision made satisfies all of them (Butler & Rothstein, 2006; Saint & Lawson, 1994). Reaching a consensus usually requires that decision makers modify their initial opinions, making them closer to each other and towards a collective opinion which must be satisfactory for all of them. Furthermore, in many real CRPs decision makers might present different attitudes towards consensus, regarding the capacity they present to modify their own preferences to achieve an agreement, as will be further studied in this paper.

The notion of consensus has been interpreted in different ways, ranging from consensus as a total agreement to more flexible approaches (Kacprzyk & Fedrizzi, 1988; Kacprzyk et al., 1992). Consensus as a total agreement, where all decision makers are aimed to achieve a mutual agreement in all their opinions, may be quite difficult to achieve in practice, and in those cases that it could be achieved, the cost derived from the CRP is usually unacceptable. Subsequently, more flexible notions of consensus have been proposed to soften the strict view of consensus as a total agreement, considering different degrees of partial agreement among decision makers to decide about the existence of consensus. One of the most widely accepted approaches for a flexible measurement of consensus is the so-called notion of *soft consensus*, proposed in Kacprzyk (1986). This approach applies the concept of fuzzy linguistic majority, which establishes that consensus exists if *most decision makers participating in a problem agree with the most important alternatives*. *Soft consensus*-based approaches have been used in different GDM problems providing satisfactory results (Herrera et al., 1996; Kacprzyk & Zadrozny, 2010; Zadrozny & Kacprzyk, 2003).

CRPs are iterative and dynamic processes consisting of several rounds of discussion. These processes are frequently coordinated by a human figure known as *moderator*, who is responsible for supervising and guiding decision makers in the overall process, as well as giving them advice to modify their opinions (Martínez & Montero, 2007). A general scheme to conduct CRPs is depicted in Fig. 1 and briefly described below:

1. *Gathering preferences*: Each decision maker provides his/her preferences.
2. *Computing the level of agreement*: The moderator obtains the level of agreement in the group by means of *consensus measures* (Kacprzyk & Fedrizzi, 1988; Kuncheva & Krishnapuram, 1995), *similarity measures*, and *aggregation operators* (Beliakov, Pradera, & Calvo, 2007).

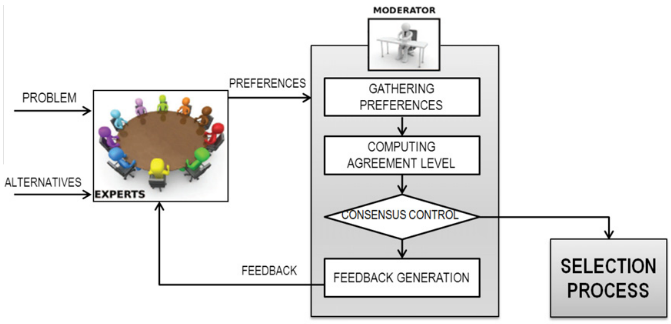

Fig. 1. General CRP scheme.

3. *Consensus control*: If the level of agreement is high enough, the group moves onto the alternatives selection process, otherwise more discussion rounds are required.
4. *Feedback generation*: The moderator identifies furthest preferences from consensus and gives decision makers some feedback, suggesting them how to modify their preferences and make them closer.

In order to deal with CRPs, a high number of theoretical consensus models have been proposed in the literature by different authors (Herrera-Viedma et al., 2005; Kacprzyk et al., 1992; Mata et al., 2009; Parreiras et al., 2010; Pedrycz et al., 2011; Saint & Lawson, 1994). Nevertheless, most consensus models do not consider the importance of replacing the human moderator (due to the subjectivity and lack of impartiality that he/she may sometimes present towards decision makers), which could make the CRP automatic to some extent. Since it would be interesting to implement the tasks defined in these consensus models into a CSS that achieves such an automation degree in practice by means of intelligent techniques, as well as eliminating the need for physical meetings and making ubiquitous CRPs possible, different CSSs have been proposed in the literature in the last few years.

2.3. Related work on CSSs

Due to the fact that in our proposal for a consensus model we consider GDM problems with a large number of decision makers who might express their preferences in different domains, it is convenient to automate such model by means of a CSS. Here, we review some CSSs presented in the literature to support decision makers in GDM problems. These CSSs designs are based on consensus models whose tasks are easily automated, therefore both the human moderator and the need for physical meetings disappear. Besides, they may facilitate dealing with large groups of decision makers, depending on the specific consensus model considered. Notice that some of the systems revised are denoted as CSS models, i.e. a proposal of a CSS scheme which may have not been implemented and put in practice yet.

Some CSSs have been developed based on the notion of *soft consensus* and fuzzy majority (Kacprzyk, 1986; Kacprzyk & Fedrizzi, 1989), such as the system presented in Zadrozny and Kacprzyk (2003), which is one of the earliest Web-based CSSs providing decision makers with a web user interface to let them insert and modify their preferences; and the one proposed in Kacprzyk and Zadrozny (2010), that applies additional techniques to manage knowledge, such as the use of ontologies.

The CSS model presented in Herrera-Viedma et al. (2005) is based on a consensus model that incorporates the use of multi-granular linguistic preference relations. Considering that decision makers with different backgrounds and level of knowledge about each problem might be users of the system, they provide their preferences by means of linguistic term sets with different granularity. In addition, the system is able to generate pieces of advice for decision makers, suggesting them how to change their preferences.

In Mata et al. (2009), a CSS model based on an adaptive consensus model was presented. Such a model adapts its behavior throughout the overall discussion process by applying different procedures to identify decision makers preferences that should be changed according to the consensus degree achieved in each round. This way, the model attempts to minimize the number of discussion rounds required to achieve a consensus, compared to other non-adaptive models.

Recently, several CSSs operating in web and mobile environments have been presented. In Alonso, Herrera-Viedma, Chiclana, and Herrera (2010), a Web-based CSS to deal with incomplete preference relations was presented. The system provides a web user interface to the decision makers involved in the GDM problem.

3. Attitude-based consensus model for heterogeneous GDM problems

In this section, we propose a new large-scale oriented consensus model for GDM problems defined in heterogeneous contexts, that deals with large groups of decision makers and is able to integrate their attitude towards consensus. The model is able to deal with heterogeneous frameworks and allows decision makers to express their opinions by using different information domains. Besides, the group's attitude provides a new vision to the CRP, since the discussion process is adapted to achieve the level of consensus required according to the decision makers' attitude.

Before developing in further detail the consensus model, we are going to present the management of heterogeneous information that our model will use, as well as the way of integrating the attitude of decision makers in the CRP.

3.1. Dealing with heterogeneous information

As previously pointed out, our interest is focused on dealing with GDM problems defined in heterogeneous frameworks in which the information provided by decision makers can be numerical, interval-valued or linguistic.

- *Numerical domain*: $p_i^{lk} = v,\ v \in [0,1]$.
- *Interval-valued domain*: $p_i^{lk} = I([0,1]) = [d,f], (d,f \in [0,1] \wedge d \leqslant f)$.
- *Linguistic domain*: Linguistic variables (Zadeh, 1975) are assessed by linguistic terms, $p_i^{lk} = s_j \in S$, where semantics is

defined by a fuzzy membership function, denoted as $\mu_{s_j}(y),\ y \in [0,1]$.

In order to deal with such heterogeneous frameworks, different solutions have been proposed (Herrera et al., 2005; Li et al., 2010; Zhang & Lu, 2003). Here, we consider the method proposed in Herrera et al. (2005) to unify information expressed in different domains, p_i^{lk} (either numerical, interval-valued or linguistic), into fuzzy sets $F(S_T)$, in a common linguistic term set $S_T = \{s_0,\ldots,s_g\}$:

$$\tau_{DS_T} : D \to F(S_T)$$
$$\tau_{DS_T}(p_i^{lk}) = \sum_{j=0}^{g} s_j/\gamma_{ij}^{lk} \tag{1}$$

where $g + 1$ is the granularity of S_T, γ_{ij}^{lk} is the membership degree of p_i^{lk} to s_j and at least $\exists\gamma_{ij}^{lk} \geqslant 0$, $j = 0,\ldots,g$.

Remark 1. The unification of heterogeneous information is conducted into fuzzy sets in a common linguistic domain to facilitate computations (see (Herrera et al., 2005) for further detail).

Once applied this unification and assuming that each fuzzy set will be represented by its membership degrees $p_i^{lk} = (\gamma_{i0}^{lk},\ldots,\gamma_{ig}^{lk})$, the preference relation P_i $i \in \{1,\ldots,m\}$ of decision maker e_i is represented as follows:

$$P_i = \begin{pmatrix} - & \cdots & p_i^{1n} = (\gamma_{i0}^{1n},\ldots,\gamma_{ig}^{1n}) \\ \vdots & \ddots & \vdots \\ p_i^{n1} = (\gamma_{i0}^{n1},\ldots,\gamma_{ig}^{n1}) & \cdots & - \end{pmatrix}$$

Subsequent computations on unified assessments p_i^{lk} are applied to a central value cv_i^{lk} computed upon them (Herrera-Viedma et al., 2005), as will be shown in Section 3.3.

3.2. Integrating attitude in consensus reaching process

The group attitude towards consensus refers to the importance given by decision makers to reach a consensus, compared to modifying their own preferences. If decision makers adopt an *optimistic attitude*, such that achieving an agreement is more important than their own preferences, then more importance is given to those positions in the group whose level of agreement is higher; on the other hand, if they adopt a *pessimistic attitude*, so that decision makers' preferences are considered more important than achieving an agreement, then those positions in the group whose level of agreement is lower are given more importance. Further detail about this concept can be found in Palomares et al. (in press).

The attitude will be integrated across the consensus process in the computation of agreement level (see Fig. 1) by means of OWA operators, due to their appropriateness to manage the attitudinal character of aggregation (Beliakov et al., 2007). To do so, we define the *Attitude-OWA*, an extension of OWA operators especially suitable for dealing with a high number of elements, h, in the aggregation process (i.e. large groups of decision makers). OWA (*Ordered Weighted Averaging*) operators are defined as follows:

Definition 1 (Yager (1988)). Let $A = \{a_1,\ldots,a_h\}$, $a_i \in R$, be a set of h values to aggregate. An *OWA operator* is a mapping $F{:}R^h \to R$, with an associated weighting vector $W = [w_1 \ldots w_h]^\top$ ($w_i \in [0,1]$, $\sum_i w_i = 1$):

$$F(a_1,\ldots,a_h) = \sum_{j=1}^{h} w_j b_j \tag{2}$$

where b_j is the jth largest of a_i values.

OWA operators can be classified according to their optimism degree, by means of a measure so-called *orness*, associated with W. This measure provides the attitudinal character of aggregation, by determining how close the operator is to the maximum (OR) function, and is defined as (Beliakov et al., 2007):

$$orness(W) = \frac{1}{h-1}\sum_{i=1}^{h}(h-i)w_i \tag{3}$$

While optimistic or OR-LIKE OWA operators are those whose *orness*$(W) > 0.5$, in pessimistic or AND-LIKE operators we have *orness*$(W) < 0.5$.

Different methods have been proposed to compute OWA weights. We consider the method proposed in Yager (1996) to compute them based on linguistic quantifiers (Zadeh, 1983), more specifically, *Regular Increasing Monotone* (RIM) quantifiers (Liu & Han, 2008), whose linear membership function $Q(r)$, $r \in [0,1]$, is defined by $\alpha,\beta \in [0,1]$ as:

$$Q(r) = \begin{cases} 0 & \text{if } r \leqslant \alpha, \\ \frac{r-\alpha}{\beta-\alpha} & \text{if } \alpha < r \leqslant \beta, \\ 1 & \text{if } r > \beta. \end{cases} \tag{4}$$

Yager proposed the following method to compute OWA weights, w_i, upon $Q(r)$ (Yager, 1988; Yager, 1996):

$$w_i = Q\left(\frac{i}{h}\right) - Q\left(\frac{i-1}{h}\right), \quad i = 1,\ldots,h \tag{5}$$

Regarding the group's attitude, it will be gathered at the beginning of the CRP by means of two *attitudinal parameters*, $\vartheta,\varphi \in [0,1]$, used to represent it:

- ϑ represents the group's attitude, which can be optimistic, pessimistic or indifferent; corresponding with a value greater, less or equal than 0.5, respectively. The higher ϑ, the more optimistic the attitude towards consensus. ϑ is also equivalent to the measure of optimism (*orness*(W)) that characterizes OWA operators.
- φ is used to indicate the amount of agreement positions which are given non-null weight in the subsequent aggregation conducted with Attitude-OWA. The higher φ, the more values are considered.

These parameters are the basis to define Attitude-OWA operator:
Definition 2 (Palomares et al (in press)). An *Attitude-OWA operator* of dimension h on a set of values $A = \{a_1,\ldots,a_h\}$ to aggregate, is an OWA operator based on two attitudinal parameters ϑ,φ given by a decision group to indicate their attitude towards consensus,

$$Attitude-OWA_W(A,\vartheta,\varphi) = \sum_{j=1}^{h} w_j b_j \tag{6}$$

where b_j is the jth largest of the a_i values, ϑ, $\varphi \in [0,1]$ are two input attitudinal parameters, and the set of weights, W, is computed by using a RIM quantifier, as shown in Eq. (5).

The attitude $\vartheta \in [0,1]$ of an Attitude-OWA operator can be determined by an associated RIM quantifier with differentiable membership function $Q(r)$, when the number of elements to aggregate h, is sufficiently large, $h \to \infty$ (i.e. when a large number of decision makers participate in the problem), as follows (considering Eqs. (3) and (5)),

$$\vartheta = \lim_{h\to\infty}\frac{1}{h-1}\sum_{i=1}^{h}(h-i)\left[Q\left(\frac{i}{h}\right) - Q\left(\frac{i-1}{h}\right)\right] = \int_0^1 Q(r)dr \tag{7}$$

see Palomares et al. (in press) for further detail. If $Q(r)$ is defined as shown in Eq. (4), then,

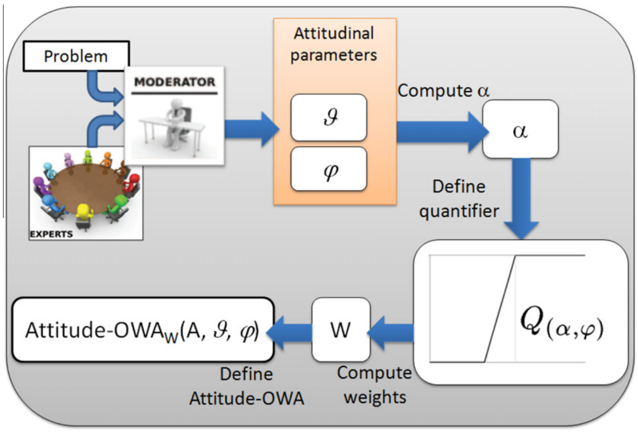

Fig. 2. Process to determine the Attitude-OWA operator based on ϑ and φ.

$$\vartheta = \int_0^1 Q(r)dr = 1 - \alpha - \frac{\varphi}{2} \tag{8}$$

where $\varphi = \beta - \alpha$. Therefore, given attitudinal parameters ϑ,φ, we can determine α,β, necessary to define $Q(r)$, as follows,

(1) $\alpha = 1 - \vartheta - \frac{\varphi}{2}$
(2) $\beta = \alpha + \varphi$

thus showing the capacity of Attitude-OWA to deal with a large number of decision makers (where $h \to \infty$) easily. Once defined the RIM quantifier Q associated to the group's attitude expressed by decision makers, weights w_i, are computed by using Eq. (5). The complete process to define an Attitude-OWA operator upon a group's attitude is shown in Fig. 2.

3.3. Consensus model

In this subsection, we describe in detail our proposal for a consensus model that extends the one shown in Fig. 1. It consists of six phases as depicted in Fig. 3.

1. *Determining group's attitude towards consensus*: The first phase consists in determining the group's attitude towards the measurement of consensus, gathered by means of attitudinal parameters ϑ,φ as explained in Section 3.2.
2. *Gathering preferences*: Each decision maker e_i provides his/her preferences on alternatives in X, by means of a preference relation P_i, consisting of a $n \times n$ matrix of assessments $p_i^{lk} \in D_i, D_i \in \{numerical, interval-valued, linguistic\}$, on each pair of alternatives (x_l,x_k), $l,k \in \{1,\ldots,n\}$.
3. *Making the information uniform*: Preferences provided by decision makers in different information domains are unified into a single common linguistic domain to facilitate the computations, as previously described in Section 3.1.
4. *Computing consensus degree*: The objective of any CRP is to reach a sufficient level of consensus among decision makers in the group. In this phase, the level of agreement among them is computed and measured as a value in [0,1]. To do so, the similarity between each pair of decision makers is measured, and these similarities are then aggregated to obtain a consensus degree at different levels. Given that our goal consists in improving the CRP taking into account the group's attitude towards consensus, as well as dealing with large groups effectively, we propose integrating such an attitude in the process to measure consensus by means of Attitude-OWA operator.
The following steps are required to compute the consensus degree:
(a) For each p_i^{lk}, $l \neq k$, a *central value* cv_i^{lk} is computed as follows

$$cv_i^{lk} = \frac{\sum_{j=0}^{g} index(s_j) \cdot \gamma_{ij}^{lk}}{\sum_{j=0}^{g} \cdot \gamma_{ij}^{lk}} \tag{9}$$

where $index(s_j) = j$ and $g + 1$ is the granularity of $S_T = \{s_0,\ldots,s_g\}$.
(b) Based on central values, a *similarity matrix* $SM_{ij} = (sm_{ij}^{lk})^{n\times n}$ is computed for each pair of decision makers e_i,e_j $(i < j)$, where each similarity value $sm_{ij}^{lk} \in [0,1]$ is computed as:

$$sm_{ij}^{lk} = 1 - \left|\frac{cv_i^{lk} - cv_j^{lk}}{g}\right| \tag{10}$$

(c) A *consensus matrix* $CM = (cm^{lk})^{n\times n}$ is obtained by aggregating similarity values at level of pairs of alternatives, using Attitude-OWA to consider the group's desired attitude towards consensus, as follows:

$$cm^{lk} = Attitude - OWA_W(SIM^{lk}, \vartheta, \varphi) \tag{11}$$

where $SIM^{lk} = \{sm_{12}^{lk},\ldots,sm_{1m}^{lk}, sm_{23}^{lk},\ldots,sm_{2m}^{lk},\ldots,sm_{(m-1)m}^{lk}\}$ is the set of all pairs of decision makers' similarities in their opinion on (x_l,x_k). Notice that the more optimistic Attitude-OWA is, the higher similarity values are rather considered in aggregation. $cm^{lk} \in [0,1]$ represents the consensus degree on the pair of alternatives (x_l,x_k).
(d) Consensus degree on each alternative x_l, ca^l, is computed as

$$ca^l = \frac{\sum_{k=1,k\neq l}^{n} cm^{lk}}{n-1} \tag{12}$$

where n is the number of alternatives.
(e) Finally, a global consensus degree, cr, is obtained as follows

$$cr = \frac{\sum_{l=1}^{n} ca^l}{n} \tag{13}$$

5. *Consensus control*
The overall level of agreement, cr, is compared with a consensus threshold $\mu \in [0,1]$ fixed a priori, according to the requirements

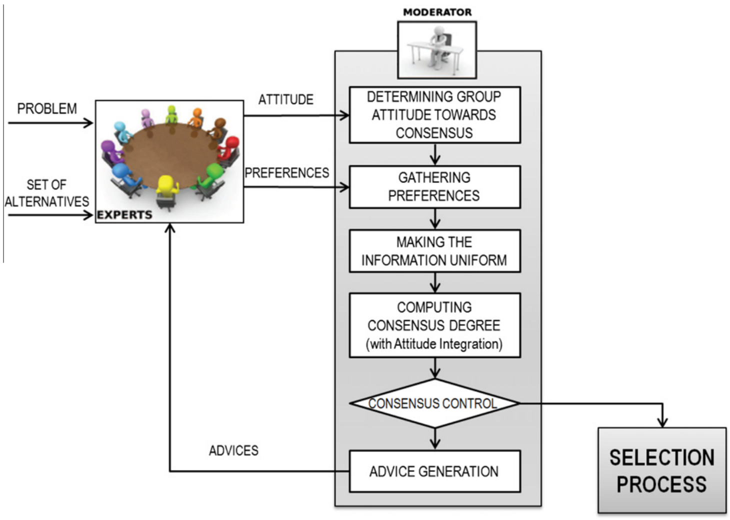

Fig. 3. Proposed consensus model scheme.

of the particular GDM problem. If $cr \geqslant \mu$, the consensus process ends and the group moves onto the selection process; otherwise, the process requires further discussion. A parameter *Maxrounds* controls the maximum number of discussion rounds allowed.

6. *Advice generation*
   When $cr < \mu$, another discussion round is required, therefore decision makers are advised to modify their preferences to make them closer to each other and increase the consensus degree in the following round. Such pieces of advice could be computed with different methods (Herrera-Viedma et al., 2005; Mata et al., 2009). In our approach, we use the method proposed in Mata et al. (2009).

4. Web-based CSS integrating group attitude towards consensus

This section presents a Web-based CSS that implements the consensus model presented in Section 3, and describes the communication and work flow that summarizes the functions of such a system. The main advantage of this CSS is the automation of the human moderator's tasks, thus eliminating any controversy caused by his/her possible subjectivity. The system also allows ubiquitous CRPs, so that no physical meetings are required anymore.

The most widely used architecture for web applications is the *client/server* architecture (see Fig. 4), in which the client is a computer. When a client sends a request to the server, it processes the request and sends a response back to the client. An advantage of using a client/server architecture is that the client users (decision makers) do not have to install the Web-based CSS application in their computer.

Regarding web technologies and programming languages considered, the application has been implemented using *Java* and *Java Server Pages* (JSP), which allow to generate dynamic web pages; *Servlets*, that control the system and carry out any necessary operation; *Javascript* and *Cascade Style Sheets*, to develop the web interface; and *MySQL*, to manage the database.

Another important feature of the Web-based CSS is its ubiquity, i.e. it can be used anytime and anywhere, which facilitates the elicitation of preferences and the overall CRP.

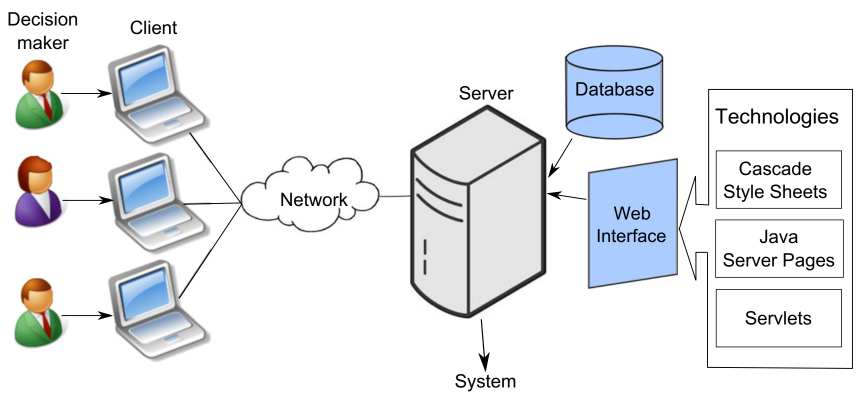

Fig. 4. Client/Server architecture.

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The performance of the Web-based CSS has been divided into two categories: client and server performances. They are briefly described below.

4.1. Client

The Web-based CSS shows the following four interfaces to the decision makers involved in a GDM problem.

- *Authentication*: The web application requests decision maker his/her username and password to log in the CSS (see Fig. 5).
- *Assigned problems*: When a decision maker logs in the system, the CSS shows him/her some information about the problems where he/she has been invited to participate (see Fig. 6).
- *Elicitation of preferences*: This interface implements the *Gathering Preferences* phase of the proposed consensus model. Decision makers use such an interface to elicit their preferences, indicating the type of information (numerical, interval-valued, linguistic) and domain (range in case of numerical or interval-valued information, and syntax of the linguistic terms in case of linguistic information) used to provide their preferences (see Fig. 7).
- *Checking current problem status*: The application shows decision makers the preferences provided in the last round. If the system has generated any recommendation for decision makers (as a result of the *Advice generation* phase), they must submit new preference values in order to increase the consensus degree. Recommendations are highlighted in the interface by means of a colored font (red color to increase and green color to decrease), as shown in Fig. 8.

4.2. Server

The server implements three main modules and manages the database that stores all the information about the defined problems, decision makers involved in each problem and the information generated during the decision process.

The communication with the client to send/receive information from/to decision makers is carried out by the Internet (see Fig. 9). The implemented modules in the server side are as follows:

- *Computing consensus degree*: Once all decision makers involved in the GDM problem have introduced their preferences, the server carries out the phases of the consensus model, *Making the information uniform* and *Computing consensus degree*. The latter one computes the consensus and similarity measures to determine the degree of agreement in the group, taking into account the decision makers' attitude towards consensus.
- *Consensus control*: This module implements the *consensus control* phase of the proposed model, checking whether the consensus level has achieved the minimum consensus level desired, in which case the CRP ends. Otherwise, more discussions rounds are required.

Fig. 5. User authentication screen.

Fig. 6. Assigned problems to a decision maker.

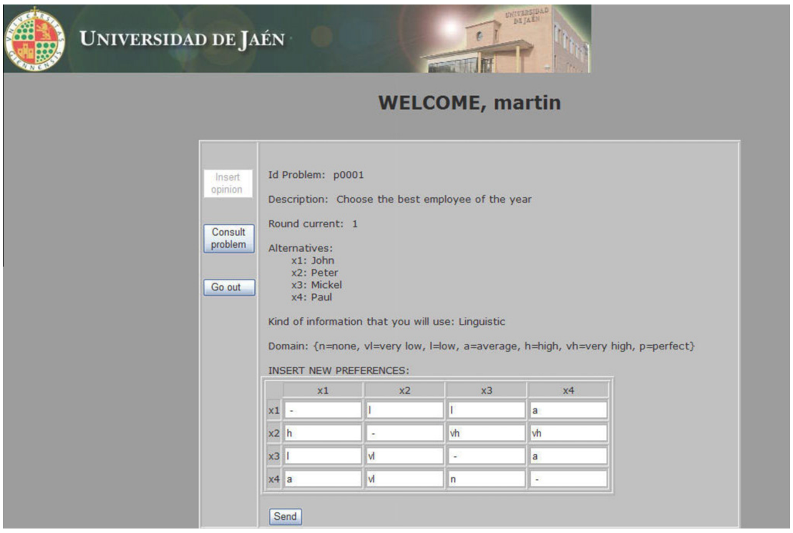

Fig. 7. A decision maker introduces his/her preferences by using linguistic information.

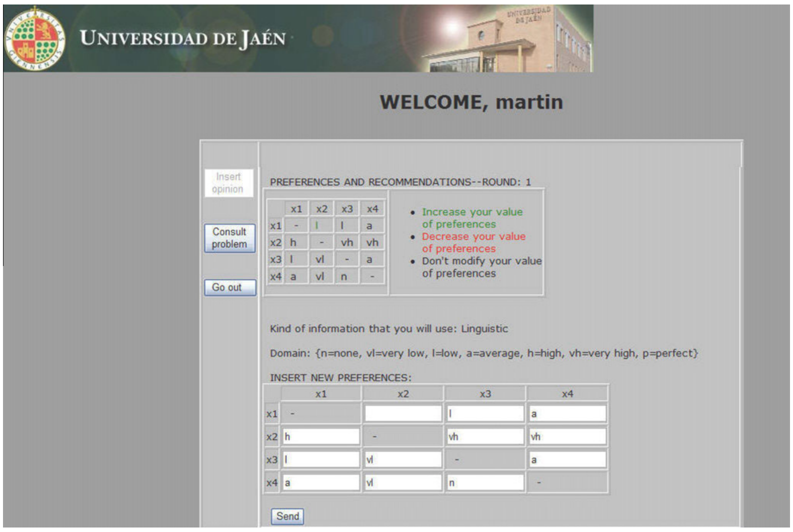

Fig. 8. A decision maker who provided his/her assessments by using linguistic terms, receives some recommendations.

- *Advice generation*: When a consensus round is conducted without having achieved the consensus threshold, the server carries out the *Advice generation* phase, which generates some recommendations to help decision makers to change their preferences on some alternatives in order to reach the consensus in the following rounds.

Once described the main functionalities of the system from the viewpoints of the client and the server, in the following we briefly show a general scheme of the work flow between them, as well as the interaction between the modules and the system's database.

1. *Initialization*: An initial step is carried out to insert in the database all the information about the GDM problem and decision makers involved in such a problem.
2. *Authentication*: When a decision maker wants to access the web application, he/she has to log in. The server checks the username and password in the database and if they are right, the

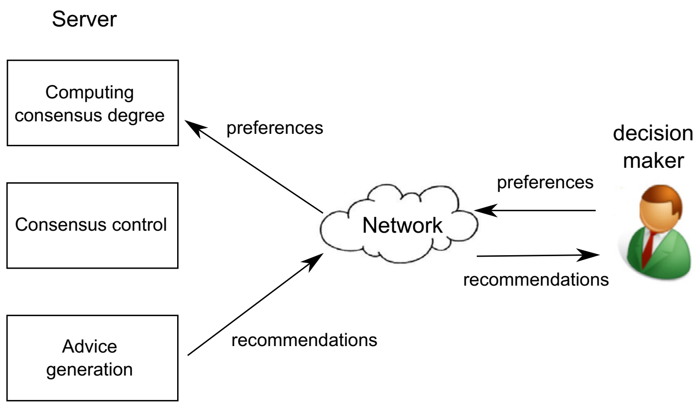

Fig. 9. Server modules.

decision maker accesses and sees the GDM problems in which he/she is involved. The decision maker can then carry out two tasks:

(a) Elicitation of preferences: decision makers can provide their preferences by using numerical, interval-value or linguistic information.
(b) Checking current problems status: decision makers can see their preferences provided in each round and their recommendations, if any.

3. *Computing consensus degree*: If all decision makers involved in the GDM problem have provided their preferences, the server starts the consensus process, makes the information uniform and computes the consensus and similarity measures.
4. *Consensus control*: The server checks if the required agreement degree has been achieved, in such a case the consensus process must finish. Otherwise, the server proceeds to step 5 before beginning a new consensus round.
5. *Advice generation*: The server generates some recommendations for decision makers to modify their preferences. In order to prevent the CRP from taking too long without having achieved an agreement, the system fixes a parameter *Maxrounds* to control the maximum number of discussion rounds allowed.

4.3. Web-based CSS performance

Once presented the main characteristics of the Web-based CSS, an example of a GDM problem is introduced and solved by using such a CSS.

Let us suppose that in a company there are 50 employees, $E = \{e_1, \ldots, e_{50}\}$ and they must choose the best employee of the year. The director of the company has selected 4 candidates $X = \{x_1 = John, x_2 = Peter, x_3 = Mickel, x_4 = Paul\}$. In the company there are 3 different departments whose employees have different backgrounds, so the type of information used for employees might

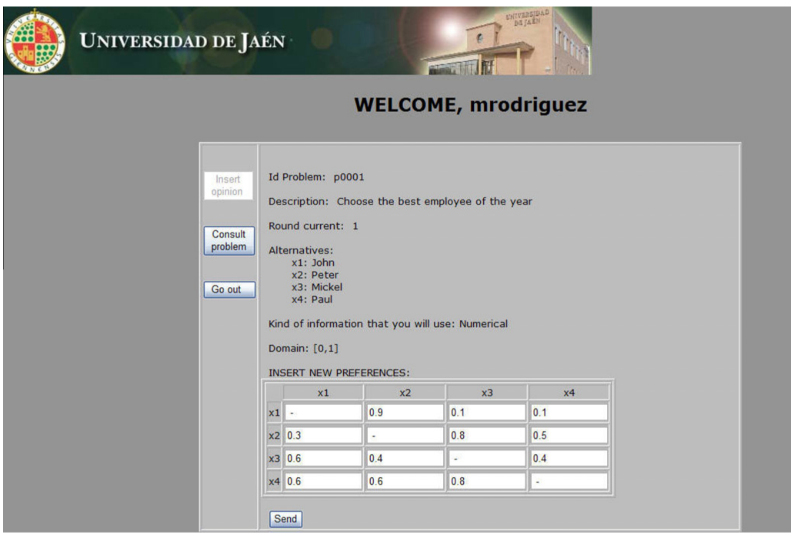

Fig. 10. The employee e_1, introduces his/her preferences by using numerical information.

be different. The employees have to reach a minimum level of agreement of $\mu = 0.85$, taking into account that employees follow an pessimistic attitude of $\vartheta = 0.25$ and the amount of agreement positions to be considered is $\varphi = 0.1$. The maximum number of discussion rounds allowed is *Maxrounds* = 10.

The employees provide their preferences by using different types of information: numerical, interval-valued and linguistic. The domains used for each type of information are:

- Numerical: $[0,1]$
- Interval-valued: $I([0,1])$
- Linguistic: $\{nothing(n), very\_low(vl), low(l), average(a), high(h), very\_high(vh), perfect(p)\}$

Fig. 10 shows the preferences provided by an employee e_1, who has used numerical information.

Once all employees have introduced their preferences, the consensus process begins with the first round, following the phases of the proposed consensus model in Fig. 3.

1. *Determining group's attitude towards consensus*
2. *Gathering preferences*
3. *Making the information uniform*
4. *Computing consensus degree*: The global consensus degree obtained in the first round is

$$cr = 0.5$$

5. *Consensus control*: As the global consensus degree, $cr = 0.5 < 0.85 = \mu$, it is then concluded that there is not enough consensus among the employees of the company, and consequently, the Web-based CSS should continue with another round.
6. *Advice generation*: Once the system verifies that the minimum level of agreement has not been reached, the system generates some recommendations for employees to modify their preferences in order to increase the level of agreement, and then the second round of discussion begins. Fig. 11 shows the recommendations generated for the employee e_1.

Table 1
Attitudinal parameters and RIM quantifiers used.

Attitude	ϑ	φ	α	$Q_{(\alpha,\varphi)}$
Pessimistic	0.25	0.1	0.7	$Q_{(0.7,0.1)}$
Indifferent	0.5	0.6	0.2	$Q_{(0.2,0.6)}$
Optimistic	0.75	0.3	0.1	$Q_{(0.1,0.3)}$

Table 2
Global consensus degree for each round.

Round	Pessimistic	Indifferent	Optimistic
1	0.5	0.7	0.87
2	0.59	0.76	
3	0.68	0.81	
4	0.77	0.86	
5	0.84		
6	0.88		

In this GDM problem, due to the choice of a pessimistic attitude, it is necessary to carry out six rounds of discussion to reach the consensus threshold $\mu = 0.85$. The proposed consensus model in Section 3 integrates the group's attitude towards consensus to deal with large groups and improve the CRP. In order to illustrate the effect of integrating different attitudes in the CRP, we will solve the GDM problem three times by using for each resolution a different attitude ϑ, including an optimistic, indifferent and pessimistic attitude, and different values for the amount of information (agreement positions between decision makers) considered in aggregation, φ.

Table 1 shows the different attitudes, given by ϑ, φ, and the different values of α and RIM quantifiers obtained, denoted as $Q_{(\alpha,\varphi)}$.

The global consensus degree obtained for each attitude and the number of necessary rounds to reach the minimum consensus level are shown in Table 2. As can be seen, the problem resolution with an optimistic attitude is the only one where the consensus has been achieved in the first round, whereas more rounds are necessary with the other two attitudes.

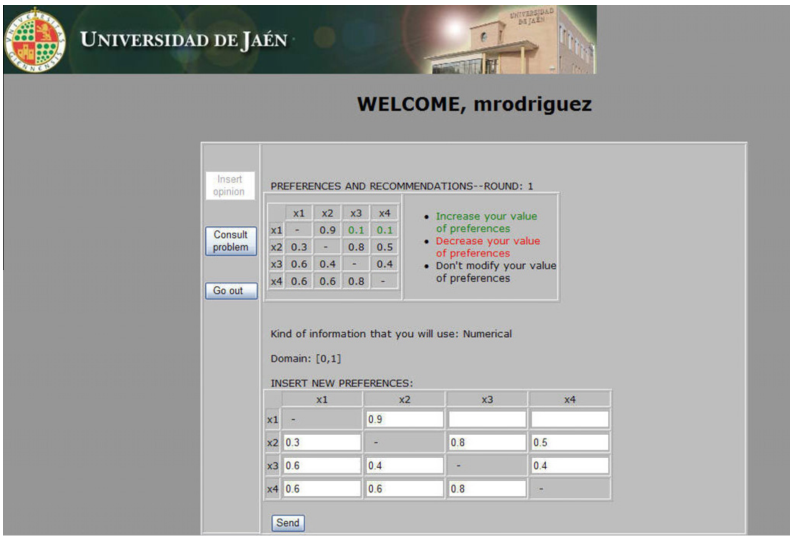

Fig. 11. The employee e_1, receives some recommendations.

The results affirm that the use of Attitude-OWA operator based on an optimistic attitude favors a greater convergence towards consensus, whereas Attitude-OWA operator based on a pessimistic attitude favors a lower convergence and a further discussion process, regardless of the proportion of values considered, φ. Therefore, depending on decision makers' priority to reach the minimum level of consensus, they can use an optimistic attitude if their priority is to achieve a consensus quickly, or a pessimistic attitude for a problem that requires further discussion.

5. Concluding remarks

The evolution of group decision making problems with increasingly larger scales of decision makers who may have different backgrounds, makes necessary to modify the present vision on current existing models. In this paper, we have presented a consensus model which deals with heterogeneous information and manages the attitude of decision makers. In addition, we have implemented a Web-based consensus support system upon such a model, that automates real consensus reaching processes. Due to its capacity to deal with large groups of decision makers, we aim to apply the system to real-life problems involving entire societies of individuals, such as e-democracy processes and social networks.

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Capítulo 5

Conclusiones y Trabajos Futuros

Finalmente, en este capítulo revisamos las principales propuestas y resultados obtenidos a lo largo de esta investigación, y proponemos las líneas de investigación y trabajos futuros más inmediatos que nos planteamos a partir de dichos resultados.

5.1. Conclusiones

Los problemas de *Toma de Decisión en Grupo* y los *Procesos de Alcance de Consenso* han cobrado especial relevancia en muchas áreas de aplicación de la vida real (tales como ciencias sociales, medicina e ingeniería), debido a la creciente necesidad de tomar decisiones en grupo con un alto nivel de acuerdo entre los expertos participantes en este tipo de problemas.

Debido a la importancia de la toma de decisión en grupo y el consenso en estas áreas, diferentes investigadores han propuesto en la literatura una amplia variedad de modelos y enfoques para soportar procesos de consenso. Dichos enfoques se han centrado normalmente en tratar con un número de expertos reducido. Sin embargo, los nuevos entornos y tecnologías que facilitan la participación de grandes grupos en procesos de decisión, tales como las redes sociales, han provocado que los llamados problemas de *Toma de Decisión en Grupo a Gran Escala* adquieran mayor interés en los últimos años. Dichos problemas plantean nuevos retos y dificultades que los enfoques de consenso actuales aún no han sido capaces de abordar:

- La necesidad de arquitecturas altamente escalables, para el desarrollo de sistemas de apoyo al consenso capaces de gestionar grandes grupos eficientemente.
- El excesivo coste temporal invertido en el proceso de discusión, debido a la gran cantidad de supervisión llevada a cabo por los expertos para revisar y modificar sus preferencias.
- La presencia de individuos o subgrupos con intereses similares, que no cooperan para alcanzar un acuerdo, dificultando el proceso de discusión o incluso intentando desviar la solución al problema a su favor.
- La dificultad para obtener conocimiento útil sobre el estado actual del problema de toma de decisión en grupo, a partir de una elevada cantidad de información sobre las preferencias de los expertos, y la necesidad de herramientas que faciliten el análisis de dicha información.
- La necesidad de conocer la actitud de los expertos hacia el alcance de consenso en cada problema, debido a que un elevado número de expertos con diferentes visiones de dicho problema podrían participar en el mismo de manera conjunta.

Teniendo en cuenta los retos que acabamos de exponer, el interés de esta investigación se ha centrado en superarlos, mediante un conjunto de propuestas orientadas a facilitar los procesos de consenso en problemas de toma de decisión en grupo a gran escala. Dichas propuestas son enumeradas a continuación:

1. Se ha desarrollado un sistema de apoyo al consenso basado en una plataforma multiagente para toma de decisión en grupo a gran escala. Dicho sistema presenta una alta escalabilidad para gestionar grandes grupos de manera eficiente, gracias a su arquitectura multiagente. Además, el sistema incorpora un novedoso modelo de autonomía semi-supervisada basado en agentes, que minimiza la supervisión requerida por los expertos humanos para revisar y modificar sus preferencias durante el proceso de discusión, reduciendo así el coste invertido en alcanzar un acuerdo.

2. Se ha propuesto e integrado en un modelo de consenso, un mecanismo para gestionar comportamientos no cooperativos en procesos de consenso, basado en técnicas de clustering difuso y en lógica difusa. El enfoque facilita la detección y gestión de individuos y subgrupos no cooperativos. Como resultado de la aplicación del modelo, se obtiene una mejora en la convergencia hacia el consenso en aquellos problemas en los que surge este tipo de comportamientos.

3. Se ha presentado una herramienta gráfica de monitorización de preferencias, que facilita un análisis visual de comportamientos no cooperativos y demás aspectos de interés en los procesos de consenso, tales como posiciones de acuerdo o desacuerdo entre expertos. La herramienta proporciona una representación visual de las preferencias de los expertos, basada en el uso de mapas autoorganizativos, facilitando así la obtención de conocimiento útil sobre el estado actual del problema, a partir de información fácilmente interpretable.

4. Se ha definido una nueva medida de consenso basada en un operador de agregación que integra la actitud de grupo hacia el consenso. Dicha medida se ha integrado en un modelo de consenso y en un sistema de apoyo al consenso Web para toma de decisión en grupo en contextos heterogéneos, y permite optimizar el proceso de consenso adaptándolo a la actitud adoptada por el grupo en cada problema.

De forma adicional, hemos llevado a cabo un estudio teórico de los modelos de consenso presentes en la literatura para toma de decisión en grupo en contextos difusos. Como resultado de dicho estudio, se ha presentado una taxonomía que revisa un gran número de modelos y los categoriza en base a sus principales características, aunando en una misma categoría aquellos modelos con características similares. Además, se ha desarrollado una aplicación software basada en simulación para el estudio y evaluación de modelos existentes.

Como podemos observar, todos los objetivos planteados al inicio de esta investigación han sido alcanzados a través de las propuestas presentadas en esta memoria.

5.2. Trabajos Futuros

A pesar de las múltiples propuestas realizadas en esta investigación, aún existe un gran número de retos en el área de investigación de la toma de decisión en grupo y los procesos de consenso, algunos de los cuales están relacionados con los problemas de toma de decisión en grupo a gran escala. Por ello, nuestros trabajos futuros se encaminan hacia las siguientes líneas de investigación:

1. Continuación y extensión de las propuestas realizadas.
   - Definición de mecanismos flexibles y mejorados para gestionar los diferentes patrones de comportamiento adoptados por cada experto durante cada etapa del proceso de consenso.
   - Estudio de nuevos métodos basados en el análisis de preferencias de expertos en grandes grupos, para detectar subgrupos de expertos con opiniones similares o en conflicto. Extensión de dichos métodos a diferentes estructuras de preferencia y dominios de expresión de la información.
2. Retos importantes y direcciones futuras para la mejora de procesos de consenso en problemas de toma de decisión en grupo a gran escala.
   - Desarrollo y puesta en marcha de una aplicación software que nos permita automatizar y simular diferentes modelos de consenso existentes, con el fin de facilitar un estudio y análisis de los mismos en la práctica.
   - Definición de nuevas métricas para evaluar el funcionamiento de los procesos de consenso. Implementación de dichas métricas en la aplicación software propuesta anteriormente, con el propósito de permitir estudios comparativos entre diferentes propuestas de modelos de consenso.
   - Aplicación de los modelos de consenso propuestos en esta investigación en entornos de comercio electrónico, mediante su integración con sistemas de recomendación para grupos o plataformas de compra on-line para grupos.

5.3. Publicaciones Adicionales y Reconocimientos

En relación a la difusión y publicación de los resultados presentados, además de las publicaciones presentadas en esta memoria destacamos las siguientes aportaciones:

- En Congresos Internacionales
    - I. Palomares, F.J. Quesada, L. Martínez, *Multi-agent based Semi-supervised Consensus Support System for Large-Scale Group Decision Making*. 8th International Conference on Intelligent Systems and Knowledge Engineering. ISKE 2013. Proceedings currently in press. Springer Berlin/ Heidelberg, 2013.
    - I. Palomares, L. Martínez, F. Herrera, *A Fuzzy Clustering Approach for Non-cooperative Behavior Detection in Consensus Reaching Processes*. 4th International Workshop on Knowledge Discovery, Knowledge Management and Decision Support. EUREKA 2013. pp. 37-44. Atlantis Press, 2013.
    - R.M. Rodríguez, I. Palomares, L. Martínez, *Attitude-based Consensus Model for Heterogeneous Group Decision Making*. 7th International Conference on Intelligent Systems and Knowledge Engineering. ISKE 2012. Advances in Intelligent and Soft Computing, vol. 214, pp. 279-288. Springer Berlin/Heidelberg, 2012.
    - I. Palomares, J. Liu, Y. Xu, L. Martínez, *Using OWA Operators to Integrate Group Attitudes towards Consensus*. 6th International Conference on Intelligent Systems and Knowledge Engineering. ISKE 2011. Advances in Intelligent and Soft Computing, vol. 123, pp. 273-282. Springer Berlin/Heidelberg, 2011.
    - I. Palomares, P.J. Sánchez, F.J. Quesada, F. Mata, L. Martínez, *COMAS: A Multi-agent System for performing Consensus Processes*. International Symposium on Distributed Computing and Artificial Intelligence. DCAI

2011. Advances in Intelligent and Soft Computing, vol. 91, pp. 125-132. Springer Berlin/Heidelberg, 2011.

- En Congresos Nacionales
  - I. Palomares, L. Martínez, *Attitude-Driven Web Consensus Support System for Large-scale Group Decision Making Problems based on Fuzzy Linguistic Approach*. XV Conferencia de la Asociación Española para la Inteligencia Artificial. CAEPIA 2013. Lecture Notes in Artificial Intelligence, vol. 8109, pp. 91-100. Springer Berlin/Heidelberg, 2013.
  - I. Palomares, J. Liu, Y. Xu, L. Martínez, *Uso de Operadores OWA para modelar la Actitud hacia el Consenso en Problemas de Toma de Decisión en Grupo*. Actas del XVI Congreso Español sobre Tecnologías y Lógica Fuzzy. ESTYLF 2012. pp. 211-216, 2012.
- Capítulos de Libros
  - I. Palomares, L. Martínez, *Attitude-based Consensus Model for Heterogeneous Multi-criteria Large-Scale Group Decision Making: Application to IT-based Services Management*. Engineering and Management of IT-based Service Systems. Intelligent Systems Reference Library, vol. 55, pp. 155-177. Springer Berlin/Heidelberg, 2014.

Además, con anterioridad a la finalización de esta investigación, una propuesta de tesis doctoral fue presentada en una sesión *Doctoral Consortium*, en la que participaron jóvenes investigadores pertenecientes al ámbito de la Inteligencia Artificial a nivel nacional. El evento fue celebrado dentro de la XV Conferencia de la Asociación Española para la Inteligencia Artificial (CAEPIA), en el marco del IV Congreso Español de Informática (CEDI 2013), durante los días 17 al 20 de Septiembre de 2013. Como resultado, la propuesta asociada a la presente tesis recibió un *Primer Premio a la Mejor Propuesta de Investigación Pre-doctoral en Inteligencia Artificial.* El diploma recibido tras obtener el citado reconocimiento se adjunta en la siguiente página.

ASOCIACIÓN ESPAÑOLA PARA LA INTELIGENCIA ARTIFICIAL (AEPIA)

DOCTORAL CONSORTIUM
Multiconferencia CAEPIA 2013

Han concedido a

Iván Palomares Carrascosa

el primer premio (ex aequo) del Doctoral Consortium celebrado en la Multiconferencia CAEPIA 2013 a la mejor propuesta de investigación por el trabajo predoctoral titulado:

"Sistema multiagente para modelar procesos de consenso en toma de decisión en grupo a gran escala"

Madrid, 20 de septiembre de 2013

Apéndice A

English Summary

This appendix covers an English summary of the thesis entitled, *Multi-agent System to model Consensus Reaching Processes in Large-Scale Group Decision Making using Soft Computing Techniques*, which is written in English language, as partial fulfilment for obtaining the International Ph.D.

Firstly, a brief introduction to the research topic and a motivation for the research conducted is shown. The objectives established in such research are then exposed, followed by the structure of chapters that compose this research memory.

A.1. Motivation

Decision Making is a usual mankind activity in daily life [8, 72]. Usually, humans face situations in which there exist several alternatives, and the most adequate one must be chosen. Group Decision Making (GDM) problems, characterized by the participation of multiple individuals or experts with different points of view in the decision process, have attained special importance and research interest within the field of Decision Making in the last decades [40, 54].

GDM problems have been traditionally solved applying an alternative selection process solely [30], so that each expert provides his/her preferences over alternatives, and the best alternative or subset of them is chosen. This resolution process does

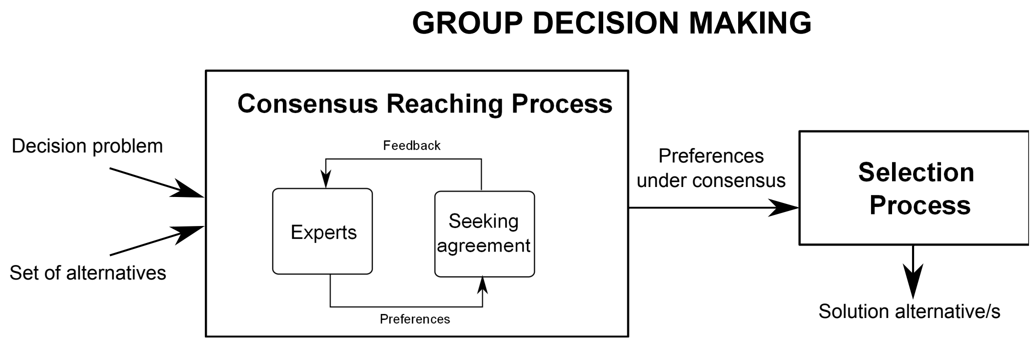

Figura A.1: Resolution process for GDM problems under consensus

not take into account the existing level of agreement between experts regarding their preferences. Consequently, it may occur that the decision made is not accepted by some experts, because they might consider that their individual preferences have not been taken into account. For this reason, the study of Consensus Reaching Processes (CRPs) to reach a collective agreement before making a group decision, has become a prominent research topic in GDM. Such processes are introduced as a new phase in the resolution process for GDM problems (see Figure A.1). CRPs are iterative processes consisting of several rounds, in which experts discuss and modify their preferences under the supervision of a human figure known as moderator, with the aim of making such preferences closer to each other and reaching a high level of agreement in the group [10, 57, 77].

As a result of the study of consensus in GDM, different works have been proposed in the literature to support CRPs, such as:

- A large number of theoretical consensus models that provide the necessary guidelines to conduct CRPs [9, 38, 60, 69, 77, 91].
- The definition of consensus measures, i.e. indicators of the level of agreement reached amongst experts [7, 32, 42]. Such measures are usually based on the use of similarity metrics and aggregation operators [1, 2].
- The development of Consensus Support Systems (CSS) based on intelligent techniques [12, 16, 19, 67], which implement the consensus models proposed. CSS are developed with the purpose of automating the human moderator task

of coordinating the discussion process, eliminating his/her possible bias due to subjective factors and, in some cases, making non-physical meetings possible when experts are physically separated (e.g. with the use of Web technologies) [45, 46].

Classically, GDM problems conducted in most organizations and institutions have taken place at a strategic level, in which decisions are usually made just by a small number of people (e.g. members at the executive level in a business environment) [10, 22, 23]. However, the recent evolution and increasing importance of new technological paradigms and environments in the last few years, is making possible the participation of a larger number of individuals in decision making processes. Some examples of these paradigms and environments are: social networks [79, 81, 97], e-democracy systems [13, 49], group recommender systems [59] and e-marketplace selection for group shopping [11], amongst others. As a result, the so-called *large-scale GDM problems*, in which a large group of experts take part in the decision problem, is becoming an important research topic that must be considered in both current and future works on GDM and consensus [17, 85].

Despite the large amount of models and approaches that have been proposed by a variety of authors to support CRPs in GDM problems, they have normally focused on problems in which a low number of experts take part. The research results obtained in this field of study up to date, are not sufficient when dealing with large-scale GDM problems: new difficulties and challenges arise, which require further study for the improvement of CRPs in which a large number of experts are involved. Some of these difficulties and challenges are described below:

- *Necessity of Scalable Consensus Support System Architectures*: Most CSS proposed up to date focused on dealing with a small number of experts only [12, 46, 103], therefore classical computer architectures are enough to develop and put them in practice successfully. However, large-scale GDM problems require more highly scalable CSS architectures to facilitate the management of large amounts of information about experts' preferences. For this reason, it is necessary to propose and develop CSS based on highly scalable architectures (e.g. multi-agent architectures [88]), which are capable of supporting this kind of GDM problems effectively.

- *Costly Supervision of Preferences*: In a CRP, experts might have to revise and modify their initial preferences at each discussion round, in order to make their opinions closer to the rest of the group and increase the level of agreement [10, 77]. When a large group of experts are involved in a CRP, the process of revising and modifying preferences may imply a higher cost, in terms of the time invested to achieve an agreement. Such increase in the cost of the CRP could even lead to the possibility that some experts experience an eventual loss of motivation and interest in the problem tackled [64].

- *Non-cooperative Behaviors of Individuals or Subgroups*: CRPs require that experts adopt a cooperating attitude towards each other, in order to reach an agreement [77]. Nevertheless, the existence of experts or subgroups of them who try to bias the discussion process, deviating the collective opinion in their favor, is frequent in many CRPs carried out during the resolution of real-life GDM problems [96]. These non-cooperative behaviors usually difficult reaching agreements. Moreover, in large-scale GDM problems, the existence of subgroups that present such behaviors is more frequent, and dealing with them might be a complex task without the aid of suitable approaches and tools to do so [65].

- *Understanding the Current State of the GDM Problem*: A general vision of the current state of the problem at each phase of the CRP, based on the positions of experts' opinions, might help to obtain useful knowledge about the level of agreement and behavior of experts. Numerical and textual information provided by the existing consensus models and CSS so far, have been enough to obtain and interpret knowledge about the state of the GDM problem easily [9, 103], due to the low number of experts who normally took part in such problems. However, the large amount of information utilized in large-scale problems increases the need for new support tools based on the visual representation of information, in order to make it more interpretable and allow groups to monitor the current state of the problem easily [66].

- *Group's Attitude towards Consensus*: The attitude of experts towards consensus is determined by the importance that they give to preserving their own preferences, compared to the importance given to the goal of reaching consensus. Knowing the vision or *attitude* of experts towards the achievement of consensus at each problem, is an important aspect to consider, in order to optimize the CRP and adapt it to such an attitude [63]. In the case of large-scale GDM problems, determining and reflecting the group's attitude towards consensus in the CRP, by means of the consensus measures utilized, might be difficult without the choice of an adequate measure that takes it into account.

The constant evolution and current challenges found in large-scale GDM problems, some of which have been described above, led to formulate the following initial hypothesis at the beginning of this research:

The existing consensus models and CSS can not satisfy the current needs present in large-scale GDM problems: the CRPs supported by these models and systems are not flexible enough to deal with large groups of experts due to several factors, as explained above. For this reason, it is necessary to make CRPs more flexible by means of the adequate approaches and tools to do so, thus facilitating an effective management of experts' tasks and behavior across discussion processes, and optimizing such processes taking into account the attitude of experts.

A.2. Objectives

Based on the current challenges of consensus in GDM and the hypothesis stated at the previous section, the initial purpose of this research is the development of a CSS based on the multi-agent systems paradigm, which is characterized by its high scalability and capabilities for distributed computing [87, 88]. Such a CSS would allow to implement different consensus models, both new and existing ones, as well as utilizing diverse soft computing techniques for improving and automating CRPs carried out in large-scale GDM problems.

Based on this initial purpose of a multi-agent based CSS, the following four objectives are considered:

1. Development of an *agent-based semi-supervised autonomy model* [64], that allows a high degree of automation in the tasks carried out by human experts, thus reducing the time cost invested by them in such tasks. The semi-supervised model must let human experts delegate into intelligent agents for modifying their preferences autonomously. The model should also request human supervision in some circumstances in which it would be more convenient, with the aim of minimizing the overall cost necessary to carry out the CRP, while preserving the sovereignty of experts to some degree.

2. Definition of mechanisms to *detect and manage non-cooperative behaviors in CRPs* [65], which allow us to deal with situations where an expert or subgroup of them with similar interests refuse to change their own initial opinions to reach a group agreement, trying to deviate the collective opinion in their favor.

3. Development of a *graphical monitoring tool* [66], that facilitates a visual analysis of preferences, evaluating diverse aspects such as positions of agreement/ disagreement between experts and non cooperating individuals throughout the CRP.

4. Integration of the *group's attitude towards consensus* [63] in the CRP. To do so, the group's attitude towards consensus should be reflected in the consensus measures utilized to determine the level of agreement, in order to optimize the CRP according to the attitude adopted by the group at each particular problem.

A.3. Structure

In order to achieve the objectives previously formulated, and in accordance with the article 23, point 3, of the current regulations for Ph.D. studies in the University of Jaén (Programme RD. 1393/2007), this research memory is presented as a compendium of articles published by the Ph.D. candidate. Such publications constitute the nucleus of the thesis, and they correspond to five scientific articles published in International Journals indexed by the JCR (*Journal Citation Reports*) database, produced by ISI (*Institute for Scientific Information*), as well as another article submitted to revision in an International Journal indexed by JCR at the time of finishing this memory.

Therefore, the memory is composed by a total of six publications, and it is structured into five chapters, as follows:

- Chapter 2: It presents an overview of GDM and consensus, reviewing the basic concepts and preliminaries related to GDM problems, preference modeling, CRPs and a brief description of the existing related works to support them: consensus measures, consensus models and CSSs. A broad view on consensus in GDM and a taxonomy that overviews and characterizes a large number of existing consensus models in the literature, are introduced at the end of the chapter and later presented in Section 4.1, by attaching the paper entitled *Consensus under a Fuzzy Context: Taxonomy, Analysis Framework and Experimental Case of Study.* The content associated to this chapter can also be found in this English summary, Section A.4.
- Chapter 3: It presents a summary of the research conducted to achieve the objectives formulated in this memory, and shows a brief discussion of the results obtained at each proposal. Such proposals are developed in the five papers that can be found from Sections 4.2 to 4.6, and they are organized around two main courses of action (as it is detailed in the chapter): *Automated and Proactive Management of Consensus Reaching Processes in Large-Scale*

GDM and *Management of Group Attitudes towards Consensus Reaching*. The content associated to this chapter can also be found in this English summary, Section A.5.

- Chapter 4: This chapter constitutes the nucleus of the thesis, and it contains the six resultant publications from this research.

- Chapter 5: In this chapter, some concluding remarks and future works are pointed out, based on the research findings obtained. The content associated to this chapter can also be found in this English summary, Section A.6.

A.4. Background on Group Decision Making and Consensus

In this section, it is revised some theoretical background necessary to understand the research presented in this memory. Firstly, some basic concepts and definitions about group decision making and consensus are introduced. Consensus reaching processes are then described. The main types of existing research works to support groups in consensus reaching processes are briefly introduced. Finally, it is briefly introduced a consensus background and taxonomy of existing consensus models, whose related publication is attached in Chapter 4.

A.4.1. Group Decision Making

Decision Making is an inherent mankind activity in daily life. Human beings must constantly face situations in which there exist several alternatives and, in some occasions, they have to decide which one is the best, or which one should be carried out. The field of Decision Making has been applied in a wide range of disciplines, such as: social sciences, economy, engineering, planning, medicine, psychology, etc. As a consequence of this variety of application fields, different Decision Making models have been defined, thus giving rise to the so-called Decision Theory [8, 54, 71–73, 76].

Classical decision problems present the following basic elements:

1. One or several objectives to solve.

2. A set of alternatives or possible decisions to make for reaching such objectives.

3. A set of factors or states of nature, that define the context in which the decision problem is formulated.

4. A set of utility values, being each one associated to an specific alternative and state of nature.

Decision Making processes can take place in different situations, depending on the context in which the decision problem is carried out:

1. *Certainty environment*: In this situation, the utility values of alternatives are accurately known.

2. *Risk environment*: This situation occurs when the knowledge about each alternative is modeled by means of a probability distribution.

3. *Uncertainty environment*: In this situation, we do not have probabilistic knowledge about alternatives, and their utility values are characterized in an approximate way.

Classical Decision Theory provides a number of methods suitable for dealing with problems defined in environments of certainty and risk. However, such methods are not adequate to manage decision problems defined under uncertainty of non-probabilistic nature, where the information about the problem is vague and imprecise [58]. These situations are also known as decision making problems in a fuzzy context, or "Fuzzy Decision Making" [3]. Fuzzy sets theory [52, 99] and the fuzzy linguistic approach [100–102], proposed by L.A. Zadeh, have proven to be an effective means to deal with uncertain information in decision problems.

Decision Making problems can be classified according to different points of view, being the following two some of them [54]:

- *Number of individuals or experts*: When only one expert takes part in the decision problem, we have an Individual Decision Making problem. On the other hand, when several experts take part in a problem together, we have a Group Decision Making Problem [10, 40].

- *Number of criteria*: Some problems require assessing each alternative based on one attribute or criterion only (Single Criterion Decision Making Problems), whilst others consider necessary to assess alternatives according to several criteria (Multi-criteria Decision Making Problems) [26, 50, 95].

This research is focused on decision making problems under uncertainty in which several experts take part, more specifically, Group Decision Making (GDM) problems. Making group decisions implies the involvement in the decision problem of several experts, each one with their own ideas, attitudes, motivations and knowledge, who attempt to make a collective decision in order to achieve a common solution to such a problem. Decision making processes in which several experts take part, might sometimes lead to better decisions than those carried out by a single expert. Formally, a GDM problem is characterized by the following elements [40]:

- The existence of a common problem to solve.

- A set X of *alternatives* or possible solutions to the problem:

$$X = \{x_1, \ldots, x_n\} \ (n \geq 2) \tag{A.1}$$

- A set E of individuals or *experts*, who express their opinions on the set of alternatives X and attempt to achieve a common solution to the problem considered:

$$E = \{e_1, \ldots, e_m\} \ (m \geq 2) \tag{A.2}$$

Each expert expresses his/her opinions over alternatives by means of a preference structure. One of the most usual preference structures in GDM problems under uncertainty is the so-called *fuzzy preference relation* [62, 68, 84]. Given a finite set of alternatives X, a fuzzy preference relation associated to expert $e_i \in E$, $i \in \{1,\dots,m\}$ is represented as a $n \times n$ matrix as follows:

$$P_i = \begin{pmatrix} - & \dots & p_i^{1n} \\ \vdots & \ddots & \vdots \\ p_i^{n1} & \dots & - \end{pmatrix}$$

being each *assessment* $p_i^{lk} = \mu_{P_i}(x_l, x_k) \in [0,1]$ the degree of preference of the alternative x_l over x_k, $l,k \in \{1,\dots,n\}$, $l \neq k$, according to e_i, interpreted as follows:

- $p_i^{lk} > 0.5$ indicates e_i's preference of the alternative x_l over x_k, and $p_i^{lk} = 1$ means that x_l is absolutely preferred over x_k.
- $p_i^{lk} < 0.5$ indicates e_i's preference of the alternative x_k over x_l, and $p_i^{lk} = 0$ means that x_k is absolutely preferred over x_l.
- $p_i^{lk} = 0.5$ indicates e_i's indifference between alternatives x_l and x_k.

Other preference structures that have been considered by some researchers in several GDM approaches are: utility vectors [9] and preference orderings [83], amongst others. Moreover, in order to deal with uncertain information, experts may utilize different information domains to provide their preferences on alternatives, depending on their knowledge area or level of expertise [24]. Some information domains frequently utilized in GDM problems under uncertainty are [36]:

- *Numerical* [9,62,104]: Assessments are represented as values in [0,1] (as occurs with fuzzy preference relations, for instance).
- *Interval-valued* [27, 93]: Assessments are represented as intervals, $I([0,1])$.
- *Linguistic* [20, 29, 35, 55, 58]: Assessments are represented as linguistic terms $s_u \in S$, $u \in \{0,\dots,g\}$, being $S = \{s_0,\dots,s_g\}$ a set of linguistic terms with granularity g [100–102].

The solution for a GDM problem can be determined by applying either a direct approach or an indirect approach [30]. In a *direct approach*, the solution is directly

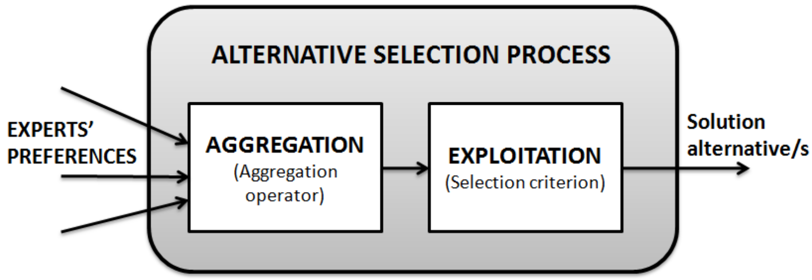

Figura A.2: Selection process for the resolution of GDM problems

obtained from the individual preferences of experts, without constructing a social opinion first [31], whereas in an *indirect approach*, a social opinion or *collective preference* is determined a priori from individual opinions, and then it is utilized to find a solution for the problem. Regardless of the approach considered, the classical selection process for reaching a solution to GDM problems is composed by two phases [76], as illustrated in Figure A.2:

(i) *Aggregation phase*: Experts' preferences are combined.

(ii) *Exploitation phase*: It consists in obtaining an alternative or subset of alternatives as the solution to the problem.

Furthermore, different situations can be found depending on the context of the GDM problem addressed, e.g. collaboration or competitiveness amongst experts, compatible or incompatible proposals involving different environments (e.g. companies and governments), and so on. For this reason, the process to find a solution to a GDM problem could be based on different guiding rules [10, 57]:

- *Majority rule*: The decision is made according to the opinions of the majority of experts involved in the problem. Once adopted the majority opinion, it must be respected by other minority positions in the group, since it is assumed that all individuals accept the use of this rule. The notion of majority admits two different modalities for its application:

1) *Absolute majority*: The majority opinion has been considered by more than half of the total number of experts in the group.
2) *Relative or Simple majority*: It only requires that the majority opinion is the one supported by the highest number of experts, even though the sum of the remaining experts is higher.

- *Minority rule*: The decision is delegated to a subgroup of individuals, since the problem requires a level of expertise that not all experts have. It is necessary that all experts accept this rule and they agree with the fact of delegating to a subgroup for making the decision.
- *Individual*: This situation takes place when the group resorts to an expert to make the decision or there exists a leader in the group.
- *Unanimity*: All experts must agree with the decision made.

A.4.2. Consensus in GDM: Consensus Reaching Processes

In most GDM situations described previously, when an alternative selection process is applied solely, it might occur that some experts do not accept the decision made, because they consider that their preferences have not been taken into account sufficiently. A high level of collective agreement becomes crucial in many real-life GDM problems, therefore it is necessary to apply a Consensus Reaching Process (CRP), which introduces an additional phase in the resolution process for GDM problems, aimed at seeking such an agreement before making a decision [10, 77].

The RAE[1] defines the term *consensus* as an agreement produced by mutual consent between all members in a group or between several groups. In [77], Saint et al. defined consensus as "a state of mutual agreement between members of a group, where all legitimate concerns of individuals have been addressed to the satisfaction of the group". These definitions assume the idea of a GDM process after which no experts disagree with the decision made, although some of them may still consider

[1] RAE: Royal Language Academy (Spain): http://rae.es

that their preferred solution would be better than the group solution. In order to achieve consensus, it is necessary that *all* experts modify their initial opinions, bringing them closer to each other, towards a collective opinion viewed as satisfactory by the group.

The concept of consensus may cause some controversy, since it can be interpreted in different ways, from a classical view of consensus as total agreement (*unanimity*) to more flexible interpretations. Consensus as unanimity [51] is normally very difficult or even impossible to achieve in practice, or it might have been achieved by intimidation or other external circumstances imposed on the group, so that no true agreement is really made (*normative consensus*) [86]. Consensus should not be understood as unanimous agreement but rather as the result of a discussion process in which the final decision made may not be in accordance with the initial positions of experts. This view of consensus is known as *cognitive consensus*, and it implies that experts modify their initial opinions after several rounds of discussion and negotiation [57]. Based on this idea, a number of more feasible and flexible approaches of consensus that consider different degrees of partial agreement, have been proposed in the literature [10,32,42]. One of the most accepted approaches to soften the classic view of consensus as unanimity, is the one so-called *soft consensus*, proposed by Kacprzyk in [40]. This approach is based on the concept of fuzzy linguistic majority. Such a concept states that there exists consensus in a group when "most of the important individuals agree as to (their testimonies concerning) almost all of the relevant options" [41, 42]. The concepts of *soft consensus* and fuzzy majority are based on fuzzy sets theory [99] and fuzzy linguistic quantifiers [98]. This approach has provided satisfactory results in many different GDM frameworks [42,43,46].

The main goal of CRPs consists in reaching a desired level of agreement before applying the alternatives selection process, after one or several rounds of discussion on preferences [77]. Therefore, a CRP is an iterative and dynamic process, which is usually coordinated by human figure known as *moderator*. The moderator is a key figure in such processes, whose main functions are [57]:

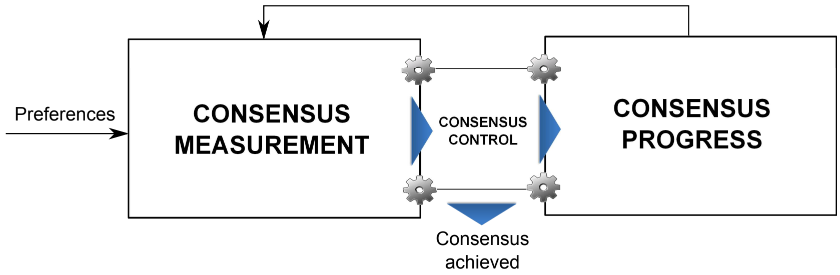

Figura A.3: General CRP scheme

- Evaluate the level of existing agreement at each discussion round.
- Identify the alternatives that hamper achieving consensus.
- Inform experts about the changes they should consider on their preferences, regarding the alternatives identified.

Before initiating a CRP, it is important that all experts understand and accept some a priori assumptions [57]:

- Every member of the group *must* understand the process carried out to achieve consensus, clarifying any possible questions or doubts before beginning it.
- Conducting a CRP implies that all experts *accept* to collaborate with each other, in order to search for a common agreed solution.
- If required, experts *should move* from their initial positions, in order to bring their preferences closer to the rest of the group.

Figure A.3 shows a general scheme followed by most existing approaches in the literature to conduct CRPs. Its main phases are described in detail below:

(1) *Consensus measurement*: Preferences of all experts over X, P_i, $i \in \{1, \ldots, m\}$, are gathered to compute the current degree of consensus in the group by means of a consensus measure, which determines how close the opinions of experts are from unanimous agreement. Further detail about consensus measures will be given later in this section.

(2) *Consensus control*: The consensus degree computed in the previous phase is checked to decide whether it is enough or not. If consensus is enough, the group moves on to the selection process. Otherwise, it is necessary to carry out another round of discussion. Two parameters, whose values are fixed a priori by the group, could be utilized in this phase:

- A consensus threshold μ, whose value indicates the minimum level of agreement required amongst members in the group. Many consensus models compute the consensus degree as a value in the unit interval [38,43,60,68], being a value of 1 interpreted as full and unanimous agreement, therefore $\mu \in [0,1]$ in such cases.
- A maximum number of discussion rounds allowed, $Maxround \in \mathbb{N}$. If the number of rounds carried out exceeds this value, then the CRP ends without having reached consensus.

(3) *Consensus Progress*: If the current degree of consensus is not enough, a procedure is applied to increase the level of agreement in the following round of the CRP. Such a procedure has been traditionally based on providing experts with some feedback, which indicates them how to modify their preferences, but some approaches that conduct this process automatically have been also proposed:

(a) *Feedback Generation*: This is the usual process carried out in classical CRPs, in which human experts discuss about their preferences, guided by a moderator. The moderator identifies the farthest experts' assessments from consensus in the current round. He/she then provides experts with some advices to modify the value of assessments previously identified, in order to bring them closer to the rest of the group and increase the consensus degree in the following round. Many existing consensus models incorporate feedback mechanisms based on this process [9,12,38,60]. Figure A.4 illustrates a general scheme for CRPs with feedback generation.

(b) *Automatic Updates*: Some other proposed consensus models do not incorporate a feedback mechanism, and instead they implement approaches

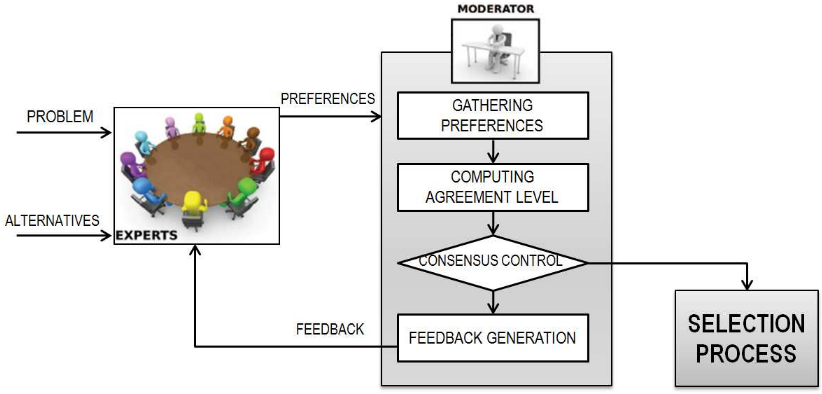

Figura A.4: General scheme of CRPs based on feedback mechanism

that update information (e.g. assessments of experts) to increase consensus in the group automatically [4, 28, 89, 90, 92, 104]. Therefore, once experts provide their initial preferences at the beginning of the CRP, they do not need to supervise them at each round.

As a result of a thorough research on consensus within the field of GDM during the last few decades, a large number of approaches have been proposed by different authors. Such results include:

i) Consensus measures [7,15,21,32,34,42,44,48,78,82], i.e. measures to compute the level of group agreement from individual preferences of experts. Consensus measures are frequently based on applying similarity or distance metrics to compute the closeness between experts' preferences, as well as aggregation operators that obtain the global level of group agreement from the aggregation of similarity degrees previously calculated [1, 2].

ii) Consensus models [9,33,37–39,47,60,68,69,77,91,92], that provide groups with the necessary guidelines to support them in CRPs, in GDM problems defined in different frameworks. A wide variety of consensus models have been proposed by several researchers to support CRPs in different GDM contexts, such as: (i) decision environments with a high degree of uncertainty that require the use of linguistic information domains suitable to express preferences [15,33,56,74,75],

(ii) GDM problems in which alternatives must be assessed taking into account several criteria [69, 92], (iii) environments in which experts would need to use different preference structures depending on their level of expertise [38], etc.

iii) Consensus Support Systems (CSS) [12, 16, 19, 45, 46, 67, 103], i.e. computer-based decision support systems to carry out CRPs, based on the implementation of different consensus models. Some of the usual advantages of using CSS are the automation of tasks carried out by the human moderator, and the possibility of conducting non-physical meetings, with the aid of appropriate means to do so, e.g. Web technologies.

A.4.3. A Taxonomy of Existing Consensus Models for GDM in a Fuzzy Context

This section aims at providing a broad view on the background and basic concepts of consensus in GDM, that must be taken into account to facilitate a better understanding of the rest of proposals in this research. Such a background includes a review of a large number of consensus models proposed in the literature, and it can be found in one of the papers attached in this research memory (Chapter 4).

Consensus has become a prominent topic of research in the field of GDM: a large number of consensus models to support CRPs have been proposed by several researchers in the last decades. Given this wide variety of existing models with different features, and the necessity of a framework of reference to categorize them, a taxonomy of consensus models for GDM problems in a fuzzy context is proposed. Such a taxonomy overviews a number of consensus models and classifies them into several categories, based on their main characteristics.

The article associated to this proposal is (see Section 4.1):

- I. Palomares, F.J. Estrella, L. Martínez, F. Herrera, *Consensus under a Fuzzy Context: Taxonomy, Analysis Framework AFRYCA and Experimental Case of Study*. Information Fusion, submitted (2014).

A.5. Discussion of Results

This section presents a summary of the main proposals considered in this research memory, and presents a brief discussion about the research findings and results obtained from each of them. The results obtained in this research are organized into two main proposals, which are in turn subdivided into several parts.

1. *Automated and Proactive Management of Consensus Reaching Processes in Large-Scale GDM.* This proposal is subdivided into the following two parts:
   (a) *Agent Semi-Supervised Autonomy Model in a Consensus Support System based on the Multi-Agent System Paradigm.*
   (b) *Management of Non-cooperative Behaviors in Consensus Reaching Processes with Large Groups.*
2. *Management of Group Attitudes towards Consensus Reaching.* This proposal is subdivided into the following two parts:
   (a) *Consensus Measure based on an Operator that allows Reflecting the Group's Attitude.*
   (b) *Integration of Group's Attitude towards Consensus in a Web-based Consensus Support System for GDM with Heterogeneous Information.*

A.5.1. Automated and Proactive Management of Consensus Reaching Processes in Large-Scale GDM

In this proposal, the main difficulties found in current consensus models and CSS when dealing with large groups of experts are discussed. Some of these difficulties are: (i) the necessity of constant supervision by human experts to revise and modify their preferences, which might cause an excessive amount of time invested in the CRP, and (ii) the presence of experts or subgroups of them with similar interests, whose behavior does not contribute to achieve an agreement in the group, because they are reluctant to move from their initial positions towards the rest of the group. In order to overcome such difficulties, we propose the following two approaches:

- An agent-based semi-supervised autonomy model that minimizes the amount of human expert supervision required in CRPs, and its integration into a multi-agent based CSS.
- A consensus model and graphical monitoring tool of experts' preferences, to facilitate the detection and management of non-cooperative behaviors in CRPs with large groups.

A.5.1.1. Agent Semi-Supervised Autonomy Model in a Consensus Support System based on the Multi-Agent System Paradigm

In this approach, the existing proposals of consensus models and CSS in the literature are briefly revised. The current achievements of these works to support CRPs in GDM problems with small groups are pointed out [5, 60, 69, 103], and the limitations and weaknesses that such works present for dealing with large-scale GDM problems are also remarked. One of these difficulties is the need for CSSs based on distributed and highly scalable architectures which are capable of managing a large amount of information about the problem efficiently. Another shortcoming is the necessity of constant supervision of preferences by human experts throughout the CRP. Such human supervision requires investing a substantial amount of time in revising and modifying preferences, therefore it may often cause an excessive time cost, or even the loss of motivation and interest of experts on the problem [57].

In order to overcome the above mentioned difficulties, it is proposed a novel semi-supervised CSS based on a multi-agent architecture, aimed at supporting CRPs in real-life large-scale GDM problems. The multi-agent system paradigm is characterized by its high scalability and distributed computing capabilities [87, 88], which facilitates managing large amounts of information about preferences in large-scale GDM problems computationally [64], hence the choice of multi-agent technology for the CSS architecture. The system incorporates a set of software agents with different roles, which are responsible for carrying out autonomously the tasks classically assumed by the human moderator of CRPs. Moreover, each human expert can delegate his/her tasks of supervising preferences into a software agent so-called expert agent, which acts on behalf of its corresponding human expert, thus automating his/her tasks to a high degree. Agents communicate with each other by means of two ontologies, exchanging information about the problem expressed under a common language and semantics [46, 80].

The main novelty of the proposed system is an agent semi-supervised autonomy approach, that minimizes the human supervision of experts to revise and modify their preferences. Such human supervision is not eliminated completely, because in some circumstances in which the system suggests critical changes on experts' assessments, it would be advisable that the human expert revises such changes and decides whether applying them or not, instead of letting an expert agent apply changes autonomously. Thus, the sovereignty of experts' is preserved to some degree, unlike it occurs in some proposals of automatic consensus models in the literature, in which such sovereignty is eliminated [90, 92]. The semi-supervised approach consists of two components: (i) a set of *change profiles* that implement different patterns adopted by expert agents to apply changes on assessments, inspired by agent-based negotiation models such as *Kasbah* [14]; and (ii) a set of *supervision rules*, that analyze the advice generated on assessments of experts to determine in which cases the software agent should request human supervision. The semi-supervised approach is integrated with a consensus model for GDM problems based on fuzzy preference relations.

An experimental case of study is conducted to show the achievements made by using the semi-supervised CSS. To do so, a large-scale GDM problem is solved twice, using the proposed semi-supervised system and another version of the system that requires full human expert supervision. By comparing the results obtained with both systems, it is demonstrated that, both the amount of supervision required, and the number of experts who needed to revise their assessments at each round of the CRP, are significantly reduced with the proposed system. Moreover, the semi-supervised CSS contributes to improve the convergence towards consensus, given by the number of necessary rounds of discussion to reach an agreement.

The article associated to this part is (Section 4.2):

- I. Palomares, L. Martínez, *A Semi-Supervised Multi-Agent System Model to support Consensus Reaching Processes*. IEEE Transactions on Fuzzy Systems, in press (2014). DOI:10.1109/TFUZZ.2013.2272588.

A.5.1.2. Management of Non-cooperative Behaviors in Consensus Reaching Processes with Large Groups

As previously stated, reaching consensus implies that experts must discuss and modify their initial preferences, moving their opinions closer to each other, towards a collective solution which satisfies the whole group [77]. In this part, it is studied the problem of dealing with experts who present a non-cooperative behavior in CRPs, because they are reluctant to modifying their initial positions and making them closer to the rest of experts in the group. The presence of individuals - or subgroups of them - whose behavior does not contribute to the achievement of consensus, is particularly frequent in large-scale GDM problems: in large groups, it is more common to find subgroups or *coalitions* of experts with similar interests. Some of these coalitions may decide not modify their preferences, or even they might modify them against the rest of experts' positions coordinately, with the aim of introducing a bias in the collective opinion in their favor [65, 96]. These behaviors would affect the CRP performance negatively, since they might difficult achieving a collective agreement significantly.

Given the necessity of detecting non-cooperative behaviors and acting accordingly to guarantee that the CRP performance is not affected by them, a methodology to detect and manage such behaviors is proposed and integrated into a consensus model for large-scale GDM. In such a model, experts have assigned importance weights, as considered in several GDM and consensus approaches [54, 69, 92, 96].

The methodology proposed to deal with non-cooperative behaviors is applied at each round of the CRP, and it utilizes a fuzzy clustering[2] method [18,70] based on the Fuzzy C-Means algorithm [6], to classify experts into different subgroups, according to the similarities amongst their preferences. Some rules based on cluster analysis and fuzzy logic [99] are then defined and applied to detect individual and subgroup non-cooperative behaviors. Once non cooperating experts have been detected, a scheme is applied to update their importance weights, in accordance with their behavior.

The consensus model is implemented and utilized to carry out an illustrative example of its usefulness in practice. The results show that detecting and managing non-cooperative behaviors by means of the proposed consensus model, improves the CRP convergence and may help to obtaining a solution which is better accepted by the group.

A visual analysis of the CRP would also be desirable to facilitate the detection of non cooperating experts and their positions with respect to the rest of the group. Classical CRPs in which few experts take part, have been usually monitored by means of supporting tools based on textual or numerical information [9,103]. However, the large amount of information utilized in large-scale GDM problems makes it necessary the use of new tools capable of providing more easily interpretable information about the state of the CRP at each round [66]. In order to overcome this limitation, a graphical monitoring tool so-called MENTOR, for visualizing experts' preferences in large-scale GDM problems, is also proposed. MENTOR is based on Self-Organizing Maps, an unsupervised learning technique widely used for data visualization purposes, characterized by projecting high dimensional data (e.g. fuzzy preference relations of experts) onto a low-dimensional space [53]. The monitoring

[2] *Clustering* and *fuzzy clustering* are unsupervised learning techniques aimed at classifying data into groups of them, based on their similarity. The latter ones are characterized by incorporating the use of fuzzy logic in the classification process [18].

tool can facilitate the detection of different aspects of interest in CRPs, such as: disagreement positions amongst experts, non cooperating experts, agreement cardinality (i.e. number of experts who strongly agree with each other on their opinions), etc. Therefore, MENTOR constitutes a useful complementary tool for the consensus model proposed above. An example of application of the monitoring tool is presented to show its usefulness, by applying a CRP to solve a large-scale GDM problem in which some subgroups of experts with different behaviors take part.

The articles associated to this part are (Sections 4.3 and 4.4):

- I. Palomares, L. Martínez, F. Herrera, *A consensus model to detect and manage non-cooperative behaviors in large scale group decision making.* IEEE Transactions on Fuzzy Systems, in press (2014). DOI:10.1109/TFUZZ.2013.2262769.
- I. Palomares, L. Martínez, F. Herrera, *MENTOR: A graphical monitoring tool of preferences evolution in large-scale group decision making.* Knowledge-based Systems, in press (2014). DOI:10.1016/j.knosys.2013.07.003.

A.5.2. Management of Group Attitudes towards Consensus Reaching

Besides the previous proposal to deal with non-cooperative behaviors of experts, it is also necessary to take into account the possible existence of experts without a common view about the GDM problem considered: not only the penalization of non cooperating individuals allows us to improve the convergence of the CRP, but also integrating the attitude of the group towards consensus in such a process would optimize it. Therefore, in this proposal the problem of integrating the attitude of experts towards consensus is considered. The proposal is divided into two parts:

- Consensus Measure based on an Operator that allows Reflecting the Group's Attitude.
- Integration of the Group's Attitude towards Consensus in a Web-based Consensus Support System for GDM with Heterogeneous Information.

A.5.2.1. Consensus Measure based on an Operator that allows Reflecting the Group's Attitude

The research in this part is focused on the concept of group's attitude towards consensus, i.e. the importance given by experts to reach consensus, with respect to the importance they give to preserving their own initial preferences. Although it is common that experts adopt different attitudes at each GDM problem in which they participate (e.g. optimistic, pessimistic or indifferent attitudes [63]), classical consensus models have not considered this aspect properly yet. In a large-scale GDM problem, where the existence of subgroups of experts with different views about the problem addressed is more frequent, knowing the attitude of experts towards consensus reaching is particularly convenient before carrying out the CRP.

In order to optimize the CRP adapting it to the group's attitude at each specific problem, it is proposed a method for integrating the group's attitude towards consensus in such a process. To do so, a weighted aggregation operator so-called Attitude-OWA, that extends OWA (*Ordered Weighted Averaging*) operators [25,94], is defined. The Attitude-OWA operator is based on two attitudinal parameters which indicate the group's attitude, and the use of a linguistic quantifier [98] to compute weights upon the values of such parameters. A flexible consensus measure that utilizes the Attitude-OWA operator to aggregate similarity degrees between experts and compute consensus degrees, is then defined. After that, a consensus model for GDM with fuzzy preference relations inspired by the ones proposed in [61,67] is extended, by incorporating the consensus measure previously defined and introducing an initial phase for determining the group's attitude.

The consensus model extended above is implemented in the prototype of multi-agent based CSS proposed in [64,67], to conduct an experimental simulation aimed at illustrating the effect of considering different group's attitudes in the CRP performance. Results of experiments show that the more optimistic the attitude, the greater the convergence towards consensus, due to the flexible behavior that the consensus measure presents depending on the attitude adopted. Finally, some guidelines are given for groups to reflect their attitude in the consensus measures appropriately, depending on their specific needs at each GDM problem.

The article associated to this part is (Section 4.5):

- I. Palomares, J. Liu, Y. Xu, L. Martínez, *Modelling experts' attitudes in group decision making.* Soft Computing, 16:10 (2012) pp. 1755-1766. DOI:10.1007/s00500-012-0859-8.

A.5.2.2. Integration of the Group's Attitude towards Consensus in a Consensus Support System for GDM with Heterogeneous Information

Some additional aspects that may require special attention in large-scale GDM, are: (i) the presence of experts with different profiles, who might prefer to express their preferences by using different information domains, according to their level of expertise or knowledge area [36]; and (ii) the necessity of CSS based on Web technologies, to make ubiquitous CRPs possible when experts are physically separated and they can not organize physical meetings. In this part, such aspects are considered, together with the problem of integrating the group's attitude towards consensus in the CRP (which was previously addressed in Section A.5.2.1).

In order to tackle the aspects described above, it is proposed a consensus model for large-scale GDM in heterogeneous contexts. Its main characteristics are, namely, an approach to manage heterogeneous information provided by experts, and the integration of the group's attitude, by means of a consensus measure based on the Attitude-OWA operator [63]. The method to deal with heterogeneous information [36] consists of unifying preferences expressed in different information domains (numerical, interval-valued and linguistic), into a common domain utilized to carry out the necessary computations in the consensus model. Once the consensus model has been presented, a Web-based CSS that implements such a model is developed. The system automates the human moderator completely, thus eliminating his/her possible subjectivity during the GDM problem, and its Web user interface facilitates ubiquitous CRPs, making physical meetings not necessary anymore. Experts introduce their preferences and receive the necessary feedback to modify them across the CRP through the Web interface.

An implementation of the Web-based CSS is utilized to illustrate its performance, with the resolution of a large-scale GDM problem in which each expert chose his/her preferred information domain to express preferences. The problem is solved multiple times with different settings for the group's attitude, in order to remark the effects of such an attitude in the convergence towards consensus.

The article associated to this part is (Section 4.6):

- I. Palomares, R.M. Rodríguez, L. Martínez, *An attitude-driven Web consensus support system for heterogeneous group decision making.* Expert Systems with Applications, 40:1 (2013) pp. 139-149. DOI:10.1016/j.eswa.2012.07.029.

A.6. Conclusions and Future Works

Finally, this section concludes this research memory, reviewing the main proposals and results obtained, and pointing out some future works.

A.6.1. Conclusions

Group Decision Making problems and *Consensus Reaching Processes* have gained special importance in many real-life application areas (such as engineering, medicine, social sciences and so on), due to the increasing need for making group decisions with a high level of agreement amongst experts in these contexts.

Given the importance of group decision making and consensus in these areas, different researchers have proposed in the literature a variety of models and approaches to support consensus reaching processes in groups. Such approaches have been normally limited to dealing with a low number of experts. However, nowadays environments and technologies that facilitate the participation of large groups in decision processes, such as social media, have caused that the so-called large-scale group decision making problems attain more interest in the last years. Large-scale group decision making problems set out new problems and challenges that the existing consensus approaches could not address yet:

- The necessity of highly scalable architectures, for the development of consensus support systems capable of managing large groups efficiently.
- The excessive time cost invested in the discussion process, due to the high amount of human supervision carried out by experts to revise and modify their preferences.
- The presence of non cooperating individuals or subgroups of them with similar interests, who might difficult reaching consensus or even try to deviate the solution to the group decision making problem in their favor.
- The difficulty to obtain useful knowledge about the current state of the group decision making problem from a large amount of information about experts' preferences, and the need for tools that facilitate the analysis of such information.
- The necessity of knowing the attitude of experts towards the achievement of consensus at each problem, since a large number of experts with different views about such a problem might participate.

Taking into account the challenges described above, the interest of this research has focused on overcoming them, by means of a number of proposals to facilitate consensus reaching processes in large-scale group decision making problems. These proposals are listed below:

1. A multi-agent based consensus support system for large-scale group decision making has been developed. The system is based on a multi-agent architecture

that allows a high scalability to support large groups efficiently, and it also incorporates a novel agent-based semi-supervised autonomy approach, aimed at minimizing the human supervision of experts to revise and modify their preferences throughout discussion, thus reducing the cost invested in reaching an agreement.

2. An approach to manage non-cooperative behaviors in consensus reaching processes has been proposed and integrated into a consensus model. Such an approach is based on fuzzy clustering techniques and fuzzy logic, and it facilitates the detection and management of non cooperating subgroups and individuals. As a result, consensus reaching processes in which these behaviors are present, have been improved by increasing the convergence towards agreement.

3. A graphical monitoring tool of preferences that facilitates a visual analysis of non-cooperative behaviors and other aspects of interest in the consensus reaching process, such as positions of agreement or disagreement amongst experts, has been also presented. The tool provides a visual representation of experts' preferences based on the use of self-organizing maps, thus facilitating the obtention of useful knowledge about the current state of the problem from easy interpretable information.

4. A novel consensus measure based on an aggregation operator that integrates the group's attitude towards consensus, has been defined and integrated in a consensus model. Moreover, a Web-based consensus support system for heterogeneous group decision making, that incorporates this consensus measure to reflect the group's attitude in the consensus reaching process, has been developed. The consensus measure allows to optimize the consensus reaching process, adapting it to the attitude adopted by the group at each problem.

Additionally, a theoretical study of consensus models present in the literature for group decision making under a fuzzy context, has been conducted. As a result, a taxonomy that categorizes and overviews the main characteristics of a large number of consensus models, grouping together those models with similar features, has been presented. In addition, a software framework based on simulation for the practical study and evaluation of existing consensus models, has been developed.

As can be seen, all the objectives pursued at the beginning of this research have been successfully achieved with the proposals presented in the research memory.

A.6.2. Future Works

Despite several proposals have been made in this research, there are still a large number of challenges in the research field of consensus and group decision making, some of which are related to large-scale group decision making problems:

1. Further research on current proposals for their extension.

    - Definition of improved and flexible mechanisms to manage different patterns of behavior adopted by each expert throughout the consensus reaching process.
    - Study of new methods based on the analysis of preferences in large groups of experts, for detecting subgroups of experts with similar or conflicting opinions, and the extension of such methods to different preference structures and information domains to express preferences.

2. Important challenges and future directions for the improvement of consensus reaching processes in large-scale group decision making problems.

    - Development of a software framework that allows us to automate different existing consensus models, in order to facilitate a study and analysis of them in practice.
    - Definition of new metrics for evaluating the performance of consensus reaching processes. Implementation of such metrics in the analysis framework proposed above, with the purpose of enabling the comparison between different proposals of consensus models.

- Application of the consensus models proposed in this research to e-commerce environments, by integrating them with group recommender systems or online group shopping platforms.

A.6.3. Additional Publications and Awards Received

Regarding the diffusion of our scientific results, besides the publications included in this memory, we highlight the following contributions:

- In International Conferences

  - I. Palomares, F.J. Quesada, L. Martínez, *Multi-agent based Semi-supervised Consensus Support System for Large-Scale Group Decision Making.* 8th International Conference on Intelligent Systems and Knowledge Engineering. ISKE 2013. Proceedings currently in press. Springer Berlin/ Heidelberg, 2013.
  - I. Palomares, L. Martínez, F. Herrera, *A Fuzzy Clustering Approach for Non-cooperative Behavior Detection in Consensus Reaching Processes.* 4th International Workshop on Knowledge Discovery, Knowledge Management and Decision Support. EUREKA 2013. pp. 37-44. Atlantis Press, 2013.
  - R.M. Rodríguez, I. Palomares, L. Martínez, *Attitude-based Consensus Model for Heterogeneous Group Decision Making.* 7th International Conference on Intelligent Systems and Knowledge Engineering. ISKE 2012. Advances in Intelligent and Soft Computing, vol. 214, pp. 279-288. Springer Berlin/Heidelberg, 2012.
  - I. Palomares, J. Liu, Y. Xu, L. Martínez, *Using OWA Operators to Integrate Group Attitudes towards Consensus.* 6th International Conference on Intelligent Systems and Knowledge Engineering. ISKE 2011. Advances in Intelligent and Soft Computing, vol. 123, pp. 273-282. Springer Berlin/Heidelberg, 2011.

- I. Palomares, P.J. Sánchez, F.J. Quesada, F. Mata, L. Martínez, *COMAS: A Multi-agent System for performing Consensus Processes.* International Symposium on Distributed Computing and Artificial Intelligence. DCAI 2011. Advances in Intelligent and Soft Computing, vol. 91, pp. 125-132. Springer Berlin/Heidelberg, 2011.

- In National Conferences

  - I. Palomares, L. Martínez, *Attitude-Driven Web Consensus Support System for Large-scale Group Decision Making Problems based on Fuzzy Linguistic Approach.* 15th Conference of the Spanish Association for Artificial Intelligence. CAEPIA 2013. Lecture Notes in Artificial Intelligence, vol. 8109, pp. 91-100. Springer Berlin/Heidelberg, 2013.
  - I. Palomares, J. Liu, Y. Xu, L. Martínez, *Uso de Operadores OWA para modelar la Actitud hacia el Consenso en Problemas de Toma de Decisión en Grupo.* Actas del XVI Congreso Español sobre Tecnologías y Lógica Fuzzy. ESTYLF 2012. pp. 211-216, 2012.

- Book Chapters

  - I. Palomares, L. Martínez, *Attitude-based Consensus Model for Heterogeneous Multi-criteria Large-Scale Group Decision Making: Application to IT-based Services Management.* Engineering and Management of IT-based Service Systems. Intelligent Systems Reference Library, vol. 55, pp. 155-177. Springer Berlin/Heidelberg, 2014.

Moreover, at a earlier stage in this research, a proposal for this Ph.D. Thesis was presented at a Doctoral Consortium session in which junior researchers in the field of Artificial Intelligence participated. The event was held as part of the XV National Conference of the Spanish Association for Artificial Intelligence (CAEPIA), within the IV Spanish Congress of Computer Science (CEDI 2013) in September 17-20th, 2013. As a result, this thesis proposal received a *First National Award to the Best Pre-Doctoral Research Proposal in the Field of Artificial Intelligence.* The diploma of such award is attached in the following page.

ASOCIACIÓN ESPAÑOLA PARA LA INTELIGENCIA ARTIFICIAL

(AEPIA)

DOCTORAL CONSORTIUM

Multiconferencia CAEPIA 2013

Han concedido a

Iván Palomares Carrascosa

el primer premio (ex aequo) del Doctoral Consortium celebrado en la Multiconferencia CAEPIA 2013 a la mejor propuesta de investigación por el trabajo predoctoral titulado:

"Sistema multiagente para modelar procesos de consenso en toma de decisión en grupo a gran escala"

Madrid, 20 de septiembre de 2013

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