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Fuzzy Tools in Recommender Systems: A Survey

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Abstract

Recommender systems are currently successful solutions for facilitating access for online users to the information that fits their preferences and needs in overloaded search spaces. In the last years several methodologies have been developed to improve their performance. This paper is focused on developing a review on the use of fuzzy tools in recommender systems, for detecting the more common research topics and also the research gaps, in order to suggest future research lines for boosting the current developments in fuzzy-based recommender systems. Specifically, it is developed an analysis of the papers focused at such aim, indexed in Thomson Reuters Web of Science database, in terms of they key features, evaluation strategies, datasets employed, and application areas.

Keywords: recommender systems, user preferences, fuzzy logic, survey

1. Introduction

One of the most used paradigms for implementing personalization processes to provide users with the information resources that best fit their preferences and needs in an overloaded digital world are recommender systems. Even though, they were initially conceived to cover e-commerce domains [106, 113], today they are successfully expanded to diverse scenarios such as e-learning [147], tourism [95], libraries [146], e-government [77], financial investment [89], and other application areas [76].

According to the available information used to generate recommendations, recommender systems can be classified into different recommendation paradigms. Initially, demographic ones were the most important approaches because such information was available, but recently the two main recommendation paradigms are the collaborative filtering-based recommendation [94] and the content-based recommendation [34].

- *Collaborative filtering systems* can generate recommendations only using users' ratings and without the necessity of additional information. In its most basic approach [37, 106], collaborative filtering focuses on suggesting to the target user the items already preferred by other users with similar preference patterns.
- *Content-based recommendation* is focused on the use of additional information beyond users' ratings (such as items' attributes) to characterize items, and therefore suggest the items with similar features to those ones that the user preferred in the past.

Beyond previous paradigms, several authors have referred to other paradigms such as social, knowledge-based or hybrid filtering [21], depending on the technique and information used for the recommendation generation.

A brief analysis of working principles of such paradigms evidences that the users' preferences play a main role in the recommendation generation process. Therefore, since 90s there have been developed a plethora of researches managing the user's preferences and also the additional users' and items' information, to obtain accurate recommendations [21, 41]. Such researches have been successfully supported by foundations taken from related research areas such as user modelling, information retrieval, computational intelligence, or machine learning; and in several cases, their contribution to recommender systems has been built from the scratch.

Our interest is focused on soft computing techniques, mainly fuzzy-based, used in recommender systems [152]. Recent reviews on recommender systems and personalization [21, 41], show relevant approaches for managing uncertainty in recommender systems (see Fig.1) such as bayesian approaches [33], markov models [111], fuzzy approaches [152], genetic algorithms [47], or neural networks [137].

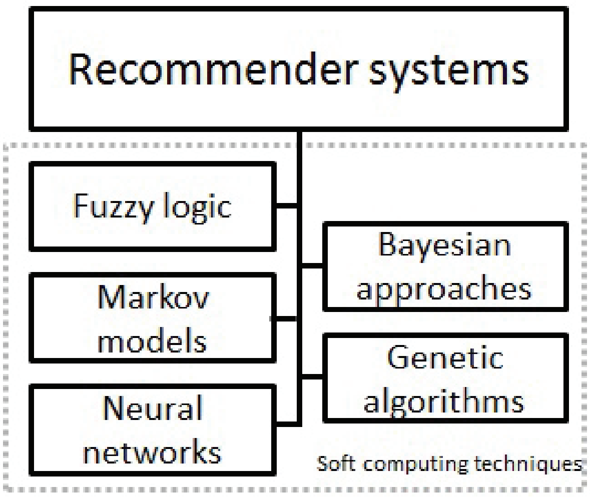

Fig. 1. Soft computing approaches in recommender systems development

In order to evaluate the relevance of such techniques in recommender systems, a search was developed in Thomson Reuters Web of Science (Core Collection) at October 2016. It was focused on finding relevant research combining such techniques for managing uncertainty, with traditional paradigms in recommender systems such as collaborative filtering-based, content-based, or demographic-based. The results obtained are shown in Fig. 5 and they suggest that it is worthy to develop a depth study focused on evaluating the current state-of-art on the use of fuzzy logic tools for improving the performance of recommender systems. Therefore, this paper is devoted to accomplish this aim.

Consequently, it is necessary to mention that even there have been developed several survey papers focused on recommender systems both regarding a wide point of view (Adomavicius and Tuzhilin [3], Konstan and Riedl [65], Bobadilla et al. [21]), and also focused on specific areas (Campos et al. [23], Klašnja-Milićević et al. [63], Abbas et al. [1], Martínez et al. [83]), according to our best knowledge (October 2016), the current paper is the first effort focused on concentrating all the research works focused on recommender systems supported by fuzzy tools.

The paper is structured as follows. Section 2 presents a brief background on recommender systems and fuzzy tools, and includes details related to content-based and collaborative filtering recommendation. Section 3 explains the survey methodology used for obtaining the research works to be considered. The main part of the contribution is developed in Section 4, by analysing the developments on fuzzy tools in recommender systems, grouped by the core approach they are based, according to the typical techniques used for building recommender systems. Section 5 pointed out future research directions for providing continuity to the current developments. Section 6 concludes the paper.

2. Background

This section is focused on presenting the necessary background for the current survey. First, a brief background on recommender systems is presented by focusing on the most widely-developed

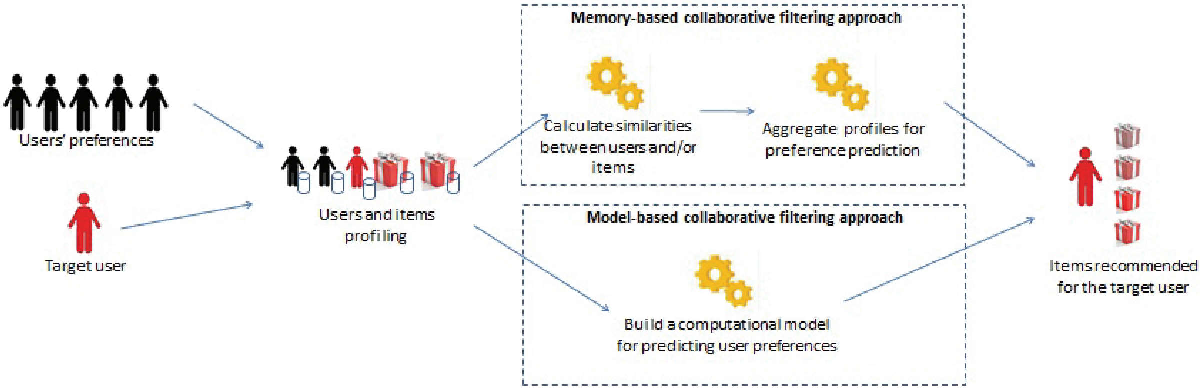

Fig. 2. Collaborative filtering-based recommendation

paradigms, specifically demographic, collaborative, content-based, and hybrid filtering. Afterwards, it is performed a quick reference to fuzzy logic concepts which are used across most of papers included in this survey.

2.1. *Recommender systems*

A recommender system is considered as "any system that produces individualized recommendations as output or has the effect of guiding the user in a personalized way to interesting or useful objects in a large space of possible options" [22]. Specifically, Gunawardana and Shani [44] have pointed out that the two more common tasks related to recommender systems are the prediction of user opinion (e.g., rating) over a set of items, ***the prediction task***, and the recommendation of a set of good (interesting, useful) items to the user, ***the recommendation task***. With such aims in mind, several recommendation approaches have been developed; and depending on their working principles, they have been classified into several categories according to the kind of information managed. One of the most popular classification has been pointed out by Bobadilla et al. [21], which groups them into a) demographic filtering, b) collaborative filtering, c) content-based filtering and d) hybrid filtering. The next subsections present a brief reference to these categories.

- **Demographic filtering** Early recommendation approaches were supported by demographic filtering, which is focused on managing user's attributes for the identification of his/her preferences and the use of such information for the recommendation generation [97]. It is supported by the principle that people with common personal attributes such as sex, age, country, and so on, may have also common preferences. Although this approach could seem simplistic at the first view regarding the current development of personalization technologies, recent works have shown its effectiveness in several recommendation scenarios [156, 157].

- **Collaborative filtering recommender systems** The most popular paradigm for developing recommendation approaches is currently collaborative filtering, which is focused on performing the typical tasks of recommender systems using only users' rating values [3]. Usually, they generate the recommendations for the current user, by exploring the preferences of other related users regarding their degree of similarity. Such an exploration is typically based on their rating patterns. In contrast to content-based recommendation, this approach does not depend on items attributes; therefore it could be used in any recommendation scenario having enough preference values (Fig. 2). Collaborative filtering systems are typically classified into *memory-based* or *model-based* approaches [3]. A comprehensive analysis of such approaches can be found in related review papers such as Su and Khoshgoftaar [120], Ekstrand et al. [37], Bobadilla et al. [21] and Ning et al. [94].

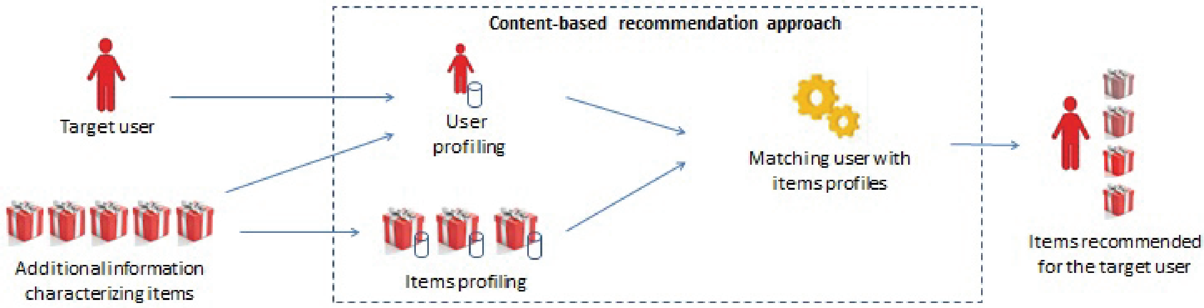

Fig. 3. Content-based recommendations

- **Content-based recommender systems**
  In the last few years, the incremental growing of several platform managing high volumes of information related with user profiles, has increased the relevance of content-based recommendation. Content-based recommender systems take as reference the item's descriptions and a profile with the interests of the active user, for suggesting items similar to those the active user already liked in the past [75]. Basically, they focus on performing a comparison between the user profile and the candidate items, to determine which items will be recommended. Items profiles are usually represented through a set of attributes that can include weights to represent the importance of each one of them [3]. Taking into account the preference degrees of users about items and such attributes values, there are usually proposed computational approaches to *learn* the user profiles in terms of the same attributes. Afterwards, it could be used several matching approaches between users and items for the recommendations generation (Fig. 3). Two relevant surveys on content-based recommendation have been presented by Lops et al. [75] and de Gemmis et al. [34].

- **Hybrid recommender systems**
  Several researches have proposed hybridizations of some of the previous approaches for simultaneously overcoming their limitations. Some popular hybridization approaches are the combination of collaborative and demographic filtering [132], or collaborative and content-based filtering [12]. Specifically, a still-updated survey developed by Burke [22], has pointed out six different techniques for hybridization of recommender systems, which are weighted, mixed, switching, feature combination, feature augmentation, and meta-level.
  Because of the diversity of information sources that are emerging nowadays, an important amount of the recommender systems developed in the last few years could be classified as hybrid systems.

2.2. *Fuzzy logic tools*

Fuzzy logic is focused on modelling some real world concepts which cannot be represented in a precise way. Specifically, the definition of a fuzzy set [151] over a universe of discourse, extends the notion of a set through the introduction of the degree of membership of the elements. It establishes a correspondence between the elements of the universe of discourse X into the interval [0, 1], given by a membership function:

$$\mu_{\widetilde{A}} : X \to [0,1] \tag{1}$$

Based on this membership function, a fuzzy set $\widetilde{A}$ defined over the domain X is represented by the set of pairs of the element x and its membership:

$$\widetilde{A} = \{\ (x, \mu_{\widetilde{A}}(x))\ \ /\ \ x \in X,\ \mu_{\widetilde{A}}(x) \in [0,1]\ \} \tag{2}$$

Let $\widetilde{A}$ be a fuzzy set. The α-cut of $\widetilde{A}$ is defined as

$$\widetilde{A}_\alpha = \{x \in \mathbb{R} \mid \mu_{\widetilde{A}}(x) \geqslant \alpha\}.$$

Also the notions of intersection and union over traditional sets are extended to be defined for fuzzy sets [151, 158], see Eqs. 3) and 4) respectively:

$$\mu_{\widetilde{A}\cap\widetilde{B}}(x) = i[\mu_{\widetilde{A}}(x), \mu_{\widetilde{B}}(x)], \qquad x \in X \quad (3)$$

$$\mu_{\widetilde{A}\cup\widetilde{B}}(x) = u[\mu_{\widetilde{A}}(x), \mu_{\widetilde{B}}(x)], \qquad x \in X, \quad (4)$$

being i and u fuzzy binary operations, usually referred in the literature as t-norms and t-conorms, respectively.

The fuzzy linguistic approach [46, 108] is useful for modelling uncertain and vague preferences in recommender systems by using the concept of linguistic variable [150]. Its use implies the selection of appropriate linguistic descriptors for the term set, and their syntax and semantics. The semantics associated to the syntax are represented by fuzzy membership functions (see Fig. 12). Different linguistic computational models have been introduced, being the Computing with words methodology the most relevant taking into account the use of fuzzy linguistic modeling [81, 82, 107, 110].

The following sections present how these tools have been extensively used for managing the uncertainty associated to recommender systems.

3. Survey methodology and initial results

Recommender systems have been involved in a high volume of research works in recent years. To obtain a relevant and representative sample of such works, we searched on a well-recognized database of high-quality scientific literature, that is Thomson Reuters Web of Science (WoS). Specifically, it was executed a query focused on retrieving the mainstream research in recommender systems (the query was *(”recommender systems” or recommendation) and (collaborative or content-based or demographic)*), which retrieved, at October 2016, exactly 1432 records. Figure 4 presents the temporal distribution of such records, suggesting an important increase of results associated to the last four years, and therefore proving the interest of the research community on recommender systems nowadays.

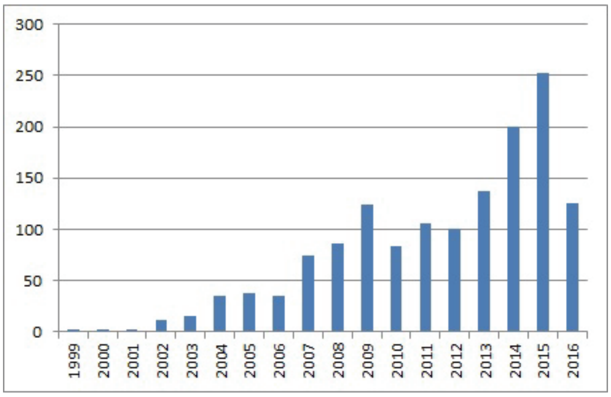

Fig. 4. Temporal distribution of papers on recommender systems.

As it was pointed out in the section 1, our interest is focused on recommender systems supported by soft computing approaches. Therefore, we refine the search results presented in Figure 4 by performing new queries *((”recommender systems” and **approach**) or (recommendation and **approach**)) and (collaborative or content-based or demographic)*, being **approach** respectively replaced by: 1) ”fuzzy”, 2) ”bayesian”, 3) ”markov”, 4) ”genetic algorithm”, and 5) ”neural network”.

Figure 5 shows the amount of results obtained for each query, being relevant the amount of papers that consider the use of fuzzy tools (152 results), over the remaining categories. This list of papers suggests the necessity of evaluating the current state-of-art on the use of fuzzy logic tools for improving the performance of recommender systems.

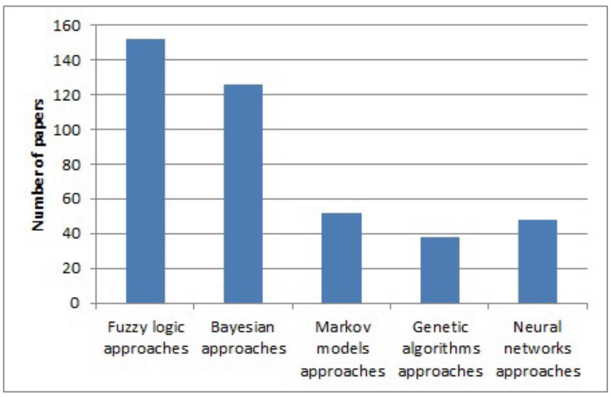

Fig. 5. Search results in Thomson Reuters WoS, for recommender systems, and different soft computing techniques.

Such a list was considered as the preliminary list of papers to be included in our survey, because in it still remains some papers that are not focused on recommender systems. Hence, we developed a manual procedure (Figure 6) to keep in the list those papers related to the recommendation system research field, which also incorporate fuzzy logic approaches in their proposals. In addition, it was also considered the fact that there could be several papers presenting the same research results, and therefore were excluded those papers that contain preliminary results extended later by other research papers already included in the list.

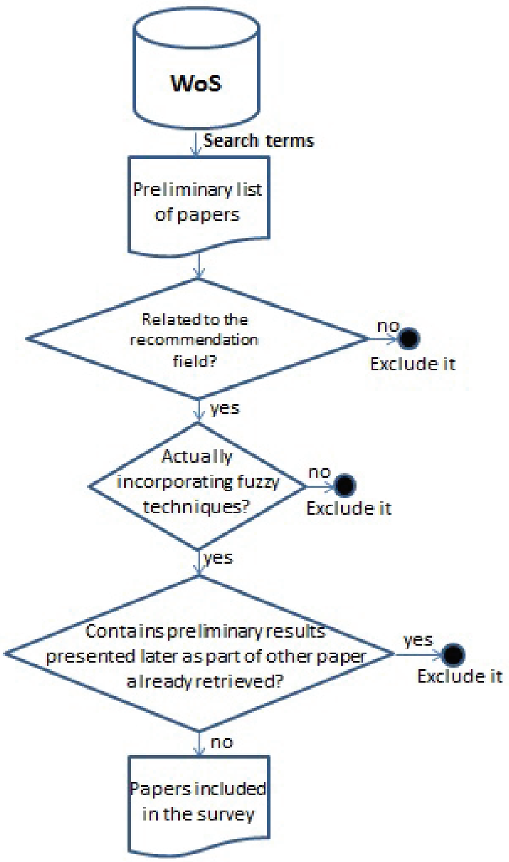

Fig. 6. Survey methodology.

This manual procedure obtains as final result a list of 108 papers, that was the definitive list to be analysed in the current paper.

In order to obtain an initial snapshot of this final list of papers, it was built a tag cloud (Figure 7 using the online tool http://tagcrowd.com/) considering the keywords associated to each paper. When papers did not contain associated keywords, their titles were used as input for the tag cloud.

Fig. 7. Tag cloud associated to the keywords of the list of papers to be analysed.

In such a tag cloud, as it was expected, the more important terms are related to *collaborative filtering, recommender systems* and *fuzzy sets*. Although *content-based* recommendation does not seem to play a relevant role here, next section will show that it is associated to an important segment of proposals. This cloud also shows other relevant terms such as *clustering, similarity*, or *linguistic*.

Figure 8 presents a temporal distribution of the papers, by differentiating journal and conference papers. It suggests that in the last years the amount of papers focused on fuzzy approaches has been increased.

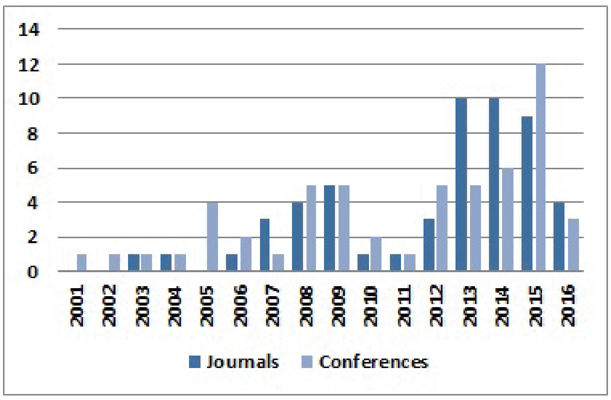

Fig. 8. Temporal distribution of the list of papers.

In addition, the distribution of main journals and conferences associated to the list of papers is shown in Figure 9. The majority of the papers were concentrated in popular journals in the field of information/intelligent systems and fuzzy logic, such as

Expert Systems with Application, *Information Sciences*, *International Journal of Intelligent Systems*, *Applied Soft Computing*, or *Fuzzy Sets and Systems*. Beyond such a figure, there are 17 journals contributing to the retrieved list with just one paper.

Unlike journal papers, the origin of the conference papers is more heterogeneous. Here, the most represented conferences were the *IEEE International Conference on Fuzzy Systems (FUZZ-IEEE)* (8 papers), *IEEE International Conference on Systems, Man, and Cybernetics (IEEE SMC)* (3 papers), and *International Conference on Intelligent Systems and Knowledge Engineering (ISKE)* (3 papers).

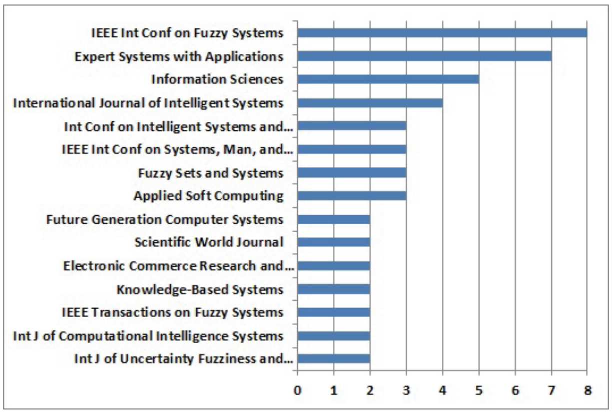

Fig. 9. Main journals and conferences distribution across the papers.

Following section introduces a comprehensive presentation about the proposals of this final list of contributions.

4. Review on fuzzy tools in recommender systems

Initially, the final list of papers is grouped by considering the typical classification of recommender systems, already presented in Section 2.1. Therefore, this section considers three big groups of fuzzy recommendation approaches: 1) the approaches focused on content-based recommendation (Section 4.1), 2) those ones focused on memory-based collaborative filtering (Section 4.2), and 3) the approaches focused on model-based collaborative filtering (Section 4.3). Regarding demographic systems, it could be pointed out that currently demographic recommenders are almost always integrated with other recommendation approaches (e.g. content-based or collaborative filtering recommendation). Therefore, it was taken as reference for including systems that use demographic features in one of the three mentioned big groups of fuzzy recommendation approaches.

It is remarkable that the hybrid nature of most of the reviewed researches implies that some of them could simultaneously belong to more than one group. In such cases the corresponding papers were added to the group related to their most important contribution according to our criteria.

The analysis of each group will conclude with a discussion subsection presenting the strengths and weaknesses in the use of fuzzy tools, associated to the corresponding group of research works.

4.1. Content-based recommendation approaches with fuzzy tools

This section is focused on presenting the contributions related to the use of fuzzy tools in content-based recommendation. First, Section 4.1.1 presents an analysis of the proposals identified in Section 3 focused on content-based recommendation, being complemented with Table 1 which presents an exhaustive survey of such works. Finally, Section 4.1.2 discusses the global strengths and weaknesses of the analysed works.

4.1.1. Proposals

The contributions related to the application of fuzzy tools to content-based recommender systems (CBRS) are reviewed here. The two main phases of a CBRS scheme (Fig. 3) are the profiling (user/item) and the matching process to suggest appropriate items to users. Fuzzy tools have been applied to both of them (see Fig. 10).

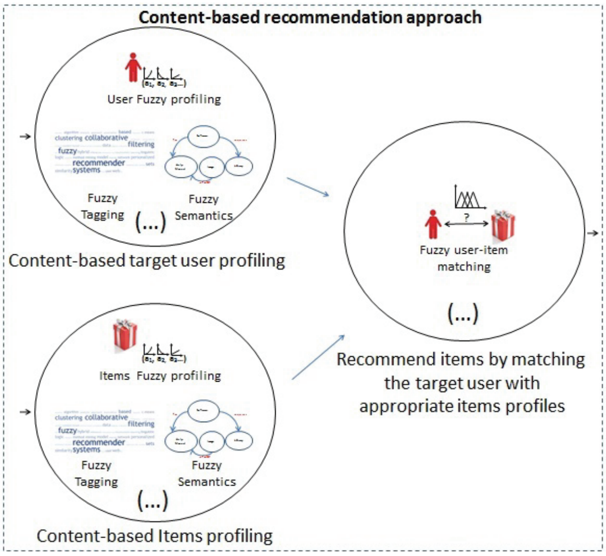

Fig. 10. Content-based recommendation using fuzzy tools

A pioneer work on the use of fuzzy logic in content-based recommendation was developed by Yager [145], that presented approaches for constructing recommender systems based on reclusive methods, closely connected to content-based recommendation. Such approaches deal with object representation, user preferences modelling, user profiling, extensionally expressed preferences, and the use of domain expert prototypes. Additionally, Karacapilidis and Hatzieleftheriou [60] also presented an early work focused on a similar direction.

More recently, Zenebe and Norcio [152] proposed a representation method for items' features and user feedback using fuzzy sets, and a content-based recommendation algorithm based on various fuzzy set theoretic similarity measures (the fuzzy set extensions of the Jaccard index, cosine, proximity or correlation similarity measures), and aggregation methods for computing recommendation confidence scores (the maximum-minimum or Weighted-sum fuzzy set theoretic aggregation methods).

In [152], $\mu_{x_i}(I_k)$ is the degree of membership of movie $I_k(k = 1,...,M)$ to genre $x_i(i = 1,...,N)$, and it is used for proposing the application of typical similarity measures for comparing items, such as fuzzy set theoretic (Eq. 5) and cosine similarities (Eq. 6).

$$S_1(I_k, I_j) = \frac{\sum_i min(\mu_{x_i}(I_k), \mu_{x_i}(I_j))}{\sum_i max(\mu_{x_i}(I_k), \mu_{x_i}(I_j))} \tag{5}$$

$$S_1(I_k, I_j) = \frac{\sum_i \mu_{x_i}(I_k) * \mu_{x_i}(I_j)}{\sqrt{\sum_i(\mu_{x_i}(I_k)^2)}\sqrt{\sum_i(\mu_{x_i}(I_j)^2)}} \tag{6}$$

In addition, they use a half triangular fuzzy number to represent the degree of positive experiences a user has in relation with an item. This function, being rating $r \in [Min, Max]$ on I_i, Min and Max the minimum and maximum rating value, and A a fuzzy set representing degree of interest, is defined as:

$$\mu_A(I_i) = (r - Min)/(Max - Min) \tag{7}$$

Consequently, a set of items E liked by a user, is defined as $E = I_i : \mu_A(I_i) > 0.5$.

Finally, they suggest several approaches to aggregate preferences for computing the recommendation confidence score, such as the weighted sum, maximum and minimum. Eq. (8) presents the weighted sum strategy, where E is the set of preferred items and $\mu_E(I_k)$ is the membership of the item k to E. $S(I_j, I_k)$ is the similarity between I_j and I_k:

$$R_1(I_j) = \sum_k \mu_E(I_k) S(I_k, I_j) \tag{8}$$

In this way, several authors have developed similar researches, such as flexible models of user preference learning from rating values in CBRS, supported by fuzzy sets [50], being some approaches empowered by bioinspired algorithms such as particle swarm optimization [136], to learn user weights on various features. Here it is worthy to note the development of tag-based user profiling methods for improving recommendations [9], where user profiles are built through a folksonomy-based approach that evaluates items according to the membership degrees to various attribute values, which are then used to compute the fuzzy user profile. Additionally, in the last few years further works on the use of fuzzy tools for modelling specific items' features in CBRS have been developed [2, 5, 14, 69, 96, 99, 104, 126].

Recently there have also been an increasing use of more sophisticated fuzzy linguistic approaches for modelling content in recommender systems, such as the 2-tuple fuzzy linguistic approach [46]. In this direction, some early works enriched the typical content-based recommendation scheme by incorporating fuzzy linguistic variables for modelling the preferences [79], subsequently considering more flexible frameworks for capturing the uncertainty of such user's preferences [80], even considering incomplete information [84]. Additionally, such sophisticated fuzzy linguistic approaches have also supported the construction of systems for recommending diverse items such as research resources [100, 115], or furniture products [42].

Beyond the direct content modelling task, there have been identified some works focused on taking benefits of fuzzy approaches for mitigating the effect of the cold start and data sparsity issues. In this way, Rodríguez et al. [109] use a fuzzy linguistic approach and incomplete preference relations to build a recommender system, where the user initially has to select a small set of favourite items, and such items are used for completing a preference relation. Wu and Hwang [140] also use users' preferences over genres to build a user-movie matrix which is transformed through fuzzy max-min operations to alleviate its sparsity.

It was also identified that several proposals such as [36, 53, 77, 88] have focused on combining fuzzy logic with semantic web technologies, in order to improve recommendations.

Christakou et al. [31] proposed the use of fuzzy aggregation operators, specifically Ordered Weighted Averaging operators (OWA), as a way for constructing hybrid recommender systems by combining the output of two recommendation components: a neural network-based content filtering, and a collaborative filtering component.

Finally, Mao et al. [78] proposed a fuzzy content matching-based recommendation approach to assist e-Commerce customers to choose their truly interested items. In that paper, users' ratings and preferences are represented using fuzzy numbers to remain uncertainties. Additionally, here tree-structured content information is transformed into a set of descriptors, and users' preferences on these descriptors are derived from fuzzy ratings by using fuzzy number operations.

Table 1 presents a further detailed exhaustive analysis of the content-based recommendation approaches supported by fuzzy tools, regarding the key features, the performed evaluation approach, the datasets used, and the application area, if proceeds.

4.1.2. Discussion

The analysis of Table 1 leads to the identification of the following strengths and weaknesses in this group of works, which could be considered for the development of future research.

Strengths:

- The presence of several works focused on managing items' attributes with fuzzy techniques, strengthening the development of uncertainty-aware content-based recommender systems.
- The use of mainstream approaches for linguistic modelling, such as the 2-tuple model, allowing the exploitation of their advantages in the recommender systems scenarios.

Weaknesses:

- The contribution of some research works is limited because they claim for contributions related to the link between content-based recommendation and fuzzy logic, already introduced in previous papers also analysed.
- Several researches (50% of the analysed works) do not incorporate experimental evaluation, notably limiting the novelty and the scope of the fuzzy logic-supported approaches.
- Most of proposals are focused on the fuzzy modelling of items' attributes, instead of the users' preferences, which are the main source of uncertainty in recommender systems.
- Some interesting research branches for uncertainty management, such as the use of OWA operators, have not received the sufficient attention in the last few years.
- Few works explore possible bridges between semantics and fuzzy logic.

Table 1. Content-based fuzzy recommendation

Papers	Key feature	Evaluation	Datasets	Application Area
Yager [145]	Constructs recommender systems through fuzzy reclusive (content-based) methods	No	No	No
Karacapilidis and Hatzieleftheriou [60]	Incorporate a fuzzy similarity measure [135]	MAE, Robustness	Movielens, PTV database	Cities to visit
Christakou et al. [31]	Use OWA operators as aggregation scheme for building hybrid recommender systems	Prec/Recall	Movielens	No
Martínez et al. [79]	Introduce a multigranular linguistic context for expressing the user preferences	Scenario of use	No	No
Martínez et al. [80]	Allow users to express their necessities in scales closer to their own knowledge, and different from the items' scale	Scenario of use	No	No
Martínez et al. [84]	Overcome the problem of lack of information in recommendation generation by completing incomplete linguistic preference relations	Scenario of use	No	No
Porcel et al. [100]	Recommendation in a technology transfer office, based on fuzzy linguistic modelling (2-tuple)	No	No	Research resources
Horváth [50]	Alternative modelling of user preference learning tasks in content-based recommendation	No	No	No
Morales-del Castillo et al. [88]	Combine semantic web technologies, fuzzy linguistic modelling techniques, and content-based and collaborative approaches	Prec/Recall/F1	Dataset extracted from an open access repository	Digital libraries
Zenebe and Norcio [152]	Content-based recommendation using various fuzzy set theoretic similarity measures and fuzzy aggregation methods	Prec/Recall/F1	Movielens	No
Recio-García et al. [104]	Bridge recommendation and case-based reasoning. Consider group recommendation task	Scenario of use	Non public	Music
Rodríguez et al. [109]	Management of incomplete preferences relations. Supported by the 2-tuple fuzzy linguistic model	Scenario of use	No	Restaurants
Serrano-Guerrero et al. [115]	Communicating researchers interested in common research lines, based on fuzzy linguistic modelling (2-tuple)	Scenario of use, Prec/Recall/F1	No	Research resources
Pinto et al. [99]	Recommendation in online stores regarding marketing concepts	Prec/Recall/F1	Non public	Online stores
Lee [69]	Recommends items or web pages suitable to the users' understanding levels	Prec/Recall/F1	Data generated by a simulator	Web pages
Bedi and Agarwal [14]	Focused on recommending relevant items at the right context, determined using fuzzy inference	No	No	Restaurants
Djaghloul et al. [36]	Formalize the balance between interest-related content matching and situation matching (context)	Cross validation regarding RMSE	Non public	E-commerce dataset
Wu and Hwang [140]	Content-based movie preference modelling supported by movie genre	Precision and time cost	Movielens	Movie
Lu et al. [77]	Combine item-based fuzzy semantic similarity and item-based fuzzy collaborative filtering similarity	MAE, Coverage, Prec/Recall/F1	Movielens, a non public dataset	Business partner recommendation
Gerogiannis et al. [42]	Helping buyers of high involvement products with the purchasing process, supported by fuzzy information modeling (2-tuple)	No	No	Furniture manufacturing
Pardines et al. [96]	System included in a mobile application that manages information related to an environmental educational program.	No	No	Environmental activities
Adnan et al. [2]	A news recommendation scenario supported by fuzzy logic	No	No	News
Anand and Mampilli [9]	Tag-based user profiling method for improving recommendations	Precision and Rank Accuracy	Movielens+HetRec	No
Wasid and Kant [136]	Use fuzzy sets for modelling user features, and particle swarm optimization for weighting it.	MAE and Coverage	Movielens	No
Al-Qaheri and Banerjee [5]	Quantify optimal social innovation-based policy recommendations	No	No	Policy recommendations
Huang et al. [53]	Ontology-based recommendation model based on a fuzzy rough set-based hybrid mechanism	No	No	No
Mao et al. [78]	Fuzzy content matching-based recommendation approach regarding tree-structured content	Coverage, nDCG	Movielens, Yelp	No
Tsai [126]	Provides suggestions for business collaboration, representing information through fuzzy vectors	AUC/F1	Yelp	Business collaboration

4.2. Memory-based collaborative filtering approaches with fuzzy tools

This section is devoted to present the contributions focused on incorporating fuzzy approaches in memory-based collaborative filtering recommendation. First, Section 4.2.1 presents an analysis of the proposals identified in Section 3, focused on memory-based collaborative filtering recommendation. This analysis is complemented with Tables 2-4, which present an exhaustive survey of such works. Finally, Section 4.2.2 discusses the global strengths and weaknesses of the analysed works.

4.2.1. Proposals

In this recommendation paradigm, the similarity functions to compare users/items play a relevant role (see Fig. 2), together the approaches for aggregating the neighbours' preferences, to obtain the final recommendations. In this way, Fig. 11 presents a synthetic overview on how fuzzy approaches are used for developing these components. Specifically, this section is focused on presenting three groups of contributions: 1) Collaborative filtering using only rating values, 2) Demographic information and items attributes to improve the memory-based collaborative filtering recommendation, and 3) Trust values into the typical recommendation scenario.

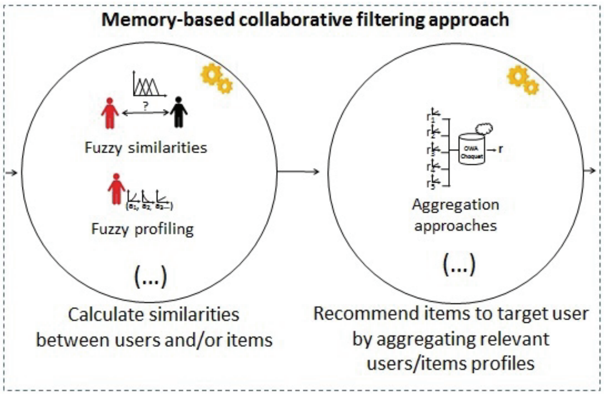

Fig. 11. Memory-based collaborative filtering recommendation using fuzzy tools

Collaborative filtering using only preferences. Table 2 presents an exhaustive analysis of the approaches that perform collaborative filtering just using rating values. Below it is highlighted the most important contributions of such works.

Aguzzoli et al. [4] proposed a fuzzy logic-based approach for collaborative filtering, showing how many-valued logic is flexible enough to perform collaborative filtering, content-based, and hybrid recommender systems.

Beyond this first work, most of the research related to memory-based collaborative filtering has been focused on proposing new similarity measures between users or items (see Fig. 2), that extend typical measures (e.g. Pearson and cosine [3]) by using fuzzy concepts. Specifically, our survey methodology detected that Al-Shamri and Al-Ashwal [6], Castellano et al. [24], Cheng and Wang [30], Cornelis et al. [32], Reformat and Yager [105], Wang et al. [134], Zhang et al. [155] and Zhang et al. [154] are focused at such aim, only using preference values and without any additional information.

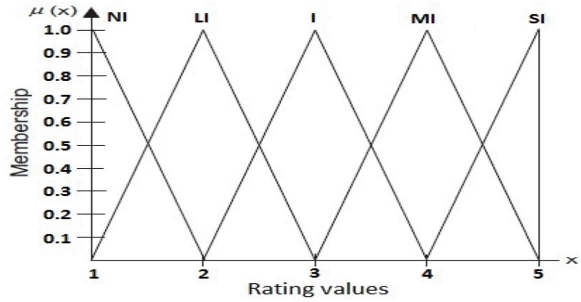

Fig. 12. Fuzzy sets and membership functions for a fuzzy collaborative filtering system.

As an example, Fig. 12 shows the fuzzy sets proposed by [155] for a fuzzy collaborative recommendation approach: {Strongly Interested (SI), More Interested (MI), Interested (I), Less Interested (LI), Not Interested (NI)}. Such research uses these linguistic terms to propose fuzzy extensions of Pearson correlation coefficient for performing user-user and item-item comparisons, and finally such similarities are combined for predicting unknown ratings. Equation (9) presents the fuzzy similarity function for comparing two items, where $S_{x,y}$ represents the set of users that rate both the items x and y; r_{x,S_α} and r_{y,S_α} represent the ratings of user s on items x and y under α-cut respectively; r^-_{x,S_α} and r^+_{x,S_α} are the left-end and the right-end of α-cut respectively, and r_{y_α}

Table 2. Memory-based collaborative filtering using only preferences

Papers	Key feature	Evaluation approach	Datasets	Application Area
Aguzzoli et al. [4]	Present a logic-based approach for recommender systems	No	No	No
Queiroz et al. [102]	Focus on recommendations for groups, based on collaborative filtering and OWA operators	Difference of mean	EachMovie	No
Cornelis et al. [32]	Use fuzzy logic for modelling one and only item recommendations	No	No	E-government
Castellano et al. [24]	Collaborative recommender system that incorporates a fuzzy linguistic approach	No	No	Academic Orientation
Al-Shamri and Al-Ashwal [6]	Fuzzy-weighted Pearson correlation coefficient for collaborative recommender systems	MAE, PCP, and Coverage	Movielens	No
Zhang et al. [155]	Hybrid recommendation approach which combines fuzzy extensions of user-based and item-based collaborative filtering	MAE	Movielens	Mobile products and services recommendation
Wang et al. [134]	Propose a new fuzzy similarity measure-based recommendation approach that only relies on rating values	MAE	Movielens	No
Zhang et al. [154]	New method to measure triangular fuzzy number, applied in collaborative filtering	MAE	Movielens	No
Cheng and Wang [30]	Fuzzy recommender system based on the integration of subjective preferences and objective information. The preferences are presented through a fuzzy linguistic model	MAE, Prec/Recall/F1	Movielens+IMDb, Yahoo Movies	No
Reformat and Yager [105]	Collaborative recommendation supported by Pythagorean fuzzy sets	No	Scenario of use	No
Ladyzynski and Grzegorzewski [67]	Quantify similarity between preferences through intuitionistic fuzzy sets	Specific accuracy metric	Semi-synthetic labelled ranking datasets	No
Son and Thong [118]	Present single-criterion and multi-criteria recommendation approaches supported by intuitionistic fuzzy sets	MAE and time cost	Well-known public datasets in the health domain	Medical diagnosis
Hu [51]	Uses the indifference relation to measure similarity in multi-criteria collaborative filtering supported by single-layer perceptron	Precision and time cost	Gathered by the authors	Group-buying website
Hu et al. [52]	Propose a similarity function that combines grey relational analysis with the Choquet fuzzy integral	Ranking accuracy, MAE, RMSE	Yahoo Movies!	No
Menhaj and Jamalzehi [85]	Propose a proximity-based similarity measure containing a fuzzy inference system that depends on homophily correlation and influence correlation	Prec/Recall/F1	Movielens	No
Castro et al. [26]	A group recommender system by combining collaborative filtering, fuzzy preference relations, and consensus reaching process	AUC and Precision	Movielens	No
Yera Toledo et al. [149]	Managing natural noise in recommender systems, by building user, item and rating profiles, and finding contradictions between them	MAE/F1	Movielens, MovieTweeting, Netflix	No

$$sim(x,y) = \frac{\sum_{s\in S_{x,y}} \int_0^1 [(r^-_{x,S_\alpha} - r^-_{x_\alpha})(r^-_{y,S_\alpha} - r^-_{y_\alpha}) + (r^+_{x,S_\alpha} - r^+_{x_\alpha})(r^+_{y,S_\alpha} - r^+_{y_\alpha})]d\alpha}{\sqrt{\sum_{s\in S_{x,y}} (\int_0^1 [(r^-_{x,S_\alpha} - r^-_{x_\alpha}) + (r^+_{x,S_\alpha} - r^+_{x_\alpha})]d\alpha)^2} \sqrt{\sum_{s\in S_{x,y}} (\int_0^1 [(r^-_{y,S_\alpha} - r^-_{y_\alpha}) + (r^+_{y,S_\alpha} - r^+_{y_\alpha})]d\alpha)^2}} \quad (9)$$

and r_{y_α} are the average rating of the users of $S_{x,y}$ on x and y respectively.

Recently, other fuzzy modelling approaches, such as intuitionistic fuzzy sets, have been used for modelling users and items similarities [67, 118]. Moreover, in the last few years, some authors have combined fuzzy concepts with other computational intelligence techniques to compose similarity measures [51, 52, 85].

In addition, there have been detected two research works focused on group recommendation supported by fuzzy concepts and regarding only user ratings, specifically proposing a Collective Fuzzy Preference Relation obtained through OWA operators [102], and proposing a consensus-driven recommender system supported by fuzzy techniques [26].

Eventually, it has been presented a fuzzy approach for managing the noise unintentionally introduced by human beings when they are eliciting preferences in collaborative recommender systems (i.e. natural noise [25, 148]), using only rating values [149]. Specifically, it focuses on building fuzzy profiles of users, items, and ratings and therefore compares such profiles in order to find and correct noisy preferences by using an associated noise degree calculated from the fuzzy profiles.

Collaborative filtering incorporating demographic information and other users' and items' features. Several works have focused on enriching similarity functions with items' attributes or demographic information. Table 3 presents an exhaustive analysis of approaches with demographic information and other users' and items' features in collaborative filtering.

In Son [117], it is presented a definition of fuzzy recommender systems as an extension of recommender systems with the fuzzy similarity calculated based on the users' demographic data instead of the crisp user-based degree. This research proposes a user-user comparison function, Eq. (10), that considers a weighted sum of the Pearson correlation value between two user profiles (HSD measure) and a similarity value considering users' demographic profiles, presented in Eq. (11) (FSD).

$$SIM(a,b) = \alpha * FSD(a,b) + \beta * HSD(a,b) \quad (10)$$

$$FSD(a,b) = 1 - \sum_{i=1}^{l} w_i * |a_i - b_i| \quad (11)$$

In Eq. (11), a_i and b_i are membership values characterizing demographic attributes (e.g. age, education, number of children, living standard), l is the number of attributes, and w_i is the corresponding attribute weight.

This scheme has been used and extended by most of contributions belonging to this group.

Eventually, new approaches have been recently proposed for modelling fuzzy tree-structured user preferences usually associated to business-to-business scenarios [138], and to an e-learning activities recommendation system [139]. These methods incorporate similarity measures for comparing trees, by considering all the information on tree structures, node attributes at the semantic level, and weights. Both cases then developed approaches for recommending such tree-structured items.

Collaborative filtering and trust. Trust networks have contributed to the success of recommender systems by users' recommendations, through the judgement of trusted sources/agents that have evaluated or experienced them (Fig. 13). Such information has been usually integrated into memory-based collaborative filtering approaches.

Table 3. Incorporating demographic information and other item's features

Papers	Key feature	Evaluation approach	Datasets	Application Area
Al-Shamri and Bharadwaj [7]	Use the fuzzy concordance/discordance principle to support the similarity function of a recommender system	MAE and Coverage	Movielens	No
Al-Shamri and Bharadwaj [8]	Fuzzy-genetic approach to recommender systems, that uses fuzzy logic for modelling the users' demographic information	MAE and Coverage	Movielens	No
Li et al. [71]	Define a semantic distance between two fuzzy sets, and apply it in a collaborative filtering	MAE	Movielens	No
Ashkezari-T and Akbarzadeh-T [10]	Use genre-based information in a hybrid fuzzy-bayesian network-based collaborative RS	MAE, Prec/Recall/F1, Coverage	Movielens	No
Kant and Bharadwaj [57]	Fuzzy collaborative filtering approach considering membership functions. Hybridized with a content-based algorithm	MAE and Coverage	Movielens	No
Son [117]	Proposes a novel hybrid user-based method that integrates fuzzy similarity degrees between users based on the demographic data	Accuracy metrics. Time cost	Movielens, Bookcrossing	Football results prediction
Wu et al. [138]	Modelling fuzzy tree-structured user preferences usually associated to business-to-business scenarios	MAE, Prec/Recall/F1	Movielens+HetRec, and a non-public dataset	Business-to-business scenarios
Wu et al. [139]	Propose an e-learning activities recommender system, composed by a fuzzy tree-structured learning activity model and a learner profile model	MAE	Movielens. Also a scenario of use	E-learning recommendation

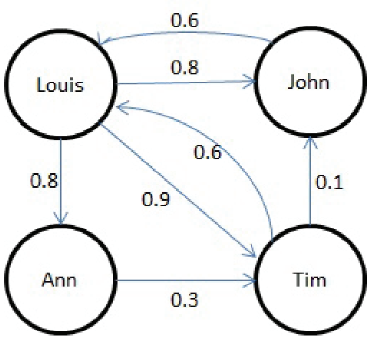

Fig. 13. Simple trust network.

Specifically, Victor et al. [130] propose the use of a user trust network model for recommender systems, in which trust is represented as a (trust,distrust)-pair. The model preserves valuable information such as gradual trust, distrust, ignorance, and inconsistency.

Bharadwaj and Al-Shamri [17] propose fuzzy computational models for trust and reputation concepts. Specifically, Fig. 14 presents the satisfied and unsatisfied fuzzy subsets presented by such authors, which are used for defining four values associated for any two users: satisfied-satisfied (SS), unsatisfied-unsatisfied (UU), satisfied-unsatisfied (SU), and unsatisfied-satisfied (US). These values are used to define agreement and disagreement between users, according to Eqs. (12) and (13), which are subsequently used for calculating reciprocity, Eq. (14).

$$agr(a_i,a_j) = \frac{SS(a_i,a_j)+UU(a_i,a_j)}{2} \quad (12)$$

$$disagr(a_i,a_j) = \frac{SU(a_i,a_j)+US(a_i,a_j)}{2} \quad (13)$$

$$rec(a_i,a_j) = (1-disagr(a_i,a_j))agr(a_i,a_j) \quad (14)$$

This reciprocity value is combined with a reliability value (also detailed by Bharadwaj and Al-Shamri [17]), to obtain a more accurate reciprocity value (Eq. 15)

$$recip(a_i,a_j) = reliab(a_i,a_j) * rec(a_i,a_j) \quad (15)$$

Such a value is combined with an experience value (Eq. (16), (17), and (18)) in order to finally calculate the trust value (Eq. 19), where n_i is the number of interactions of the user i, and M is the number of users.

$$conf_{a_i}(a_j) = \frac{n_j}{max(n_i, n_j)} \tag{16}$$

$$ex(a_j) = \frac{N_j}{max(n_1, ..., n_M)} \tag{17}$$

$$Exper_{a_i}(a_j) = conf_{a_i}(a_j) * ex_{a_j} \tag{18}$$

$$trust_{a_i}(a_j) = \frac{2 * Exper_{a_i}(a_j) * reci(a_i, a_j)}{Exper_{a_i}(a_j) + recip(a_i, a_j)} \tag{19}$$

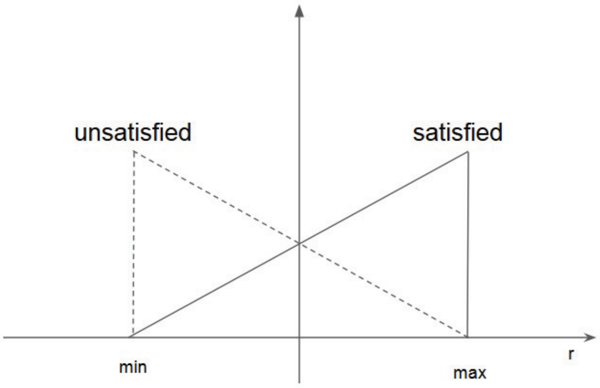

Fig. 14. Membership functions of satisfied and unsatisfied fuzzy subsets, defined by Bharadwaj and Al-Shamri [17].

Additionally, the reputation is modelled as a fuzzy extension of the beta reputation model [56], combined with an OWA operator [144].

This approach has been further extended by considering fuzzy distrust propagation operators [58], and hybridizations with case-based reasoning [127], and with context-aware recommendation [72].

Similarly, other approaches considering trust in memory-based collaborative filtering scenarios were developed at Birtolo and Ronca [19] and Bedi and Vashisth [15].

Table 4 presents further analysis of the referred research works, by additionally including other important aspects such as evaluation approaches, used datasets, and application areas.

4.2.2. Discussion

The previous analysis leads to the identification of the following strengths and weaknesses related to the use of fuzzy tools in memory-based collaborative filtering.

Strengths:

- The works present suitable evaluation protocols, by using public and popular datasets and implementing diverse evaluation metrics, proving the effectiveness of the fuzzy modelling in recommendation scenarios. Some works (around 30%) have been also focused on practical scenarios.
- Research associated to trees as a data structure has been mainstream since several decades ago. Therefore, it could be a fruitful source of new ideas for extending the presented works focused on recommending tree-structured items.

Weaknesses:

- More than 60% of the works that use public datasets for experimentation, are focused only on Movielens dataset, discarding other well-known datasets for the rating prediction task with larger rating ranges (e.g. MovieTweeting, Jester), where the use of fuzzy logic could lead to a larger improvement of the recommendation approaches.
- The development of fuzzy extensions for similarity measures between users/items, takes as basis very traditional measures such as Pearson and cosine, having a lack of works on fuzzy extensions for emerging similarity approaches well-received by the research community recently [20, 74].
- Around 50% of the works assume the availability of additional information beyond preference values, which is not always available, for building fuzzy-supported similarity measures.
- There is a lack of suitable datasets that fully contain tree-structured items.
- Only one research work uses alternatives evaluation metrics such as diversity and serendipity.

Table 4. Collaborative filtering with trust

Papers	Key feature	Evaluation approach	Datasets	Application Area
Victor et al. [130]	Propose a user trust network model preserving valuable information such as gradual trust, distrust, ignorance, and inconsistency	No	No	No
Bharadwaj and Al-Shamri [17]	Propose fuzzy computational models for trust and reputation concepts	MAE, Coverage	Movielens	No
Tyagi and Bharadwaj [127]	Combine case-based reasoning, collaborative filtering, and the model proposed by Bharadwaj and Al-Shamri [17]	MAE, RMSE, Coverage	Movielens	No
Birtolo and Ronca [19]	Integrate trust relationships between users, into a fuzzy c-means scenario	RMSE, Coverage	Movielens, Jester, Epinions, and a non-public e-commerce dataset	No
Kant and Bharadwaj [58]	Extend Bharadwaj and Al-Shamri [17] by considering fuzzy distrust propagation operators	MAE, Coverage	Movielens	No
Bedi and Vashisth [15]	Propose a new fuzzy and argumentation-based trust model integrated within the practical reasoning of agents in a recommender systems scenario	Prec/Recall/F1, Fall-Out, EPC (Novelty)	Gathered by the authors	Books
Linda and Bharadwaj [72]	Exploit fuzzy trust among users, by incorporating a context-aware approach	MAE, Coverage	Two non-public context-aware datasets	No

4.3. Model-based collaborative filtering with fuzzy tools

This section is devoted to present the contributions focused on incorporating fuzzy approaches in model-based collaborative filtering recommendation. Additionally, it also discusses the global strengths and weaknesses of the analysed works.

4.3.1. Proposals

Unlike memory-based approaches, it is more difficult the generalization of a common scheme for model-based collaborative filtering (see Fig. 2), because it depends on the computational model used for recommendation generation. In this way, Fig. 15 presents a synthetic overview on the computational models that, regarding fuzzy logic, are used to complete this task. Specifically, the research works presented at this section will be composed in four categories: 1) Fuzzy clustering, 2) Fuzzy inference-based approaches, 3) Fuzzy association rules and 4) Fuzzy bayesian approaches.

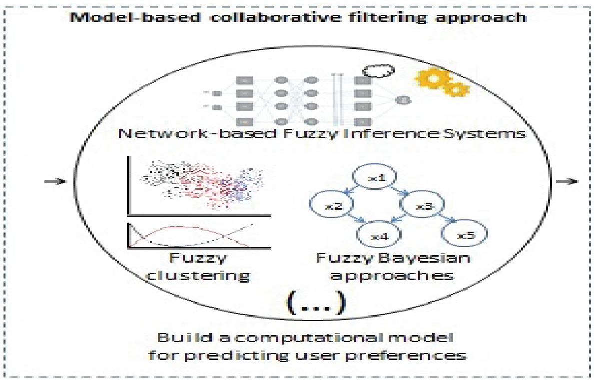

Fig. 15. Fuzzy model-based collaborative filtering

Fuzzy clustering. Several research works use the fuzzy clustering inside the recommendation approach as an intermediate step (see Tables 5 and 6), for later applying traditional algorithms such as item-item memory-based collaborative filtering [112] on specific clusters, in order to improve some recommendation performance measures such as accuracy, response time, or diversity.

In this way, a popular clustering algorithm widely employed in such scenarios is the fuzzy c-means algorithm (see Fig. 16) [16]. This algo-

Table 5. Fuzzy clustering in recommender systems (1)

Papers	Key feature	Evaluation protocol	Datasets	Application Area
Suryavanshi et al. [121]	Use relational fuzzy subtractive clustering and then mine association rules within individual clusters	F1	Web server logs (non-public)	Web usage data
Schlecht and Gaul [114]	Combine collaborative filtering with fuzzy two-mode clustering	Average absolute deviation	Movielens	No
Min and Han [86]	Consider the time dimension in data for finding fuzzy clusters at different time frames	MAE	EachMovie	No
Honda et al. [48]	Estimation of local linear models that performs a simultaneous application of fuzzy clustering and principal component analysis	MAE and ROC analysis	Movielens	No
Wang [133]	Combines a smooth filling technique with fuzzy c-means clustering	MAE	Movielens	No
Chen et al. [29]	Fill unknown ratings based on rough set theory, and use fuzzy clustering to compute user similarity and obtain nearest neighbourhoods	MAE	Movielens	No
Mittal et al. [87]	Apply first fuzzy c-means for clustering data based on attributes, and then k-means for clustering users based on ratings	No	Scenario of use	No
Liu and Yin [73]	Use fuzzy c-means to generate multiple recommendation agents which take the place of the active user's neighbours	MAE	Movielens	No
Honda et al. [49]	User-item co-clusters are extracted in a sequential way via a structural balancing technique	Cluster validation techniques	Movielens	No
Fang and Zheng [39]	Collaborative filtering recommendation based on fuzzy formal concept analysis, for conceptual clustering	MAE and RMSE	Movielens	No
Fenza et al. [40]	Use fuzzy c-means for clustering users and points-of-interest (POI), for POI recommendation	MAE and RMSE	Data gathered by the researchers	Tourist guidance
Treerattanapitak and Jaruskulchai [125]	New exponential fuzzy clustering (XFCM) algorithm by reformulating the clustering objective function with an exponential equation	MAE	Movielens	No
Esfahani and Alhan [38]	Use items' features in a content-based scenario to cluster items and users using fuzzy c-means method	No	No	Book recommendation
Verma et al. [129]	Use fuzzy c-means clustering as a previous step for the application of the item-based collaborative filtering algorithm	MAE and RMSE	Movielens	No
Son et al. [119]	Use of the fuzzy geographically clustering to solve the cold-start problem in recommender systems	MAE and RMSE	Movielens	No
Devi and Venkatesh [35]	Combine a kernel fuzzy c-means clustering approach with a Radial Basis Function Network	MAE, Prec/Recall	Movielens	No
Bilge and Polat [18]	Show how to apply clustering schemes in collaborative filtering, preserving users' confidentiality	MAE, Online time	Movielens	No
Komkhao et al. [64]	Incremental approach including fuzzy clustering, where membership degrees to clusters are expressed by the Mahalanobis radial basis function.	MAE	Movielens	Movie recommendation
Xu and Watada [143]	Proposal of new membership functions for user profile fuzzification, for alternative rating scales	MAE	Movielens	No
Wu et al. [141]	Use fuzzy clustering as a previous step for the user-based collaborative filtering algorithm	MAE	Movielens	No
Xu et al. [142]	Collaborative filtering algorithm based on user fuzzy clustering to generate optimized stock set, based on money flow model	Performance measures related to the corresponding domain	Real stock market data	Stock set recommendation

Table 6. Fuzzy clustering in recommender systems (2)

Papers	Key feature	Evaluation approach	Datasets	Application Area
Thong and Son [123]	Combine picture fuzzy clustering and intuitionistic recommender systems for medical diagnosis	MAE and time cost	Well-known public datasets in the health domain	Medical diagnosis
Bai et al. [11]	Apply fuzzy clustering for calculating users' confidence score	RMSE and MAP (Mean Average Precision)	DBLP and Microsoft Academic citation data	Research papers recommendation
Veloso et al. [128]	Incorporate user-based fuzzy c-means clustering to improve scalability	MAE, RMSE, Prec/Recall/F1	Movielens+HetRec	Media content recommendation
Vimali and Taj [131]	Use fuzzy c-mean algorithm for consolidating services' data before the application of a memory-based collaborative filtering algorithm	No	No	Service recommendation
Qiao and Zhang [101]	Propose a recommendation algorithm based on user context clustering, regarding timeliness	MAE	Movielens	No
He and Fan [45]	Propose an improved collaborative filtering recommendation based on co-clustering of users and items	No	No	No
Guan et al. [43]	Propose the Intuitionistic Fuzzy Agglomerative Hierarchical Clustering (IFAHC) algorithm for recommendation using social tagging	Scenario of use	No	No
Koohi and Kiani [66]	Apply fuzzy c-means clustering to user-based collaborative filtering	Prec/Recall, Accuracy	Movielens	No
Ramezani and Yaghmaee [103]	Group the vectors of each video by k-means and fuzzy c-means clustering	Accuracy	KTH, UCF YouTube (UCFYT), UCF Sport and HMDB action datasets	Video recommendation
Katarya and Verma [61]	Combine the particle swarm optimization technique with fuzzy c-means clustering to find a more precise neighbourhood for the active user	MAE	Movielens	No

rithm works on a matrix U_{NxC} containing the membership degree for all objects in the set N, in relation with the clusters' centroids in C. Once such a matrix is initialized (which could be done through diverse strategies, including randomly), at each k^{th}-step, each centroid c_j is calculated according to Eq (20). Afterwards, the membership values u_{ij} (object i for center c_j) are updated according to the new calculated centroids , Eq. (21). This process is repeated (see loop in Fig. 16) until matrix U does not change substantially in relation to the previous iteration, Eq. (22). In the equations, ε is a stop threshold, m is a fuzziness exponent, and $||*||$ is a norm expressing the similarity between any measured datum and the centroid.

$$c_j = \frac{\sum_{i=1}^{N} u_{ij}^m * x_i}{\sum_{i=1}^{N} u_{ij}^m} \tag{20}$$

$$u_{ij} = \frac{1}{\sum_{o=1}^{C} \left(\frac{||x_i - c_j||}{||x_i - c_o||}\right)^{\frac{2}{m-1}}} \tag{21}$$

$$||U^{(k+1)} - U^{(k)}|| < \varepsilon \tag{22}$$

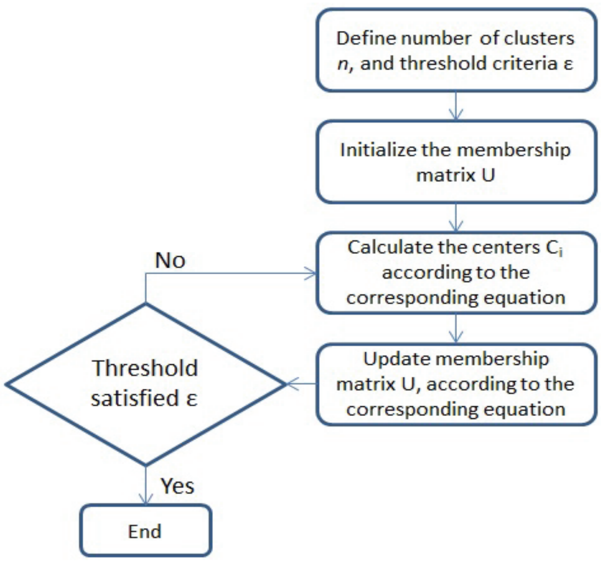

Fig. 16. Overview of the fuzzy c-means algorithm

Regarding both ratings and additional items' information, several works have employed the fuzzy c-means clustering [11, 38, 40, 66, 73, 87, 101, 103, 128, 129, 131, 141, 142], and also similar approaches such as relational fuzzy subtractive clustering [121], co-clustering [45, 49, 114, 133], picture fuzzy clustering [123], folksonomy-focused intuitionistic fuzzy agglomerative hierarchical clustering [43], fuzzy geographical clustering [119], linear fuzzy clustering [48], and other fuzzy clustering approaches [18, 29, 35, 39, 61, 64, 143].

In this way, some research works should be remarked because they take advantages of specific features related to the data in recommender systems such as the temporal information or neighbourhood models, for the development of the clustering approaches. A relevant approach is the inclusion of the time dimension to the original input data of collaborative filtering for finding the fuzzy cluster at different time frames, proposing a dynamic membership degree and determining the neighbourhood for a given user based on the dynamic fuzzy cluster [86]. On the other hand, Treerattanapitak and Jaruskulchai [125] propose a new exponential fuzzy clustering (XFCM) algorithm by reformulating the clustering objective function with an exponential equation in order to improve the method in relation to membership calculation. This transformation allows a more aggressive exclusion of irrelevant data from the clusters, improving in this way other fuzzy c-means alternatives.

Tables 5 and 6 present a deeper analysis of the particularities of the research works referred in this section.

Fuzzy inference-based approaches. This subsection is devoted to present some proposals based on fuzzy inference-based approaches, some of them network-supported approaches (see Table 7), which can be classified as model-based collaborative filtering, according to Bobadilla et al. [21].

Kim et al. [62] present an early work proposing a collaborative filtering approach that is based on improved fuzzy associative memories. The approach initially asks users to rate a gauge sets of items, processes the users' ratings, and therefore suggests a set of suitable items as a recommendation output. The proposal is based on the readjustment of the connection weights between the nodes of the fuzzy associative memory using error back propagation, for simplifying the fuzzy rules. The proposal was tested in the domain of retrieving technical papers.

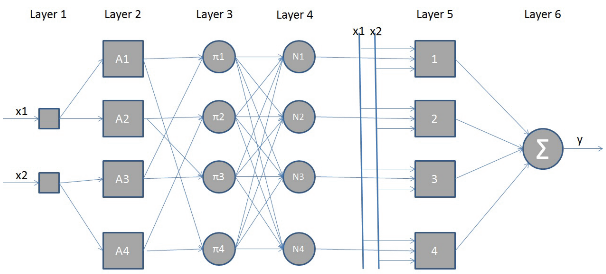

Fig. 17. ANFIS. layer 1: Input, layer 2:fuzzification, layer 3: rule, layer 4: normalization, layer 5: defuzzification, layer 6: output.

Afterwards, a more advanced approach were developed by Nilashi et al. [91], that presents a new model for multi-criteria collaborative filtering using an Adaptive-Network-based Fuzzy Inference System [54] (ANFIS, see Fig. 17) combined with subtractive clustering and high order singular value decomposition. The input parameters of the ANFIS model are the criteria over the movies, specifically acting, directing, story and visuals; and the overall rating stands for output, that is defined as the overall preference. In this direction, later these authors developed extensions of this work by implementing alternative clustering approaches, such as Self-Organization Maps [93], and the Expectation Maximization algorithm [92].

Beyond these two key researches, other authors such as Chao et al. [27], Sobecki et al. [116], Jeon et al. [55], Nguyen and Duong [90] and Tiwari and Kaushik [124], have also developed more application-oriented recommendation approaches supported by fuzzy inference processes.

Fuzzy association rules-based approaches. In a different direction, it was also detected a group of works focused on the use of fuzzy association rule mining for supporting recommendation (Table 8). Such group is composed of the researches developed by Chen and Tai [28], Pinho Lucas et al. [98]

Table 7. Fuzzy inference-based approaches

Papers	Key feature	Evaluation approach	Datasets	Application Area
Kim et al. [62]	Readjust the connection weights between the nodes of the fuzzy associative memory using error back propagation, for simplifying the fuzzy rules used for recommendation generation	MAE	Gathered by the authors	Retrieval of technical papers
Chao et al. [27]	Recommendation agent with a fuzzy inference engine for recommending e-learning resources	No	No	e-learning platforms
Sobecki et al. [116]	Use demographic information for building membership functions, and construct fuzzy inference rules	No	No	Cooking assistance
Jeon et al. [55]	Manage the personal propensity of the users, and include a fuzzy inference system	RMSE	Netflix	No
Nilashi et al. [91]	Multi-criteria CF using ANFIS combined with subtractive clustering and higher order singular value decomposition	MAE, RMSE, Prec/Recall/F1, Coverage	Yahoo Movies, Movielens	No
Nilashi et al. [93]	Multi-criteria CF using self-organization maps	MAE, RMSE, Prec/Recall/F1, Coverage	Yahoo Movies, Movielens	No
Nilashi et al. [92]	Multi-criteria CF using expectation maximization algorithm	MAE, RMSE, Prec/Recall/F1, Coverage	Yahoo Movies, Movielens	Tourism domain
Tiwari and Kaushik [124]	Use a fuzzy inference system that manages dimensions such as traffic conditions, security, or suitable transportation	No	Gathered by the authors	Tourist spots
Nguyen and Duong [90]	Model-based collaborative filtering using a fuzzy neural network to learn user's behaviours for video recommendation	MAE, RMSE	Netflix	Video recommendation

Leung et al. [70], and Teng et al. [122]. Specifically, Leung et al. [70] introduce a collaborative filtering approach based on fuzzy association rules and multiple-level similarity. With this purpose, they fuzzify numeric ratings into three sets Like, Neutral and Dislike, and also incorporate rule's interestingness measures such as fuzzy support and fuzzy confidence.

Eventually, Banda and Bharadwaj [13] propose a novel collaborative tagging-based page recommendation algorithm using a fuzzy classifier. Specifically, they calculate the similarity of users in selecting tags and therefore use this information for finding the nearest neighbours of each user, and clustering them. The priority of tags and items for each user is then calculated for constructing a Nominal Label Matrix and Nominal Page Matrix; which are used for obtaining fuzzy rules that generate page recommendation.

Fuzzy bayesian approaches. de Campos et al. [33] propose a collaborative recommender system that combines probabilistic inference and fuzzy observations. Specifically, it involves three components: a mapping of the fuzzy ratings (input) to a probabilistic distribution; the use of probabilistic reasoning to compute the probability distribution over the expected vote; and the calculation of the user's vote (a fuzzy set).

Other approaches that also integrate fuzzy and bayesian concepts, have been proposed by Kant and Bharadwaj [59] and Zhang et al. [153].

Table 8 presents a further analysis of these works.

4.3.2. *Discussion*

The previous analysis leads to the identification of the following strengths and weaknesses related to fuzzy model-based collaborative filtering.

Strengths:

- A relatively high amount of research works within model-based collaborative filtering approaches

Table 8. Fuzzy association rules-based and fuzzy bayesian approaches

Papers	Key feature	Evaluation approach	Datasets	Application Area
		Fuzzy association rules-based approaches		
Chen and Tai [28]	Use fuzzy association rules for classifying users	No	Scenario of use	No
Leung et al. [70]	Introduce a collaborative filtering approach based on fuzzy association rules and multiple-level similarity	Recall	Movielens, Jester, EachMovie	No
Pinho Lucas et al. [98]	Fuzzy associative classification approach, focused on obtaining the possible groups to which the active user owns to	Classification accuracy, false positive rates	Movielens, Bookcrossing	No
Banda and Bharadwaj [13]	Novel collaborative tagging-based page recommendation algorithm using a fuzzy classifier. Find the nearest neighbours depending on the similarity of users regarding selected tasks.	Hit ratio, recall	Movielens + HetRec	No
Teng et al. [122]	Fuzzy Analytical Hierarchy Process (FAHP)-based recommendation method by fusing apriori rule mining	Precision, Frequency	Gathered by the authors	books
		Fuzzy bayesian approaches		
de Campos et al. [33]	Combine probabilistic inference and fuzzy observations for proposing a collaborative recommender system	MAE, Prec/Recall/F1	Movielens	No
Kant and Bharadwaj [59]	A recommendation approach based on collaborative filtering and reclusive methods, that incorporates a fuzzy naïve bayesian classifier	Prec/Recall/F1	Movielens+IMDb	No
Zhang et al. [153]	Combine item-based collaborative filtering and a bayesian approach for selecting suitable services, represented through trapezoidal fuzzy number	No	Scenario of use	e-government

with fuzzy tools (the 44% of all the reviewed papers), have proved that fuzzy versions of traditional learning paradigms, can be useful in recommendation scenarios.

- The opportunity for complementing the presented work focused on network-based fuzzy inference systems, with currently popular network-related paradigms such as deep learning [68].

Weaknesses:

- Diversity of approaches focused on very different recommendation scenarios. This fact makes difficult the development of a fair experimental comparison between the proposals.
- Most of the research works in fuzzy clustering (around 80%), are limited to the direct application of a general-purposed fuzzy clustering algorithm (e.g. fuzzy c-means), disregarding the particularities of the recommender systems data and its possible influence on the development of the clustering approach.
- It seems that the success of fuzzy rules and fuzzy inference systems in recommender systems is usually associated to the management of users' and items' attributes, being only 14% of the works belonging to such groups, directly related to the management of preference values. This fact could decrease the impact of these approaches in relation with other kind of recommendation methods.

5. Future research directions

The previous sections discussed several weaknesses in the development of research works focused on the use of fuzzy approaches in recommender systems. Such weaknesses show some research gaps that can be considered as possible future trends and challenges for fuzzy based recommendation. Specifically, we identify four challenging areas that should be expanded in the coming future, for a better exploitation of fuzzy tools to improve the performance of recommender systems:

- **Fuzzy common framework for further researches in recommender systems.** Previously, we have referred several works such as [117, 134, 152] that have made important contributions to this aim, but only present a partial solution of the problem because they have a lack of generalization in relation to typical recommendation scenarios, and in other cases the use of fuzzy tools is insufficient regarding its flexibility to represent user preferences. In addition, the absence of a common framework implies that emerging researches just reproduce previous results, because there is not a clear reference point to work with.

- **Evaluation scenarios focused on fuzzy recommender systems.** Even though many revised papers evaluate their proposals using public well-known datasets, there are still works that need to gather their own data, and also works that did not perform any kind of experimental evaluation. In many cases this fact occurs either because of the lack of suitable datasets in which the fuzzy approaches could be directly applied to or because their benefits are not appreciated.
- **New fuzzy approaches focused in new trends for recommender systems.** It is necessary to apply fuzzy to solving emerging problems such as group recommendation, context-aware recommendation or natural noise management, for improving the management of the uncertainty associated to such problems.
- **New fuzzy approaches focused on using emergent information sources.** In this way, although this survey has shown that there have been developed some works that consider the fuzzy-supported uncertain information management in emergent sources such as social networks information, tagging systems or complex items (e.g. tree-structured items), further works are necessary in this direction. Overall, it is necessary proposals that could become starting-points for subsequent researches.

6. Conclusions

This survey analysed more than a hundred papers focused on the use of fuzzy techniques for supporting recommender systems. At first, these papers were arranged in three big groups according to three different recommendation paradigms (content-based, memory-based collaborative filtering, and model-based collaborative filtering), additionally including several subgroups regarding the core computational approaches used in the corresponding works. Afterwards, it was developed an exhaustive analysis of each contribution by considering, their key features, evaluation strategies, and application areas. Particularly, it includes a deeper analysis of the works identified as relevant inside its corresponding subgroup.

Eventually it has been pointed out future research avenues in fuzzy recommender systems, mainly focused on the development of a common framework, the development of evaluation scenarios centred on recommender systems based on fuzzy information management, and the development of new approaches for emergent information sources (e. g. social networks) and for solving new problems in recommender systems research (e. g. group recommendation, natural noise management).

We hope that this modest survey would be useful for the recommender systems research community, as a starting point for the development of further contributions to the emergent research field related to the development of recommender systems supported by fuzzy tools.

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