A representation model for CLEs based on type-2 fuzzy set

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***Abstract*—In real-life linguistic decision making, participates may hesitate when they are requested to provide evaluations on alternatives due to the uncertainty and vagueness of the information. Single terms are usually not flexible enough for experts to express their opinion, thus more elaborated linguistic expressions, such as comparative linguistic expressions (CLEs), are needed. However, because of the lack of researches on fuzzy representation models of CLEs, when CLEs are adopted in decision making, it is difficult to carry out computing with words (CWW). To fill this gap, we come up with a new T2F-representation model for CLEs in this research by introducing a new T2FE of hesitant fuzzy linguistic term sets (HFLTSs). The proposed model decreases the information losing during the CWW processes when CLEs are used in decision making, by applying T2FSs to represent linguistic information. Therefore, it contributes to increase the flexibility for experts to extract and express linguistic information.**

***Index Terms*—CLEs, decision making, computing with words, type-2 fuzzy set (T2FS)**

I. Introduction

In 1996, Zadeh proposed the CWW paradigm [1], CWW is an information processing method based on words or linguistic propositions derived from natural language [2]. CWW is also an important technology for decision making, reasoning and computing when a problem goes along with linguistic information. In order to realize CWW, many researchers have adopted type-1 fuzzy set theory to convert linguistic information to numerical information. However, the meaning of the same word may be different for different people, because linguistic information contain uncertainty. We need more powerful fuzzy models that can describe, reflect and process linguistic uncertainties. T2FS can process linguistic uncertainty. Interval type-2 fuzzy set (IT2FS) is a special T2FS which is widely used in the literature, since it has a simpler structure keeping the main feature of T2FS.

The appearance of decision-making problems is often accompanied by uncertainty. In uncertain decision-making contexts decision makers tend to use linguistic information to express their assessment over alternatives. The reasons could be summarized as follows: on the one hand, linguistic information is closer to human-being's thinking habits; on the other hand, linguistic information is more reliable when the context is too vague for decision makers to elicit crisp numerical information. Therefore, more and more researchers have paid attention to linguistic information processing methods, which is of great significance to the researches on linguistic decision-making.

In uncertain decision contexts, sometimes it is difficult for decision maker to use single word to provide evaluations on alternatives, which brings the demands for more effective and flexible representations of linguistic information than single words. Rodríguez et al. [3] introduced CLEs to model experts' hesitation. Compared with other linguistic expression forms, CLEs has the following advantages:

1) CLEs are built formally benefit from a context free grammar.
2) CLEs resemble human being's cognitive model and can simulate the human thinking. Therefore, it is suitable for participates to express their opinions when they feel hesitation.
3) CLEs are related to HFLTSs that is a linguistic representation model. A transformation function [4] was introduced to gain HFLTSs from CLEs.

For CWW with CLE, it is natural to apply envelope of HFLTS. The envelope of HFLTS was originally proposed by Rodríguez et al. [5]. Up to present, researchers have constructed three typical envelopes for HFLTSs:

1) Linguistic interval envelope [3], [5]. The model uses linguistic intervals to represent HFLTSs. This model deviates from the fuzzy linguistic approach and it can not reflect the fuzziness of linguistic information.
2) Type-1 fuzzy envelope (T1FE) [6]. This model uses type-1 fuzzy sets to represent HFLTSs. Compared with envelope in form of linguistic interval, fuzzy envelope has the advantage of reflecting the fuzziness of linguistic information, but its disadvantage is that it can not deal with linguistic uncertainty.
3) Type-2 fuzzy envelope (T2FE). Recently, Liu et al. [7] have developed a novel representation model for CLEs by constructing a T2FE of HFLTSs. Its main feature is that it can decrease information lost during CWW.

Taking into account that to construct the T2FE of HFLTSs, it is necessary to define a function that satisfies some specific principles [7] (the function should not be unique), in the current paper we introduce a new T2FE by developing a new function, which is developed based on a piecewise function defined on interval $[0,1]$. The current research can enrich the methodologies to construct the T2FE of HFLTS, in order to go a step further to facilitate CWW with CLEs in linguistic decision-making.

We encourage researchers to explore various approaches to build representation models for more types of linguistic expressions and perceptions. The current work plays effects to attract more valuable opinions and approaches. The remain content is organized as follows: context-free grammar, HFLTS, T2FSs and other related concepts are reviewed in Section 2. A new approach to construct the T2F-representation of CLE is provided in Section 3. Section 4 shows some numerical examples for the computation of type-2 fuzzy representation (T2F-representation) of CLE. And finally, section 5 points out some conclusions.

II. Preliminaries

Some necessary concepts which will be used in the current research are reviewed in this section.

A. HFLTS and CLE

Researchers have carried out an "impossible to be ignored" amount of works on both the theory development and the use of HFLTSs in decision making [8] to model experts' hesitation.

Definition 1. [5] *Let $S=\{l_0,l_1,\ldots,l_g\}$ be a linguistic term set. A HFLTS, H_S, is an ordered finite subset of consecutive linguistic terms of S.*

$$H_S=\{l_i,l_{i+1},\ldots,l_j\}, l_k\in S, k\in\{i,\ldots,j\}.$$

However, people rare use HFLTS to elicit their opinions, but linguistic expressions more complex. To do so, Rodríguez et at. introduced a context-free grammar G_H (see [4] for details) to generate CLEs more richer than single terms. These CLEs are based on HFLTSs.

By using context free grammar G_H, three types of CLEs could be generated, such as 'at least l_i', 'between l_i and l_j', 'at most l_i' ($i,j\in\{0,\ldots,g\}$, $i\le j$). Primary terms, i.e., single term l_i in S could be viewed as special CLE 'between l_i and l_i'.

In order to obtain HFLTS from CLE, a transformation function was defined:

Definition 2. [5] *Let E_{G_H} be a function which can convert the CLE, $ll\in S_{ll}$ obtained by G_H, into $HFLTS$, H_S:*

$$E_{G_H}: S_{ll}\rightarrow H_S \tag{1}$$

being G_H generates S_{ll} as expression domain.

Different kinds of CLEs generated from G_H could be converted into HFLTSs by using E_{G_H}. For instance, $E_{G_H}(\text{between } l_i \text{ and } l_j)=\{l_k|l_i\le l_k\le l_j \text{ and } l_k\in S\}$.

B. T2FS and IT2FS

Here we only review some fundamental concepts about T2FSs and IT2FSs, for further detail see [9], [10].

A T2FS $\tilde{A}$ is defined with a function $\mu_{\tilde{A}}(o,v)$ [10]:

$$\tilde{A}=\{((o,v),\mu_{\tilde{A}}(o,v))|o\in U, v\in[0,1]\}, \tag{2}$$

here, $0\le\mu_{\tilde{A}}(o,v)\le 1$.

Compared with type-1 fuzzy set, for each $o\in U$, the fuzzy membership of o in a T2FS is not a crisp value, but another fuzzy set.

If the value of $\mu_{\tilde{A}}(o,v)$ is always 1 for each $o\in U, v\in[0,1]$, the T2FS $\tilde{A}$ is named an IT2FS, and it could be characterized by

$$\tilde{A}=\int_{o\in U}\int_{v\in J_o}1/(o,v)=\int_{o\in U}[\int_{v\in J_o}1/v]/o, \quad J_o\subseteq[0,1], \tag{3}$$

where J_o is called the primary membership of x, which is defined by

$$J_o=\{(o,v): v\in[\underline{\mu}_{\tilde{A}}(o),\overline{\mu}_{\tilde{A}}(o)]\}. \tag{4}$$

An IT2FS $\tilde{A}$ could be determined by its footprint of uncertainty (FOU), i.e.,

$$FOU(\tilde{A})=\{(o,v): v\in J_o\subseteq[0,1]\}. \tag{5}$$

An IT2FS $\tilde{A}$ can also be characterized by

$$\tilde{A}=1/FOU(\tilde{A}). \tag{6}$$

C. Entropy measures of HFLTS

A necessary prerequisite for developing T2FE of HFLTS is to measure its contained uncertainty properly. Entropy measure of HFLTS is an important tool to evaluate such uncertainty, so they are briefly revised here. Up to present, three main types of entropy measures have been proposed by researchers, i.e., hesitant entropy E_h, fuzzy entropy E_f and comprehensive entropy E_c [7], [11]. Among these, comprehensive entropy is capable to reflect and describe the uncertainty contained in a HFLTS relative completely.

Only the axiomatic definition for comprehensive entropy is reviewed here, for further details see [7], [11].

Definition 3. [7], [11] *Suppose that $S=\{l_0,l_1,\ldots,l_g\}$ is a linguistic term set, $H_S=\{l_{\gamma_1},l_{\gamma_2},\ldots,l_{\gamma_l}\}$ is an HFLTS defined on S, and $\mathscr{H}(S)$ stands for all the HFLTSs built on S. Let $E_c:\mathscr{H}(S)\longrightarrow[0,1]$ satisfy four conditions:*
(1) *$E_c(H_S)=0$ iff $H_S=\{l_0\}$, $H_S=\{l_g\}$;*
(2) *$E_c(H_S)=1$ iff $H_S=\{l_{\frac{g}{2}}\}$;*
(3) *Let $H_S^1=\{l_{\gamma_1},l_{\gamma_2},\ldots,l_{\gamma_l}\}$, H_S^2 be two HFLTSs. H_S^2 is obtained by switching any element $l_{\gamma_i}(i=1,2,\ldots,l)$ in H_S^1 to $l_{\gamma_i'}$. If $|I(l_{\gamma_i})-\frac{g}{2}|\ge|I(l_{\gamma_i'})-\frac{g}{2}|$ and $\eta(H_S^1)\le\eta(H_S^2)$, then $E_c(H_S^1)\le E_c(H_S^2)$. Here $\eta(H_S)=\frac{2}{l(l-1)}\sum_{i=1}^{l-1}\sum_{j=i+1}^{l}(I(l_{\gamma_j})-I(l_{\gamma_i}))$, where $I(l_{\gamma_i})$ is the index of term l_{γ_i};*
(4) *$E_c(H_S)=E_c(Neg(H_S))$,*
E_c is a comprehensive entropy of H_S.

III. A T2FE OF HFLTS

In [7], we originally provided the framework to obtain the T2F-representation of CLE. It contains three main procedures:

1) Compute the T1FE of HFLTS that is transformed from the corresponding CLE.
2) Estimate the uncertainty contained in HFLTS by using the comprehensive entropy of HFLTS.
3) Apply the value of comprehensive entropy as the width of footprint of uncertainty to achieve the T2FE of HFLTS. Suppose that a CLE, S_{ll}, could be transformed into a HLFTS, H_S, by using a transformation function, then the T2FE of H_S will be used as the T2F-representation of S_{ll}.

The generation process for T2F-representation of a CLE could be shown by Fig. 1.

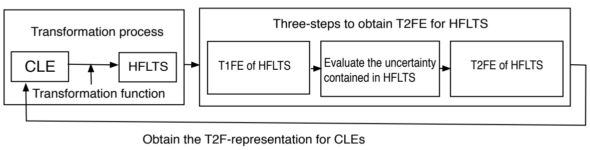

Fig. 1. General process for T2F-representation of CLEs

In the following, we will provide the detailed method to construct T2F-representation model of CLE.

A. Compute the T1FE of HFLTS.

The approach in [6] will be adopted in this proposal to compute T1FE of HFLTS considering the specific meaning of corresponding CLE.

The following discussions are all constructed based on the assumption that $S = \{l_0, l_1, \ldots, l_g\}$ is a linguistic term set, in which terms are characterized by using a triangular membership function.

Supposed that fuzzy representation of a single term could be given as $l_i = (a_L^i, a_M^i, a_R^i)$. To build fuzzy representation of a HFLTS $\{l_i, \ldots, l_j\}$, the values need to be proceeded as $V = \{a_L^i, a_M^i, a_M^{i+1}, \ldots, a_M^j, a_R^j\}$. The T1FE for H_S could be represented as a trapezoidal fuzzy number $F_{H_S} = T(e_1, e_2, e_3, e_4)$, where the parameters $e_1 - e_4$ are computed by using elements in V. Usually e_1 and e_4 are the smallest and largest value in V, respectively. The other parameters e_2 and e_3 can be got by aggregating the rest of values in V using OWA operators defined in [12].

B. Evaluate the uncertainties contained in HFLTS

According to our research in [7], the uncertainty contained in HFLTS H_S could be evaluated by its comprehensive entropy, which is computed by:

$$E_c(H_S) = \frac{E_g(H_S) + \beta(H_S)E_h(H_S)}{1 + \beta(H_S)E_h(H_S)}, \tag{7}$$

In this equation, $\beta(H_S)$ is the key component that need to be determined. This parameter is used to control the different effectiveness of fuzzy uncertainty and hesitant uncertainty when computing the overall uncertainty. The smaller $\beta(H_S)$ is, the less hesitancy will be considered when computing the overall uncertainty. Several principles have been proposed in [7] to determine this parameter:

1) $\beta(H_S) = 1$ if $H_S = \{l_0, l_1, \ldots, l_g\}$;
2) $\beta(H_S) = 0$ if $H_S = \{l_i\}$, $l_i \in S$;
3) $\beta(H_S) < \beta(H'_S)$ if "all terms in H_S are contained in H'_S" and "the number of terms in H_S is smaller than the number of terms in H'_S", i.e., $|H_S| < |H'_S|$;
4) The change of $\beta(H_S)$ is positively related to fuzziness of the term added in/deleted from H_S.
5) $\beta(H_S) = \beta(Neg(H_S))$, where if $H_S = \{l_i, \ldots, l_j\}$, then $Neg(H_S) = \{l_{g-j}, \ldots, l_{g-i}\}$.

In [7], we have provided several specific functions to compute $\beta(H_S)$, which is built based on trigonometric functions. However, the functions should not be unique. Researchers could provide different functions to determine this parameter $\beta(H_S)$, as long as the functions satisfy the above five principles.

The following new functions are introduced to determine the parameter $\beta(H_S)$:

Let $l_k = (a_L^k, a_M^k, a_R^k)$ $(0 \le k \le g)$, for any HFLTS $H_S = \{l_i, l_{i+1}, \ldots, l_j\}$ $(0 \le i \le j \le g)$,

$$\beta(H_S) = \begin{cases} \frac{\sum_{k=i}^{j} g(a_M^k)}{\sum_{k=0}^{g} g(a_M^k)}, & \text{if } i < j; \\ 0, & \text{if } i = j. \end{cases} \tag{8}$$

where

$$g(t) = \begin{cases} t+1 & t \in [0, 0.5]; \\ 2-t, & t \in [0.5, 1]. \end{cases} \tag{9}$$

The above function is provided here as a feasible example, we hope that researchers could find more suitable functions to compute $\beta(H_S)$ satisfy 1)-5) and to enrich schemes for constructing envelopes hereafter.

Theorem 1. *The $\beta(H_S)$ defined by Eq. (9) satisfies principles 1-5.*

Proof. (1) When $H_S = \{l_0, l_1, \ldots, l_g\}$, obtain that $\sum_{k=i}^{j} g(a_M^k) = \sum_{k=0}^{g} g(a_M^k)$. Therefore, $\beta(H_S) = 1$.
(2) When $H_S = \{l_i\}$, it can be viewed as $H_S = \{l_i, l_{i+1}, \ldots, l_j\}$ and $i = j$, in this way, $\beta(H_S) = 0$.
(3) Suppose that $H_S = \{l_i, l_{i+1}, \ldots, l_j\}$ and $H'_S = \{l_{i'}, \ldots, l_{j'}\}$, If $H_S \subseteq H'_S$ and $|H_S| < |H'_S|$, then $\sum_{k=i}^{j} g(a_M^k) < \sum_{k=i'}^{j'} g(a_M^k)$, therefore $\frac{\sum_{k=i}^{j} g(a_M^k)}{\sum_{k=0}^{g} g(a_M^k)} < \frac{\sum_{k=i'}^{j'} g(a_M^k)}{\sum_{k=0}^{g} g(a_M^k)}$, that is, $\beta(H_S) < \beta(H'_S)$.
(4) Suppose that $l_q = (a_L^q, a_M^q, a_R^q)$ is the linguistic term which is deleted from or added to a HFLTS H_S. According to the axiom definition for fuzziness of single term [11], it is easy to obtain that fuzziness of term in the middle position in S is the largest, while fuzzy degree of terms at the far right or at the far left are the smallest.

On the one hand, from the structure of function $g(t)$ in Eq. (9), $g(a_M^q)$ will reach the maximum value when a_M^q is 0.5, that is, l_q is at the middle position of the linguistic term set. Meanwhile, $g(a_M^q)$ will reach the minimum value when a_M^q is

0 or 1, that is, l_q is at the far right or the far left position of the linguistic term set. Therefore, $g(a_M^q)$ is positively related to the fuzziness of l_q. From another aspect, from Eq. (8), the change value of $\beta(H_S)$ is positively related to $g(a_M^q)$. From both the above two aspects, we know that $\beta(H_S)$ is positively related to fuzziness of l_q.
(5) See distribution of terms in S, we have $a_M^i = 1 - a_M^{g-i}$ for any $l_i \in H_S$. Suppose that $H_S = \{l_i, l_{i+1}, \ldots, l_j\}$, then we have $Neg(H_S) = \{l_{g-j}, l_{g-j+1}, \ldots, l_{g-i}\}$. From Eq.(9), therefore we have $g(a_M^i) = g(1 - a_M^i) = g(a_M^{g-i})$, so we get $\sum_{k=i}^{j} g(a_M^k) = \sum_{k=i}^{j} g(a_M^{g-k}) = \sum_{k=g-j}^{g-i} g(a_M^k)$, hence from Eq.(8) we obtain that $\beta(H_S) = \beta(Neg(H_S))$. □

C. Compute the T2F-representation of CLEs.

Suppose that the T1FE of a HFLTS H_S is $F(H_S)$, the comprehensive entropy of H_S is denoted by $E_c(H_S)$, then the T2FE of H_S, denoted by $\tilde{F}(H_S)$, could be characterized by

$$\tilde{F}(H_S) = 1/\{(o,v) : o \in U, v \in [F(H_S)(o) - E_c(H_S), F(H_S)(o)]\} \tag{10}$$

where the denominator is the footprint of uncertainty, that could be denoted by $FOU(\tilde{F}(H_S))$. T2FE of H_S is the T2F-representation of CLE it is converted from.

IV. Numerical examples: T2F-representation of CLEs

An example will be given to show how to obtain the T2F-representation of a CLE based on the proposed approach in Section III.

Example 1 In a decision making situation, suppose that a decision maker offers evaluation on "possibility to win" for a football team by using a CLE built on $S = \{l_0$: "very rare", l_1 : "rare", l_2 : "relatively rare", l_3 : "middle", l_4 : "relatively large", l_5 : "large", l_6 : "very large"$\}$ (see Fig. 2).

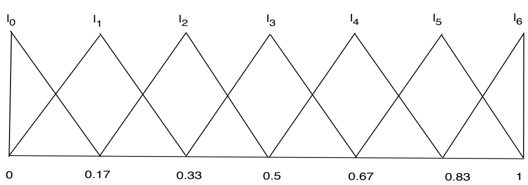

Fig. 2. A linguistic term set

The decision maker's evaluation is a CLE (between 'middle' and 'large'), i.e., (between l_3 and l_5). To obtain the T2F-representation for the CLE, we carry out the following procedures:

1) By using E_{G_H}, we have $E_{G_H}(\text{between } l_3 \text{ and } l_5) = \{l_3, l_4, l_5\}$.
2) We need to compute the T1FE of HFLTS, $E_{G_H}(\text{between } l_3 \text{ and } l_5) = \{l_3, l_4, l_5\}$. By using the approach in [6], it is $F_{E_{G_H}(\text{between } l_3 \text{ and } l_5)} = T(0.33, 0.64, 0.70, 1)$.
3) Compute the comprehensive uncertainty contained in $E_{G_H}(\text{between } l_3 \text{ and } l_5)$. In this work, we still adopt the same fuzzy entropy and hesitant entropy which is adopted in [7], then
$E_g(E_{G_H}(\text{between } l_3 \text{ and } l_5)) \approx 0.82$,
$E_h(E_{G_H}(\text{between } l_3 \text{ and } l_5)) \approx 0.22$.
By using Eqs. (8)-(9), we have $\beta(E_{G_H}(\text{between } l_3 \text{ and } l_5)) = \frac{g(0.5)+g(0.67)+g(0.83)}{g(0)+g(0.17)+g(0.33)+g(0.5)+g(0.67)+g(0.83)+g(1)} \approx 0.47$.
Therefore, $E_c(E_{G_H}(\text{between } l_3 \text{ and } l_5)) = \frac{0.82+0.47\times0.22}{1+0.47\times0.22} \approx 0.837$ by Eq. (7).
4) Finally, we obtain the T2F-representation of the CLE (between l_3 and l_5) (see Fig. 3) as
$\tilde{F}(E_{G_H}(\text{between } l_3 \text{ and } l_5))$
$= 1/\{(o,v) : o \in [0,1], v \in [F_{E_{G_H}(\text{between } l_3 \text{ and } l_5)}(o) - 0.837, F_{E_{G_H}(\text{between } l_3 \text{ and } l_5)}(o)]\}$
where
$$F_{E_{G_H}(\text{between } l_3 \text{ and } l_5)}(o) = \begin{cases} 0, & o \in [0, 0.33]; \\ \frac{1}{0.31}o - \frac{0.33}{0.31}, & o \in [0.33, 0.64]; \\ 1, & o \in [0.64, 0.70]; \\ -\frac{1}{0.3}o + \frac{1}{0.3}, & o \in [0.7, 1], \end{cases}$$
which could easily be obtained from its parameterized representation $F_{E_{G_H}(\text{between } l_3 \text{ and } l_5)} = T(0.33, 0.64, 0.70, 1)$.

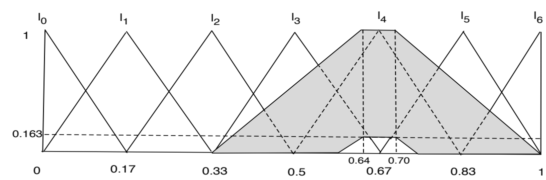

Fig. 3. T2F-representation for CLE, between l_3 and l_5.

V. Conclusion

By introducing a new function to control the effectiveness of hesitancy during the process of evaluating the overall uncertainty contained in a HFLTS, a new T2FE of HFLTS is developed. Furthermore, considering the relationship between CLE and HFLTS, a new T2F-representation model of CLE is obtained. The researches on the new construction methodologies of T2F-representation models of CLE will facilitate CWW in linguistic decision making. In the coming future, these works will be carried out:

- The applications of the proposed model in real life linguistic decision making will be further discussed, it is planed to be applied in both decision making and group decision making situations.
- CLE is suitable to be used as a tool to characterize complex linguistic preference in some specific linguistic decision making situations, however the tool should not be limited. We will explore more tools to realize such a flexible characterization by introducing new context-free grammars, grammars relying on specific circumstance, and construct fuzzy representations of new linguistic expression forms.

- We will explore application methodologies of T2F-representation model of CLE in other fields where CWW with CLE is necessary.

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