A Comparative Performance Analysis of Consensus Models Based on a Minimum Cost Metric

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Abstract. Consensus reaching processes (CRPs) are key in the resolution of many group decision making problems, since they guarantee a solution in which most of decision makers agree. For this reason, a great number of consensus models have been proposed in the specialized literature, being it difficult to make a proper comparison among all of them and determine which model best fits a given problem. Recently, a new cost metric based on comprehensive minimum cost consensus models has been proposed, which compares the solution obtained by a consensus model with the agreed one with minimum cost. Therefore, this contribution aims to carry out a reliable comparative analysis of the performance of several consensus models by using the cost metric in order to identify their strong points and drawbacks.

Keywords: Consensus reaching process · Minimum cost consensus · Cost metric

1 Introduction

Nowadays, it is pretty common to find decision making problems in which several decision makers with different points of view provide their opinions with the aim of obtaining a common solution, this kind of problems are called, group decision making (GDM) problems. Traditionally, a GDM problem is solved by a selection process [2] ignoring the agreement among decision makers. Consequently, decision makers could feel that their preferences were not considered to obtain the best alternative as solution of the problem [1]. To overcome this situation, a consensus reaching process (CRP) is included in the resolution scheme of a GDM problem. A CRP is an iterative process in which decision makers discuss and modify their initial preferences to achieve an agreement and an acceptable solution for all of them. There are different consensus models in the literature [3,4,8] and they can be classified according to the taxonomy introduced in [8]:

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- Consensus with feedback and without feedback
- Consensus measures based on distances to the collective opinions and based on distances among decision makers.

Usually, authors propose new consensus models assuring that their models provide better results in comparison with others, but they do not use any metric to support that statement. The first attempt to analyse and compare the performance of different CRPs was introduced in AFRYCA [5,8]. AFRYCA is a tool that implements the most widely used CRPs and considers several measures to evaluate CRPs such as, number of rounds necessary to reach the consensus, the number of changes carried out across the CRP and several consistency measures. Nevertheless, these criteria are quite simple and cannot objectively measure the performance of different consensus models. To cope this challenge, recently Labella et al. [4] have proposed a metric based on the cost of modifying decision makers' preferences. It uses a comprehensive minimum cost model to compute the optimal consensus solution, the one that involves the least possible changes in decision makers' preferences, and compare it with the one obtained by the analyzed consensus model. Therefore, the aim of this contribution is to analyze the performance of several consensus models by means of the cost metric and point out their advantages and disadvantages. To do so, we will use AFRYCA that implements some well-known CRPs and the cost metric mentioned.

The structure of this contribution is as follows: Sect. 2 revises some basic concepts about GDM, CRP and the cost metric. Section 3 shows the comparative analysis among the consensus models by using AFRYCA. Finally, Sect. 4 points out the conclusions and future works.

2 Preliminaries

This section revises some preliminary concepts about GDM, CRP and a cost metric used to evaluate the performance of the consensus models.

2.1 Group Decision Making and Consensus Reaching Process

Usually, there are several decision makers with different points of view and knowledge involved in decision making problems with the aim of obtaining a common solution, this leads to GDM [6]. In a GDM problem a set of decision makers $E = \{e_1, \ldots, e_m\}$ provide their preferences over a set of alternatives $X = \{x_1, \ldots, x_n\}$ by means of a fuzzy preference relation (FPR), because it is simple and easy to build.

Definition 1. *[7] A FPR P^k, associated to a decision maker e_k on a set of alternatives X, is a fuzzy set on $X \times X$, characterized by the membership function $\mu_{P^k} : X \times X \rightarrow [0, 1]$.*

$$P^k = \begin{pmatrix} p^k_{11} & \cdots & p^k_{1n} \\ \vdots & \ddots & \vdots \\ p^k_{n1} & \cdots & p^k_{nn} \end{pmatrix},$$

where each assessment p_{ij}^k *represents the degree of preference of the alternative* x_i *over* x_j *according to decision* e_k*. The FPR is usually assumed to be reciprocal, i.e.,* $p_{ij}^k + p_{ji}^k = 1,\ \forall i, j = 1, 2, \ldots, n, k = 1, 2, \ldots, m$.

Traditionally, a GDM problem is solved by a selection process which is divided into two phases: (i) aggregation in which decision makers' preferences are fused to get a collective opinion, and (ii) exploitation in which the best alternative is selected according to the collective opinion. Nevertheless, this process does not take into account the agreement among decision makers which implies that some of them could feel that their preferences were not considered. In real world problems, disagreement among decision makers is inevitable, for this reason, it is important to remove the disagreement among decision makers and obtain solutions more appreciated and accepted by the group. To do so, a CRP is included in the solving scheme of GDM problems. A CRP is a dynamic and iterative process in which decision makers discuss and modify their preferences to reach a collective opinion which is accepted by the whole group. A general scheme of a CRP is explained below:

- Consensus measurement: the consensus level is calculated by consensus measures based on aggregation operators and distance measures.
- Consensus control: the consensus level obtained is compared with the consensus threshold $\mu \in [0, 1]$ set a priori. If the consensus level is greater than the threshold, a selection process is applied, otherwise, another round is necessary.
- Consensus progress: the decision makers who are far away from the collective opinion are identified and some advises are generated to change their preferences and increase the consensus level in the next round.

The computation of the consensus level is very important. According to Palomares et al. in [8] the consensus measures can be classified in two types:

- Consensus measure based on the distance of each decision maker to the collective opinion.
- Consensus measure based on the distances among decision makers.

2.2 Metric Based on Minimum Cost

There are many consensus models [3,4,8], however, when a new consensus model is proposed, authors affirm that it is better than previous ones, but there is not any measure that analyzes its performance in comparison with others to know which one is better. A first attempt was introduced in AFRYCA [5,8], which considers several criteria to evaluate the performance such as, the number of rounds necessary to achieve the consensus, the number of changes carried out across the consensus process, and some consistency measures. Nevertheless, these measures are quite simple and cannot compare consensus models in a proper way. Recently, Labella et al. [4] have introduced a metric based on the cost of changing decision makers' preferences to evaluate CRPs. This metric uses an optimal solution obtained of a comprehensive minimum cost consensus model defined as well.

Two MCC models were introduced depending on the consensus measure used.

Let P^k be the FPR provided by a decision maker, and it is adjusted to $\overline{P}^k = (\overline{p}_{ij}^k)_{n\times n}, k = 1,\ldots,m$, to reach a solution accepted by the group, and $\overline{P} = (\overline{p}_{ij})_{n\times n}$ is the adjusted FPR of the collective FPR.

- Consensus measure based on the distance between decision makers' preferences and the collective opinion.

$$(\mathbf{M-1})$$

$$\min \sum_{k=1}^{m}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} c_k|p_{ij}^k - \overline{p}_{ij}^k|$$

$$s.t. \begin{cases} \overline{p}_{ij} = \sum_{k=1}^{m} w_k \overline{p}_{ij}^k \\ |\overline{p}_{ij}^k - \overline{p}_{ij}| \le \varepsilon, k = 1,\ldots,m, i = 1,\ldots,n-1, j = i+1,\ldots,n \\ \frac{2}{n(n-1)}\sum_{k=1}^{m}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} w_k|\overline{p}_{ij}^k - \overline{p}_{ij}| \le \gamma. \end{cases}$$

  where $(c_1,\ldots,c_m)$ are the cost of moving each decision maker' opinion 1 unit, $\varepsilon \in [0,1]$ is the maximum acceptable distance of each decision maker to the collective opinion, $\gamma = 1-\mu$, being μ the consensus threshold and $w_i \in [0,1]$ is the decision maker' weight, $\sum_{i=0}^{m} = 1$.
- Consensus measure based on the distance among decision makers.

$$(\mathbf{M-2})$$

$$\min \sum_{k=1}^{m}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} c_k|p_{ij}^k - \overline{p}_{ij}^k|$$

$$s.t. \begin{cases} \overline{p}_{ij} = \sum_{k=1}^{m} w_k \overline{p}_{ij}^k \\ |\overline{p}_{ij}^k - \overline{p}_{ij}| \le \varepsilon, k = 1,\ldots,m, i = 1,\ldots,n-1, j = i+1,\ldots,n \\ \frac{2}{n(n-1)}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\sum_{k=1}^{m-1}\sum_{l=k+1}^{m} \frac{w_k+w_l}{m-1}|\overline{p}_{ij}^k - \overline{p}_{ij}^l| \le \gamma. \end{cases}$$

The metric computes the difference in cost between the MCC model (M−1 or M−2) and the solution obtained with the consensus models to evaluate.

Let $P = (P^1,\ldots,P^m)$ be the initial decision makers' preferences, and $\overline{P} = (\overline{P}^1,\ldots,\overline{P}^m)$ the optimal adjusted FPRs of the MCC model (M−1) or (M−2), where P^k and $\overline{P}^k$ are the initial and adjusted FPRs of the decision maker e_k, $k = 1,2,\ldots,m$, respectively. The distance between P^k and $\overline{P}^k$ is calculated as

$$d\left(P^k,\overline{P}^k\right) = \frac{2}{n(n-1)}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\left|p_{ij}^k - \overline{p}_{ij}^k\right|, k = 1,\ldots,m. \quad (1)$$

and the relative distance between P and $\overline{P}$, is defined as follows:

$$D\left(P,\overline{P}\right) = \sum_{k=1}^{m} d\left(P^k,\overline{P}^k\right). \quad (2)$$

In similar manner, let $\widehat{P} = (\widehat{P}^1, \widehat{P}^2, \ldots, \widehat{P}^m)$ be the agreed solution obtained in a consensus model, the distance between P^k and $\widehat{P}^k$ is calculated as

$$d\left(P^k, \widehat{P}^k\right) = \frac{2}{n(n-1)} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \left|p_{ij}^k - \widehat{p}_{ij}^k\right|, k = 1, \ldots, m. \tag{3}$$

and

$$D\left(P, \widehat{P}\right) = \sum_{k=1}^{m} d\left(P^k, \widehat{P}^k\right). \tag{4}$$

Finally, the cost metric is defined as follows:

$$\phi\left(\widehat{P}, \overline{P}\right) = \begin{cases} 1 - \frac{D\left(P,\widehat{P}\right)}{D\left(P,\overline{P}\right)}, if\ D\left(P, \widehat{P}\right) \leq D\left(P, \overline{P}\right) \\ \frac{D\left(P,\overline{P}\right)}{D\left(P,\widehat{P}\right)} - 1, if\ D\left(P, \widehat{P}\right) > D\left(P, \overline{P}\right). \end{cases} \tag{5}$$

$\phi\left(\widehat{P}, \overline{P}\right) \in [-1, 1]$, when $\phi\left(\widehat{P}, \overline{P}\right) < 0$ means there is excessive changes in decision makers' preferences, on the contrary $\phi\left(\widehat{P}, \overline{P}\right) > 0$ means decision makers' preferences can be closer each other. If $\widehat{P} = \overline{P}$, the CRP solution is the best.

3 Comparative Analysis

This section shows a comparative analysis among several CRPs by using the cost metric proposed by Labella et al. [4] together other metrics. Three CRPs have been selected, Palomares et al. [9], Quesada et al. [10] and Rodríguez et al. [11]. Such a selection is based on two reasons:

- The similarity between them in terms of consensus computation, since all of them use the consensus measure based on the distance among decision makers that facilitates to carry out a fair comparative analysis
- The consensus models present strengths and drawbacks in different aspects in their performances, which implies a deep analysis that cannot be carried out with simple metrics.

3.1 Description of the Problem and Parameters

The previous CRPs are used to solve a GDM problem composed by 30 decision makers, $E = \{e_1, \ldots, e_{30}\}$, who provide their preferences by means of FPRs, over 3 alternatives $X = \{x_1, x_2, x_3\}$. The consensus threshold is predefined as $\mu = 0.85$ and the maximum numbers of rounds to reach consensus is $h = 15$. The parameter ε related to the comprehensive MCC models is $\varepsilon = 0.1$. Finally, each consensus model uses different parameters to simulate the CRP. For sake of space, the definition of the parameters is not introduced here, but their values are represented in Table 1 (see [9,10] and [11] for further details).

Table 1. CRPs' parameters value.

Palomares et al. [9]	Quesada et al. [10]	Rodríguez et al. [11]
$\varepsilon^* = 0.05$	$\varepsilon^* = 0.05$	$\delta = 0.7$
$\alpha = 0.2$	$\alpha = 0.2$	$a = 3$
$\beta = 0.6$	$\beta = 0.6$	$b = 10$
$increment = 0.1$	$increment = 0.1$	$\beta^* = 1.8$
	$h_{start} = 4$	
	$\eta = 0.5$	
	$g = 0.5$	

Remark 1. Note that we keep the notation of the CRPs' parameters used by the authors. For this reason, ε^* and β^* have been represented with the symbol $*$ in Table 1 in order to avoid confusion with other parameters.

Remark 2. For the CRPs simulations, we have considered that all the experts accept always the suggestions provided by the consensus models to guarantee a fair comparison among them.

3.2 Comparison

A comparative analysis about the CRPs performance is carried out in this section. Firstly, the GDM problem is solved with the different consensus models by using AFRYCA. The results obtained are represented in Table 2.

Table 2. CRPs' parameters value.

	Palomares et al. [9]	Quesada et al. [10]	Rodríguez et al. [11]
Number of rounds	6	9	12
Number of changes	143	198	345
Cost metric	0.41	0.46	0.27
Consensus level	0.87	0.86	0.86
Ranking	$x_3 \succ x_1 \succ x_2$	$x_3 \succ x_1 \succ x_2$	$x_3 \succ x_1 \succ x_2$

Table 2 provides relevant information about the CRPs. Regarding the ranking of the alternatives, all of them choose alternative x_3 as the best solution of the problem. However, Palomares et al.'s model needs just 6 rounds to achieve the predefined consensus, being the fastest. The consensus level achieved for each model is quite similar, although the Palomares et al.'s model reaches such a level with a less number of changes in the decision makers' preferences (143). According to the previous issues, it would seem logical to think that Palomares et al. is the best analyzed consensus model because it reaches a higher level of consensus with less changes. On the contrary, Rodríguez et al.'s model might seem the worst, since it needs more rounds and changes than any other model. Nevertheless, the cost metric shows that Rodríguez et al.'s model is much closer to the optimal solution than the other consensus models. This issue is graphically represented in Fig. 1, which shows the evolution of the decision makers' preferences for each CRP and the visualization of the solution with minimum cost.

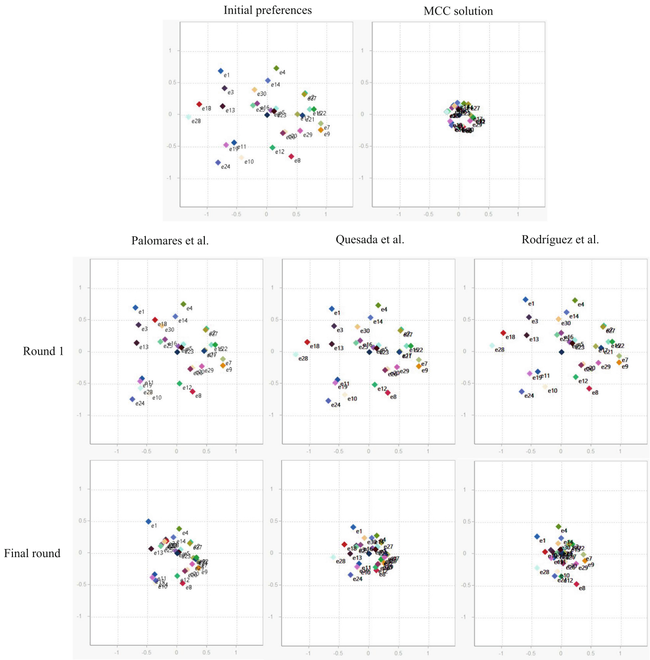

Fig. 1. Decision makers' preferences evolution.

Figure 1 shows the optimal cost solution for the problem on the top of the figure, noted as *MCC solution*, which is the agreed solution provided by model (M−2). If we compare such a solution with the one provided by the Palomares et al.'s consensus model, the decision makers' preferences are dispersed. This means that it achieves the desired consensus because it compensates the decision makers whose opinions are furthest away with others whose opinions are quite similar. Obviously, it implies that the consensus model needs less changes to achieve the consensus, but the solution is not homogeneous. On the other hand, the models of Quesada et al. and Rodríguez et al. present more homogeneous solutions in which decision makers' preferences are closer each other, but Rodríguez et al. model presents clearly the closest one to the optimal solution.

Therefore, although at first sight it might seem that the Palomares et al.'s model is the best one by number of rounds and changes required, it is also necessary to take into account other metrics, such as the cost metric, that allows to measure other characteristics of the performance of consensus models and carried out a more reliable and deeper analysis.

4 Conclusions

The increasing key role of CRPs in GDM problems has resulted in many consensus approaches with their own peculiarities to support such processes. For this reason, it is difficult to make proper comparisons among consensus models to choose the best suitable for a given problem. Several authors use simple metrics to justify that their models are better than other but, often they are not enough to guarantee such a statement in an objective way. Recently, a new cost metric based on comprehensive MCC models has been proposed, which allows to measure properly the performance of consensus models.

This contribution has carried out a comparative performance analysis of several consensus models by using the cost metric in order to show the importance of using proper metrics to compare consensus models and identify their advantages and drawbacks. To do so, AFRYCA that implements some consensus models, has been used to solve the GDM problem. As future research, we intend to optimize the MCC models in order to evaluate the performance of CRPs on large-scale group decision making problems with hundreds of experts.

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