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Soft Computing Applications for Group Decision-making and Consensus Modeling

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A Feedback Mechanism Based on Granular Computing to Improve Consensus in GDM

Francisco Javier Cabrerizo, Francisco Chiclana, Ignacio Javier Pérez, Francisco Mata, Sergio Alonso and Enrique Herrera-Viedma

Abstract Group decision making is an important task in real world activities. It consists in obtaining the best solution to a particular problem according to the opinions given by a set of decision makers. In such a situation, an important issue is the level of consensus achieved among the decision makers before making a decision. For this reason, different feedback mechanisms, which help decision makers for reaching the highest degree of consensus possible, have been proposed in the literature. In this contribution, we present a new feedback mechanism based on granular computing to improve consensus in group decision making problems. Granular computing is a framework of designing, processing, and interpretation of information granules, which can be used to obtain a required flexibility to improve the level of consensus within the group of decision makers.

Keywords Group decision making · Consensus · Feedback mechanism · Granular computing

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1 Introduction

Group decision making (GDM) is utilized to get the best solution or solutions for a given problem using the preferences or opinions expressed by a group of decision makers [11, 18, 44]. In such a situation, each decision maker usually approaches the decision process from a different point of view. However, the decision makers have a common interest in obtaining a consensus or agreement before making the decision. In particular, in a GDM situation, there is a set of different alternatives to solve the problem and a group of decision makers that are usually required to express their opinions about the alternatives by means of a particular preference structure [13, 16].

An important issue in a GDM situation is the level of consensus achieved among the decision makers before making the decision. Usually, when decisions are made by a group of individuals, it is advisable that the decision makers are involved in a discussion process in which they talk about their reasons for making decisions with the aim of arriving at a sufficient level of consensus acceptable to all [6, 27]. If this discussion process is not carried out, solutions which are not well accepted by some decision makers could be obtained [6, 45], and therefore the decision makers might reject them. Due to it, a consensus process is usually carried out before obtaining a final solution in a GDM situation [1, 10, 15, 17, 27, 35, 51].

In a consensus process, an important step is the recommendations provided to the decision makers to improve the level of consensus. From this point of view, the first consensus approaches presented by the researchers of the GDM field can be considered as basic approaches because they are based on a moderator who gives the advice to the decision makers [5, 19, 20, 28–30]. The objective of the moderator in each discussion round is to address the consensus process towards success by achieving the highest consensus degree and reducing the number of decision makers outside of the agreement. However, a drawback of these approaches is that the moderator can introduce some subjectivity in the discussion process. To overcome it, new consensus approaches have been presented by providing to the moderator with better analysis tools or substituting the moderator figure. It makes more effective and efficient the discussion process.

In consensus approaches incorporating a feedback mechanism, which substitutes the moderator's actions, proximity measures are computed to evaluate the proximity between individual decision makers' opinions and the collective one [7, 22, 24, 25, 49]. These proximity measures are utilized to identify the opinions given by the decision makers which are contributing less to reach a high consensus level. The goal of the feedback mechanism is to give advice to those decision makers to find out the modifications they need to make in their preferences to achieve a solution with better consensus.

On the other hand, a novel data mining tool [31], the so called action rules [37], has been incorporated in consensus approaches to support and stimulate the discussion in the group. The aim of an action rule is to show how a subset of flexible attributes should be modified to achieve an expected change of the decision attribute for a subset of objects characterized by some values of the subset of stable attributes. In such a way, these action rules are utilized to suggest and indicate to the moderator with which decision makers and with respect to which preferences it may be expedient to deal.

In any case, decisions makers have to allow a certain degree of flexibility and be ready to make changes on their first opinions to obtain a higher level of consensus. In such a situation, information granularity [38, 40, 41, 50] may become relevant because it gives to the decision makers a level of flexibility using some first opinions that can be adjusted in order to improve the consensus level among the decision makers.

The objective of this contribution is to develop a new feedback mechanism based on granular computing to improve the consensus achieved among the decision makers in a GDM situation. Granular computing is a paradigm that represents and processes information in form of information granules [2, 38], that are complex information entities arising in the process of abstraction of data and derivation of knowledge from information [4]. In particular, an allocation of information granularity is used in the feedback mechanism as a key component to suggest advice to the decision makers in order to improve the consensus.

This contribution is organized as follows. In Sect. 2, we introduce the description of a GDM situation and describe the process carried out to solve it. Section 3 presents the feedback mechanism based on granular computing proposed here to improve the consensus achieve among the decision makers involved in a GDM situation. An example of application of the feedback mechanism is illustrated in Sect. 4. Finally, some conclusions and future work are pointed out in Sect. 5.

2 GDM Process

A GDM process is defined as a situation in which a group of two or more decision makers, $E = \{e_1, e_2, \dots, e_m\}$ $(m \geq 2)$, provide their opinions or preferences about a solution set of possible alternatives, $X = \{x_1, x_2, \dots, x_n\}$ $(n \geq 2)$, to achieve a common solution [11, 18, 27]. In particular, if the decision process is defined in a fuzzy context, the goal is to rank the alternatives from best to worst, associating with the alternatives some degrees of preferences given in the unit interval.

In the literature we can find different representation structures in which the decision makers can convey their judgments [13, 14]. Among them, the fuzzy preference relation [34, 47, 52] has been widely utilized by the researchers because this representation structure offers a very expressive representation and, in addition, it presents good properties allowing to operate with it easily [13, 23].

Definition 1 A fuzzy preference relation PR on a set of alternatives X is a fuzzy set on the Cartesian product $X \times X$, i.e., it is characterized by a membership function $\mu_{PR} : X \times X \to [0, 1]$.

A fuzzy preference relation PR is usually represented by the $n \times n$ matrix $PR = (pr_{ij})$, being $pr_{ij} = \mu_{PR}(x_i, x_j)$ ($\forall i, j \in \{1, \ldots, n\}$) interpreted as the preference degree or intensity of the alternative x_i over x_j: $pr_{ij} = 0.5$ indicates indifference between x_i and x_j ($x_i \sim x_j$), $pr_{ij} = 1$ indicates that x_i is absolutely preferred to x_j, and $pr_{ij} > 0.5$ indicates that x_i is preferred to x_j ($x_i \succ x_j$). Based on this interpretation, we have that $pr_{ii} = 0.5 \; \forall i \in \{1, \ldots, n\}$ ($x_i \sim x_i$). Since pr_{ii}'s (as well as the corresponding elements on the main diagonal in some other matrices) do not matter, it will be written as '–' instead of 0.5 [25, 28].

GDM processes are usually faced by carrying out two processes before a final solution can be provided [1, 30]:

- A consensus process referring to how to get the highest degree of agreement among the decision makers.
- A selection process obtaining the final solution using the opinions expressed by the group of decision makers.

In the following subsections, both the consensus process and the selection process are described in detail.

2.1 Consensus Process

A consensus process is an iterative and a dynamic discussion process carried out among the members of a group, coordinated by a moderator who helps them bring their preferences closer. On the one hand, if the agreement among the decision makers is lower than a threshold, the moderator would urge them to discuss their preferences further in an effort to bring them closer. On the other hand, if the consensus level is higher than the threshold, the moderator would apply the selection process with the aim of obtaining the final consensus solution to the problem [27, 36].

An important step of a consensus process is the assessment of the agreement achieved among the group of decision makers. To obtain it, coincidence existing among the decision makers is computed [8, 21]. Consensus approaches usually obtain consensus degrees, utilized to evaluate the current level of agreement among the decision makers' preferences, given at three different levels of a fuzzy preference relation [8, 19]: pairs of alternatives, alternatives, and relation. According to it, the computation of the consensus degrees is performed as follows once the fuzzy preference relations have been provided by all the decision makers within the group [8, 25, 49]:

1. For each pair of decision makers (e_k, e_l) ($k = 1, \ldots, m-1,\; l = k+1, \ldots, m$) a similarity matrix, $SM^{kl} = (sm_{ij}^{kl})$, is defined as:

$$sm_{ij}^{kl} = 1 - |pr_{ij}^k - pr_{ij}^l| \tag{1}$$

2. Then, a consensus matrix, $CM = (cm_{ij})$, is calculated by aggregating all the $(m-1) \times (m-2)$ similarity matrices using an aggregation function, ϕ:

$$cm_{ij} = \phi(sm_{ij}^{kl}),\ k = 1, \dots, m-1,\ l = k+1, \dots, m \tag{2}$$

   Here, the arithmetic mean is utilized as aggregation function. However, different aggregation operators could be utilized according to the particular properties that we want to implement.
3. Once the consensus matrix has been calculated, the consensus degrees are obtained at the three different levels of a fuzzy preference relation:

   a. *Consensus degree on the pairs of alternatives.* The consensus degree on a pair of alternatives (x_i, x_j), called cp_{ij}, is defined to measure the consensus degree among all the decision makers on that pair of alternatives. In this case, this is expressed by the element of the collective similarity matrix CM:

$$cp_{ij} = cm_{ij} \tag{3}$$

   The closer cp_{ij} to 1, the greater the agreement among all the decision makers on the pair of alternatives (x_i, x_j).

   b. *Consensus degree on the alternatives.* The consensus degree on the alternative x_i, called ca_i, is defined to measure the consensus degree among all the decision makers on that alternative:

$$ca_i = \frac{\sum_{j=1;j\neq i}^{n}(cp_{ij} + cp_{ji})}{2(n-1)} \tag{4}$$

   c. *Consensus degree on the relation.* The consensus degree on the relation, called cr, expresses the global consensus degree among all the decision makers' opinions. It is computed as the average of all the consensus degree for the alternatives:

$$cr = \frac{\sum_{i=1}^{n} ca_i}{n} \tag{5}$$

The consensus degree of the relation, cr, is the value used to control the consensus state. The closer cr is to 1, the greater the agreement among all the decision makers' preferences.

2.2 Selection Process

Once the consensus level is higher than a specified threshold, the selection process is carried out in two sequential steps:

1. Aggregation step defining a collective fuzzy preference relation that indicates the global preference between every pair of alternatives.
2. Exploitation step transforming the global information about the alternatives into a global ranking of them, from which a set of alternatives is derived.

In what follows, we present in more detail both the aggregation step and the exploitation step of a selection process.

2.2.1 Aggregation Step

The aim of this step is to obtain a collective fuzzy preference relation, $PR^c = (pr^c_{ij})$, by aggregating all individual fuzzy preference relations, $\{PR^1, \ldots, PR^m\}$, given by the decision makers involved in the problem. Each value pr^c_{ij} represents the preference of the alternative x_i over the alternative x_j according to the majority of the decision makers' assessments. To do so, an OWA operator is used [53].

Definition 2 An OWA operator of dimension n is a function $\phi : [0,1]^n \longrightarrow [0,1]$, that has a weighting vector associated with it, $W = (w_1, \ldots, w_n)$, with $w_i \in [0,1]$, $\sum_{i=1}^n w_i = 1$, and it is defined according to the following expression:

$$\phi_W(a_1, \ldots, a_n) = W \cdot B^T = \sum_{i=1}^{m} w_i \cdot a_{\sigma(i)} \tag{6}$$

being $\sigma : \{1, \ldots, n\} \longrightarrow \{1, \ldots, n\}$ a permutation such that $p_{\sigma(i)} \geq a_{\sigma(i+1)}$, $\forall i = 1, \ldots, n-1$, i.e., $a_{\sigma(i)}$ is the i-highest value in the set $\{a_1, \ldots, a_n\}$.

OWA operators fill the gap between the operators Min and Max. It can be immediately verified that OWA operators are commutative, increasing monotonous and idempotent, but in general not associative.

In order to classify OWA aggregation operators with regards to their localization between "or" and "and", Yager [53] introduced the measure of *orness* associated with any vector W expressed as:

$$\text{orness}(W) = \frac{1}{n-1} \sum_{i=1}^{n} (n-i) w_i \tag{7}$$

This measure, which lies in the unit interval, characterizes the degree to which the aggregation is like an "or" (Max) operation. Note that the nearer W is to an "or", the closer its measure is to one; while the nearer it is to an "and", the closer is to zero. As we move weight up the vector we increase the orness(W), while moving weight down causes us to decrease orness(W). Therefore, an OWA operator with much of nonzero weights near the top will be an "orlike" operator (orness(W) ≥ 0.5), and when much of the weights are nonzero near the bottom, the OWA operator will be "andlike" (orness(W) < 0.5).

A natural question in the definition of the OWA operator is how to obtain the associated weighting vector. In [53], it was defined an expression to obtain W that allows to represent the concept of fuzzy majority [28] by means of a fuzzy linguistic non-decreasing quantifier Q [58]:

$$w_i = Q\left(\frac{i}{n}\right) - Q\left(\frac{i-1}{n}\right),\ i = 1, \dots, n \tag{8}$$

The membership function of Q is given by Eq. (9), with $a, b, r \in [0, 1]$. Some examples of non-decreasing proportional fuzzy linguistic quantifiers are: "most" (0.3, 0.8), "at least half" (0, 0.5), and "as many as possible" (0.5, 1).

$$Q(r) = \begin{cases} 0 & \text{if } r < a \\ \frac{r-a}{b-a} & \text{if } a \le r \le b \\ 1 & \text{if } r > a \end{cases} \tag{9}$$

When a fuzzy quantifier Q is used to compute the weights of the OWA operator ϕ, it is symbolized by ϕ_Q.

2.2.2 Exploitation Step

The aim of this step is to obtain a rank of the alternatives. To do so, the concept of fuzzy majority (of alternatives) and the OWA operator are used to compute two choice degrees of alternatives: the quantifier-guided dominance degree (*QGDD*) and the quantifier-guided non-dominance degree (*QGNDD*) [9, 26]. They will act over the collective preference relation resulting in a global ranking of the alternatives, from which the solution will be obtained.

- $QGDD_i$: It quantifies the dominance that one alternative has over all the others in a fuzzy majority sense. It is obtained as follows:

$$QGDD_i = \phi_Q(pr^c_{i1}, pr^c_{i2}, \dots, pr^c_{i(i-1)}, pr^c_{i(i+1)}, \dots, pr^c_{in}) \tag{10}$$

- $QGNDD_i$: It gives the degree in which each alternative is not dominated by a fuzzy majority of the remaining alternatives. It is obtained as follows:

$$QGNDD_i = \phi_Q(1 - p^s_{1i}, 1 - p^s_{2i}, \dots, 1 - p^s_{(i-1)i}, 1 - p^s_{(i+1)i}, \dots, 1 - p^s_{ni}) \tag{11}$$

where $p^s_{ji} = max\{pr^c_{ji} - pr^c_{ij}, 0\}$ represents the degree in which x_i is strictly dominated by x_j. When the fuzzy quantifier represents the statement "all", whose algebraic aggregation corresponds to the conjunction operator Min, this non-dominance degree coincides with Orlovski's non-dominated alternative concept [34].

Two different policies can be used to carry out the application of both choice degrees: a sequential policy or a conjunctive policy [12, 26]. In the sequential policy, one of the choice degrees is selected and applied to the set of alternatives according to the opinions given by the decision makers, obtaining a selection set of alternatives. If there is more than one alternative in this selection set, then, the other choice degree is applied to select the alternative of this set with the best second choice degree. In the conjunctive policy, both choice degrees are applied to the set of alternatives, obtaining two selection sets of alternatives. The final selection set of alternatives is obtained as the intersection of these two selection sets of alternatives. As it is possible to get and empty selection set, the latter conjunction selection process is more restrictive than the former sequential selection process.

3 A Feedback Mechanism Based on Granular Computing

In the discussion process, if the consensus achieved among the decision makers is lower than a consensus threshold, the decision makers must discuss and modify their opinions. It is done by a feedback mechanism, which gives advice to the decision makers on how to change their preferences in order to increase the consensus. In addition, the feedback mechanism usually substitutes the moderators' actions with the aim of avoiding the subjectivity that the moderator can introduce in the discussion process.

In order to improve the consensus, the decision makers have to accept some modifications in their initial preferences by allowing a certain flexibility. If fuzzy preference relations are used to represent the assessments provided by the decision makers, this flexibility could be brought by allowing the fuzzy preference relations to be granular rather than numeric. That is, the feedback mechanism proposed here assumes that the entries of a fuzzy preference relation are information granules instead of plain numbers. In such a way, the feedback mechanism elevates the fuzzy preference relations to their granular format.

To emphasize that the feedback mechanism uses granular fuzzy preference relations, the notation $\boldsymbol{G}(PR)$ is employed. Here, $\boldsymbol{G}(.)$ represents a specific granular formalism being utilized. For example, as information granules we could use fuzzy sets [54–57], rough sets [46], probability density functions [59], intervals [3], and others. In particular, information granularity is used here by the feedback mechanism as an important computational and conceptual resource being exploited as a means to give advice to the decision makers in order to improve the consensus among them. That is, granularity is used as synonymous of flexibility. It facilitates the increase of the agreement achieved among the group of decision makers.

In this contribution, the feedback mechanism uses intervals to articulate the granularity of information. Therefore, the length of the intervals can be sought as a level of granularity α. In addition, because interval-valued fuzzy preference relations are used, $\boldsymbol{G}(PR) = \boldsymbol{P}(PR)$, where $\boldsymbol{P}(.)$ denotes a family of intervals.

The concept of interval-valued fuzzy preference relations is employed by the feedback mechanism to generate recommendations to the decision makers in order to improve the consensus among them. Specifically, the level of consensus achieved among the decision makers is used as a performance index.

In what follows, we give the details both the performance index to be optimized and its optimization, which, given the nature of the required task, is carried out by the Particle Swarm Optimization (PSO) framework [32].

3.1 The Performance Index

The level of granularity is used by the feedback mechanism to improve the agreement achieved among the decision makers by generating recommendations in order to bring all preferences close to each other. Decision makers should feel comfortable when accepting the modifications provided by the feedback mechanism located within the bounds established by the fixed level of granularity α.

Advice is generated by the feedback mechanism by maximizing the global consensus degree among the decision makers. It is calculated in term of the consensus degree on the relation (see Sect. 2.1):

$$O = cr \tag{12}$$

The optimization problem reads as follows:

$$\text{Max}_{PR^1, PR^2, \ldots, PR^m \in \boldsymbol{P}(PR)} O \tag{13}$$

This maximization problem is performed by the feedback mechanism for all interval-valued fuzzy preference relations that are possible according to the fixed level of information granularity α. This truth is emphasized by incorporating the granular form of the fuzzy preference relations, that is, $PR^1, \ldots, PR^m$, are elements of the family of interval-valued fuzzy preference relations, $\boldsymbol{P}(PR)$.

Due to the nature of the not straight relationship between the optimized fuzzy preference relations, this optimization problem is not an easy task. The optimized fuzzy preference relations are chosen from a quite large search space formed by $\boldsymbol{P}(PR)$ and, therefore, it requires the use of an advanced technique of global optimization.

Among the different techniques of global optimization, the PSO framework [32] is used in this contribution because it does not come with a prohibitively high level of computational overhead as this is the case of other global optimization techniques and it offers a substantial level of optimization flexibility, being a viable alternative for this problem. However, it should be noted that other techniques as genetic algorithms, evolutionary optimization, simulated annealing, and so on, could be also used.

3.2 PSO Framework

As aforementioned, the PSO environment is employed in the feedback mechanism to optimize the fuzzy preference relations coming from the space of interval-valued fuzzy preference relations because this technique is a viable alternative for the problem at hand.

PSO is an evolutionary computational method based on the social behavior metaphor, which was developed by Kennedy and Eberhart [32, 33]. In this technique, a population of random candidate solutions, called particles, is initialized. Then, a randomized velocity is assigned to each particle, which is iteratively moved though the search-space according to simple mathematical formulae over the particle's velocity and position. The movement of each particle is attracted towards the position of the best fitness achieved so far by the particle itself (z_p) and by the position of the best fitness achieved so far across the whole population (z_g) [32, 48] (see Fig. 1).

An important issue in the PSO framework is how to find a suitable mapping between the representation of the particle and the problem solution. In a GDM context, each particle represents a vector in which the elements are located in the unit interval. That is, if the GDM problem is set up with a group of m decision makers and a set of n alternatives, the number of elements of the particle will be $m \cdot n(n-1)$.

Let us consider an element pr_{ij} and assume a level of granularity α located in the [0, 1] interval. If we use an initial fuzzy preference relation expressed by a decision maker, the interval of admissible values of this element of $\boldsymbol{P}(PR)$ is equal to:

$$[a, b] = [\max(0, pr_{ij} - \alpha/2), \min(1, pr_{ij} + \alpha/2)] \tag{14}$$

As an example, if we have $pr_{ij} = 0.8$, being the level of granularity α equal to 0.2 and the corresponding element of the particle x equal to 0.3, then, the corresponding interval of the interval-valued fuzzy preference relation calculated using Eq. (14) is $[a, b] = [0.70, 0.90]$. Using the expression $z = a + (b - a)x$, the modified value of pr_{ij} becomes equal to 0.76.

Another important question in the PSO framework is how to assess the performance of each particle during its movement. To do so, a performance index or fitness function is used. In the GDM context considered in this contribution, the PSO aims to maximize the level of agreement achieved among the decision makers involved in the problem. Hence, the following fitness function f will be used:

$$f = O \tag{15}$$

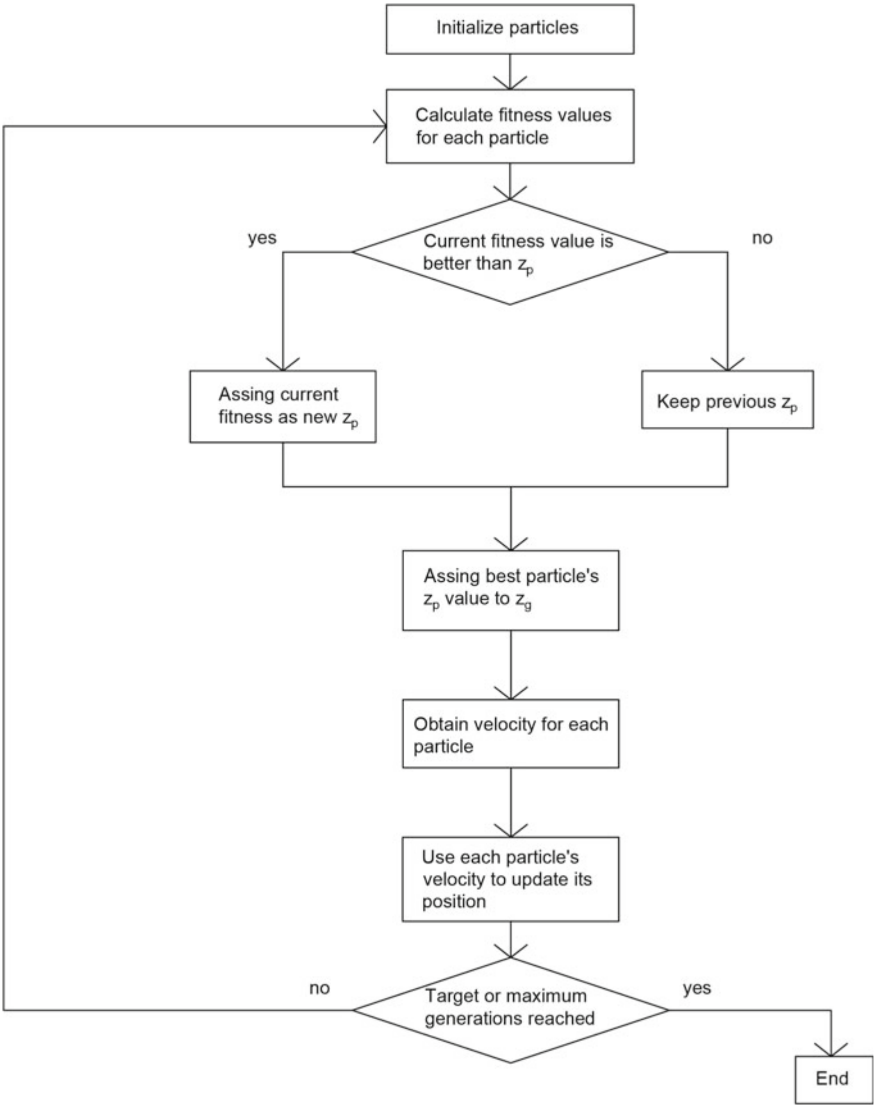

Fig. 1 PSO flowchart

where O is the optimization criterion presented previously. Here, the higher the value of f is, the better the particle is.

It should be pointed out that the generic form of the PSO framework is employed in this contribution. Therefore, the updates of the velocity of a particle are performed in the form $\mathbf{v}(\mathrm{t}+1) = w \times \mathbf{v}(\mathrm{t}) + c_1\mathbf{a} \cdot (\mathbf{z}_p - \mathbf{z}) + c_2\mathbf{b} \cdot (\mathbf{z}_g - \mathbf{z})$. Here, $\cdot$ means a vector multiplication carried out coordinate-wise, "t" is an index of the generation, $\mathbf{z}_g$

denotes the best position overall and developed so far across the swarm, and $\mathbf{z}_p$ is the best position obtained so far for the particle under study. The inertia component, called w, scales the actual velocity $\mathbf{v}(\mathrm{t})$ and stresses some effect of resistance to modify the actual velocity. Its value is usually 0.2 and it is kept constant through the process [39]. On the other hand, **a** and **b** represents vectors of random numbers that are drawn from the uniform distribution over the unit interval. These vectors help from a proper mix of the components of the velocity. Finally, in iteration "t + 1", the particle's position is calculated as: $\mathbf{z}(\mathrm{t}+1) = \mathbf{z}(\mathrm{t}) + \mathbf{v}(\mathrm{t}+1)$.

Once the PSO algorithm has optimized the fuzzy preference relations coming from the space of interval-valued fuzzy preference relation, the feedback mechanism advise the decision makers the modifications that they should put into practice in their opinions in order to improve the consensus among them.

4 An Illustrative Example

An example of application of the proposed feedback mechanism is presented in this section. It helps quantifying the improvement of the consensus when the feedback mechanism is applied.

Let us suppose that a patient presents some symptoms, being all of them common to several diseases, and some doctors, who are specialist in different diagnosis, have to jointly diagnose the disease that the patient has contracted. This situation can be defined as a GDM problem in which there are a set of four possible diseases (alternatives), $\{x_1, x_2, x_3, x_4\}$, and a set of four doctors (decision makers), $\{e_1, e_2, e_3, e_4\}$.

4.1 First Consensus Round

At the first stage of the discussion process, the four doctors express the following fuzzy preference relations:

$$PR^1 = \begin{pmatrix} - & 0.30 & 0.70 & 0.50 \\ 0.70 & - & 0.70 & 0.60 \\ 0.40 & 0.20 & - & 0.30 \\ 0.70 & 0.30 & 0.80 & - \end{pmatrix} \quad PR^2 = \begin{pmatrix} - & 0.30 & 0.60 & 0.70 \\ 0.80 & - & 0.70 & 0.20 \\ 0.20 & 0.40 & - & 0.50 \\ 0.20 & 0.60 & 0.50 & - \end{pmatrix}$$

$$PR^3 = \begin{pmatrix} - & 0.80 & 0.50 & 0.20 \\ 0.20 & - & 0.60 & 0.90 \\ 0.50 & 0.30 & - & 0.70 \\ 0.60 & 0.20 & 0.20 & - \end{pmatrix} \quad PR^4 = \begin{pmatrix} - & 0.90 & 0.20 & 0.70 \\ 0.30 & - & 0.60 & 0.30 \\ 0.90 & 0.40 & - & 0.50 \\ 0.40 & 0.90 & 0.50 & - \end{pmatrix}$$

4.1.1 Consensus Measures

Once the doctors have provided their opinions, the consensus measures are calculated as described in Sect. 2.1.

The consensus matrix is equal to:

$$CM = \begin{pmatrix} - & 0.62 & 0.73 & 0.72 \\ 0.63 & - & 0.93 & 0.60 \\ 0.63 & 0.88 & - & 0.80 \\ 0.72 & 0.60 & 0.70 & - \end{pmatrix}$$

The element (i, j) of the consensus matrix represents the consensus degrees on the pair of alternatives (x_i, x_j).

The consensus degrees on the alternatives are:

$$\begin{aligned} ca_1 &= 0.67 \\ ca_2 &= 0.71 \\ ca_3 &= 0.78 \\ ca_4 &= 0.69 \end{aligned}$$

And the consensus on the relation is:

$$cr = 0.71$$

Assuming a minimum consensus threshold equal to 0.75, the selection process cannot be applied because the consensus achieved among the doctors is lower than the minimum consensus threshold. Therefore, the feedback mechanism has to be applied in order to improve the agreement.

4.1.2 Feedback Mechanism

The aim of the feedback mechanism is to support the doctors' changes in their fuzzy preference relations in order to increase the consensus.

First, it should be pointed out that, as a result of an intensive experimentation, the following values of the parameters were selected in the PSO algorithm:

- 50 particles formed the swarm. This value was found to obtain stable results. That is, identical or similar results were obtained in successive runs of the PSO algorithm.
- 200 generations or iterations were carried out as it was observed that were no further modifications of the values of the fitness functions after this number of iterations.
- c_1 and c_2 were set as 2 because these values are commonly found in the existing literature.

Considering a given level of granularity $\alpha = 0.4$, the recommended fuzzy preference relations generated by the feedback mechanism are as follows:

$$PR^1 = \begin{pmatrix} - & 0.31 & 0.60 & 0.57 \\ 0.63 & - & 0.58 & 0.63 \\ 0.35 & 0.22 & - & 0.31 \\ 0.73 & 0.40 & 0.80 & - \end{pmatrix} \quad PR^2 = \begin{pmatrix} - & 0.33 & 0.61 & 0.69 \\ 0.85 & - & 0.60 & 0.27 \\ 0.26 & 0.39 & - & 0.50 \\ 0.15 & 0.61 & 0.50 & - \end{pmatrix}$$

$$PR^3 = \begin{pmatrix} - & 0.78 & 0.45 & 0.13 \\ 0.19 & - & 0.49 & 0.78 \\ 0.46 & 0.35 & - & 0.65 \\ 0.58 & 0.13 & 0.30 & - \end{pmatrix} \quad PR^4 = \begin{pmatrix} - & 0.88 & 0.20 & 0.66 \\ 0.25 & - & 0.65 & 0.36 \\ 0.81 & 0.53 & - & 0.46 \\ 0.48 & 0.86 & 0.50 & - \end{pmatrix}$$

4.2 Second Consensus Round

In the second consensus round, we assume that the doctors agree the advice generated by the feedback mechanism. Then, the consensus measures are computed again.

4.2.1 Consensus Measures

The consensus matrix is equal to:

$$CM = \begin{pmatrix} - & 0.64 & 0.77 & 0.71 \\ 0.60 & - & 0.92 & 0.70 \\ 0.71 & 0.84 & - & 0.82 \\ 0.69 & 0.60 & 0.75 & - \end{pmatrix}$$

The element (i,j) of the consensus matrix represents the consensus degrees on the pair of alternatives (x_i, x_j).

The consensus degrees on the alternatives are:

$$ca_1 = 0.68$$
$$ca_2 = 0.72$$
$$ca_3 = 0.80$$
$$ca_4 = 0.71$$

And the consensus on the relation is:

$$cr = 0.73$$

Because the consensus achieved among the doctors is lower than the minimum consensus threshold, the feedback mechanism has to be applied again in order to increase the consensus.

4.2.2 Feedback Mechanism

Considering the same values of the parameters in the PSO algorithm as in the first round, the new recommended fuzzy preference relations provided by the feedback mechanism are the following:

$$PR^1 = \begin{pmatrix} - & 0.31 & 0.60 & 0.57 \\ 0.63 & - & 0.58 & 0.63 \\ 0.35 & 0.22 & - & 0.31 \\ 0.73 & 0.40 & 0.80 & - \end{pmatrix} \quad PR^2 = \begin{pmatrix} - & 0.33 & 0.61 & 0.69 \\ 0.85 & - & 0.60 & 0.27 \\ 0.26 & 0.39 & - & 0.50 \\ 0.15 & 0.61 & 0.50 & - \end{pmatrix}$$

$$PR^3 = \begin{pmatrix} - & 0.78 & 0.45 & 0.13 \\ 0.19 & - & 0.49 & 0.78 \\ 0.46 & 0.35 & - & 0.65 \\ 0.58 & 0.13 & 0.30 & - \end{pmatrix} \quad PR^4 = \begin{pmatrix} - & 0.88 & 0.20 & 0.66 \\ 0.25 & - & 0.65 & 0.36 \\ 0.81 & 0.53 & - & 0.46 \\ 0.48 & 0.86 & 0.50 & - \end{pmatrix}$$

4.3 *Third Consensus Round*

As in the above round, it is assumed that the doctors accept the preferences generated by the feedback mechanism and, therefore, the consensus measures are obtained again.

4.3.1 Consensus Measures

The consensus matrix is equal to:

$$CM = \begin{pmatrix} - & 0.65 & 0.78 & 0.71 \\ 0.60 & - & 0.90 & 0.82 \\ 0.68 & 0.84 & - & 0.82 \\ 0.69 & 0.64 & 0.78 & - \end{pmatrix}$$

The element (i,j) of the consensus matrix represents the consensus degrees on the pair of alternatives (x_i, x_j).

The consensus degrees on the alternatives are:

$$\begin{aligned} ca_1 &= 0.69 \\ ca_2 &= 0.75 \\ ca_3 &= 0.81 \\ ca_4 &= 0.75 \end{aligned}$$

And the consensus on the relation is:

$$cr = 0.75$$

In this round, the consensus is equal to the minimum consensus threshold and, therefore, the selection process can be applied in order to rank the alternatives.

4.4 Selection Process

The goal of the selection process is to obtain a ranking of the alternatives from best to worst according to the preferences given by the doctors. To do so, an aggregation step and an exploitation step are carried out.

4.4.1 Aggregation

The OWA operator is used to aggregation the fuzzy preference relations given by the doctors. We make use of the linguistic quantifier "most", defined in Sect. 2.2.1, which, applying Eq. (8), generates a weighting vector of four values to obtain each collective preference value pr^c_{ij}. As example, the collective preference value pr^c_{12} is computed as follows:

$$\begin{aligned} & w_1 = Q(1/4) - Q(0) = 0 - 0 = 0 \\ & w_2 = Q(2/4) - Q(1/4) = 0.4 - 0 = 0.4 \\ & w_3 = Q(3/4) - Q(2/4) = 0.9 - 0.4 = 0.5 \\ & w_4 = Q(1) - Q(3/4) = 1 - 0.9 = 0.1 \\ & pr^c_{12} = w_1 \cdot pr^4_{12} + w_2 \cdot pr^3_{12} + w_3 \cdot pr^2_{12} + w_4 \cdot pr^1_{12} = 0.51 \end{aligned}$$

Then, the collective fuzzy preference relation is:

$$PR^c = \begin{pmatrix} - & 0.51 & 0.48 & 0.56 \\ 0.40 & - & 0.58 & 0.46 \\ 0.38 & 0.35 & - & 0.46 \\ 0.49 & 0.46 & 0.48 & - \end{pmatrix}$$

4.4.2 Exploitation

Using again the same linguistic quantifier "most" and Eq. (8), we obtain the following weighting vector $W = (w_1, w_2, w_3)$:

$$
\begin{aligned}
w_1 &= Q(1/3) - Q(0) = 0.07 - 0 = 0.07 \\
w_2 &= Q(2/3) - Q(1/3) = 0.73 - 0.07 = 0.66 \\
w_3 &= Q(1) - Q(2/3) = 1 - 0.73 = 0.27
\end{aligned}
$$

Using, for example, the quantifier guided dominance degree, $QGDD_i$, we obtain the following values:

$$
\begin{aligned}
QGDD_1 &= 0.51 \\
QGDD_2 &= 0.45 \\
QGDD_3 &= 0.38 \\
QGDD_4 &= 0.47
\end{aligned}
$$

Finally, applying the sequential policy with the quantifier guided dominance degree, the following ranking of alternatives is obtained:

$$x_1 \succ x_4 \succ x_2 \succ x_3$$

Therefore, according to the doctors' judgments, the patient's symptoms correspond to the first disease.

Finally, it should be pointed out that here a granularity level of 0.4 has been used. However, the higher the level of granularity is, the higher the level of flexibility is and, hence, the possibility of obtaining a higher consensus. Anyway, if the level of granularity is very high, the fuzzy preference relations generated by the feedback mechanism could be very different in comparison with those provided by the decision makers and, in such a way, they could reject them.

5 Conclusions and Future Work

In this contribution, we have presented a feedback mechanism based on granular computing to improve the consensus achieved among the decision makers in a GDM situation. The feedback mechanism assumes the concept of granular fuzzy preference relation and accentuates the role of information granularity, which is regarded as an important resource to be exploited as a means to improve the consensus achieved among the decision makers involved in the problem. In particular, the granularity level has been treated as synonymous of flexibility, which has been used to optimize a certain optimization criterion capturing the essence of reconciliation of the individual preferences. It has also been shown that the PSO environment is a suitable optimization framework for this purpose. However, it should be noted that the

PSO optimizes the fitness function but there is no guarantee that the result is optimal rather that the solution is the best one being formed by the PSO environment.

In the future, it is worth continuing this research in several directions:

- In this contribution, intervals have been used as information granules in the granular representation of the fuzzy preference relations. However, other formalism as, for instance, rough sets or fuzzy sets, could be utilized in the granular representation of the preferences.
- The feedback mechanism has been proposed in a fixed framework, that is, in a situation in which the decision makers and the alternatives do not change during the decision making process. However, with the aim of making the process more realistic, the approach should be able to deal with changeable elements. In such a way, the feedback mechanism should be able to deal with a dynamic environment [42, 43].

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