UNIVERSIDAD DE JAÉN

Departamento de Informática

Procesos de Alcance de Consenso ante Nuevos Retos en Toma de Decisión en Grupo:

Grandes Grupos y Recomendación

Memoria de Tesis presentada por

Francisco José Quesada Real

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para optar al grado de Doctor en Informática

Director:
Dr. Luis Martínez López

Jaén, 29 de abril de 2019

Llegados a este punto, culmen del itinerario académico, quisiera dedicar esta tesis doctoral a mis abuelos, que a pesar de carecer de formación reglada, con su sencillez, humildad y esfuerzo me han legado lecciones para la vida más valiosas que cualquier enseñanza oficial.

La memoria titulada *Procesos de Alcance de Consenso ante Nuevos Retos en Toma de Decisión en Grupo: Grandes Grupos y Recomendación*, que presenta D. Francisco José Quesada Real para optar al grado de doctor, ha sido realizada dentro del Programa de Doctorado en Tecnologías de la Información y la Comunicación de la Universidad de Jaén bajo la dirección del Dr. D. Luis Martínez López. Para su evaluación, esta memoria se presenta como un conjunto de trabajos publicados, acogiéndose y ajustándose a lo establecido en el punto 2 del artículo 25 del *Reglamento de los Estudios de Doctorado de la Universidad de Jaén*, aprobado en febrero de 2012 y modificado en febrero de 2019.

En Jaén, a 29 de abril de 2019

El Doctorando	El Director
Fdo: D. Francisco José Quesada Real	Fdo: Dr. D. Luis Martínez López

Agradecimientos

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A todos ellos, mil gracias.

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Capítulo 1

Introducción

En este capítulo se realiza una introducción general a la memoria de tesis doctoral titulada: *Procesos de Alcance de Consenso ante Nuevos Retos en Toma de Decisión en Grupo: Grandes Grupos y Recomendación*. En él, queda recogida la motivación que justifica la investigación realizada, así como los objetivos planteados a partir de dicha motivación. Por último, se describe la estructura y la organización de los contenidos de esta memoria.

1.1. Motivación

La Toma de Decisión (TD) es un proceso habitual dentro de las actividades cotidianas de los seres humanos, que consiste en elegir la mejor opción de entre un conjunto de alternativas posibles dentro de un entorno determinado [111]. Concretamente, la mayoría de procesos de TD reales se desarrollan en ambientes de incertidumbre [83], lo que hace necesario el uso de enfoques y herramientas que permitan adaptarse a estos ambientes, entre los que destaca la Lógica Difusa [148]. Por otra parte, la frecuente necesidad de múltiples puntos de vista en el proceso de TD ha dado lugar a los llamados problemas de Toma de Decisión en Grupo (TDG), donde varios expertos deben alcanzar una solución común a un problema de decisión [17].

Tradicionalmente, los problemas de TDG bajo incertidumbre han sido resueltos mediante un proceso de selección de alternativas [47, 116] en el que la mejor alternativa era seleccionada. Sin embargo, el hecho de llegar a una decisión sin llevar a cabo un proceso de negociación entre los participantes, podía dar lugar a que no hubiese un nivel de acuerdo aceptable en torno a dicha solución, conllevando a que ésta no fuese aceptada por todos los miembros del grupo de expertos [118]. Para solventar este problema surgen los Procesos de Alcance de Consenso (PACs). En estos procesos cada uno de los expertos participantes se compromete a cumplir un contrato de colaboración [87] con el objetivo de que todos los participantes pongan todo de su parte para conseguir un alto nivel de acuerdo antes de tomar una decisión. Para lograr este objetivo, se lleva a cabo un proceso de discusión entre los expertos, que sirve para que éstos puedan ir modificando sus preferencias de acuerdo a las necesidades del PAC, estando habitualmente guiados bajo la supervisión de un moderador humano [17, 54, 118].

A lo largo de las últimas décadas han sido numerosas las contribuciones que se han realizado en el área de los PACs, destacando por su relevancia algunos modelos de consenso tales como los recogidos en [33,53,129]. Del mismo modo existen propuestas con la finalidad de que los PACs puedan ser integrados en sistemas de soporte a la decisión [3, 101, 102, 154].

El concepto de *consenso* ha evolucionado a lo largo del tiempo. Inicialmente se consideraba una visión clásica, entendiéndolo como un acuerdo total entre todos los expertos en todas sus preferencias. En la práctica, normalmente, este acuerdo era difícil de alcanzar. Por este motivo, la visión clásica fue evolucionando hasta enfoques más flexibles [59, 64] donde se utilizan medidas de consenso difusas para evaluar diferentes niveles de acuerdo parciales. Entre dichos enfoques destaca la idea de "*soft consensus*" propuesta por J. Kacprzyk y basada en el concepto de mayoría difusa [59]. De este modo, existe consenso cuando "la mayoría de expertos participantes en el problema, está de acuerdo en sus opiniones sobre las alternativas importantes". Un aspecto fundamental de los modelos de consenso bajo el punto de vista de estos nuevos enfoques, es la elección de medidas de consenso adecuadas para evaluar el nivel de acuerdo global alcanzado [54]. Estas medidas de consenso están normalmente basadas en el uso de medidas de similitud entre expertos, y operadores de agregación [7, 18, 157].

Clásicamente, los problemas de TDG eran resueltos por un número reducido de expertos, que reunidos en un mismo lugar llevaban a cabo el proceso de TD. Sin embargo, hoy en día esto ha cambiado considerablemente gracias al auge de Internet y al desarrollo de la tecnología móvil, permitiendo así llevar a cabo procesos de TD sin necesidad de que los participantes tengan que estar físicamente presentes en un mismo lugar. La eliminación de la barrera de la presencialidad física así como el desarrollo de las redes sociales, han abierto un amplio abanico de nuevos escenarios en el ámbito de la TD, destacando la posibilidad de incrementar notablemente el número de participantes en estos procesos. Sin embargo, estos avances también han conllevado la aparición de nuevos retos en Toma de Decisión en Grupo con Grandes Grupos (TDGGG), también denominados como problemas de Toma de Decisión en Grupo a Gran Escala (TDGGE) [21, 22, 159], donde el número de participantes es muy superior a la visión clásica en la que sólo se contemplaba un número reducido de expertos [13, 17–19, 21]. En [24] se propone considerar TDGGE cuando el número de expertos es superior a 20, aunque la intención es que puedan ser cientos o miles de expertos. Ejemplos de este tipo de problemas los podemos encontrar en empresas multinacionales, democracia electrónica [66], redes sociales [128], o gestión de crisis [65] y situaciones de emergencia [138] entre otros. Liu y otros clasificaron en [82] las principales líneas de investigación relacionadas con los problemas de TDGGE, destacando las siguientes categorías: (i) TDGGE basados en métodos de cluster, (ii) TDGGE aplicados a PACs, (iii) métodos de TDGGE y (iv) sistemas de soporte para la TDGGE.

En estos problemas, el incremento del número de expertos participantes añade complejidad a la hora de controlar y supervisar el proceso de TD, a diferencia de los problemas de TDG clásicos donde esta gestión resulta más sencilla. Labella y otros realizaron una comparativa entre modelos clásicos de consenso para pocos expertos aplicándolos a problemas de TDGGE en [75] mediante el software AFRYCA 2.0 [74], concluyendo que en general el funcionamiento de estos modelos se ve directamente afectado cuando el PAC es aplicado en problemas con un gran número de expertos.

Otro aspecto a tener en cuenta es que a mayor número de expertos es más probable que exista una mayor diversidad en cuanto a los perfiles y a los objetivos de los expertos participantes. Este hecho dificulta el control y la supervisión del PAC, lo cual puede ser aprovechado por determinados expertos para intentar desviar la solución del PAC hacia sus propios intereses, incumpliendo así el contrato de colaboración. Por este motivo, surge la necesidad de gestionar los comportamientos de estos expertos, de manera que se evite la manipulación de los PACs. En la actualidad son pocas las propuestas que consideran la gestión de este tipo de comportamientos derivados de incumplir el contrato de colaboración. La primera propuesta, y a la vez la más drástica, fue formulada por R. Yager en [142], proponiendo penalizar a los expertos que se nieguen a cooperar, eliminándolos así del proceso de TD. Palomares y otros proponen aplicar una penalización a los expertos que no obedezcan a las recomendaciones del moderador del PAC, de manera que el peso de su opinión en el momento de calcular el grado de consenso sea reducido [98]. La principal desventaja de esta propuesta reside en que los expertos que han sido penalizados, nunca pueden recuperar el peso de su opinión, aunque éstos adopten un comportamiento cooperativo en futuras rondas. Esto puede dar lugar a que el experto no se sienta satisfecho con la solución pese a haber corregido su comportamiento o que reconsidere su cooperación si no obtiene ninguna contrapartida a cambio.

La satisfacción de expertos o usuarios con respecto a una determinada solución, puede verse afectada de manera drástica cuando hablamos de grandes grupos. Un ejemplo lo podemos encontrar en los Sistemas de Recomendación a Grupos (SRG).

Los SRG al contrario que los Sistemas de Recomendación (SR) deben considerar las preferencias de todos los miembros del grupo para obtener recomendaciones que satisfagan a todo el grupo y no únicamente a un individuo. Actualmente, los SRG se basan en modelos de agregación que promedian la satisfacción o minimizan la disatisfacción de los miembros del grupo, sin embargo, la mayor expectativa de los usuarios de un SRG es maximizar su satisfacción como grupo.

Ante las problemáticas planteadas anteriormente sobre PAC en TDGGE y SRG, la principal motivación de nuestra investigación ha estado centrada en la superación de los siguientes retos:

- Actualmente, la gestión de comportamientos no cooperativos en PACs con grandes grupos no es capaz de generar soluciones satisfactorias debido a su visión personalizada en la que un cambio de actitud por parte de los usuarios no se ve reflejado en el resultado.
- Los actuales SRG sólo consideran las recomendaciones individuales de los miembros del grupo para calcular la recomendación grupal mediante procesos simples de agregación que no buscan maximizar la satisfacción grupal. Esto hace que exista una necesidad real en la mejora de la satisfacción del grupo en Procesos de Personalización de Grupo (PPGs).

1.2. Objetivos

Partiendo de la motivación y las consideraciones previas, el propósito general de esta investigación se centra la mejora de los PACs en TDGGE prestando especial atención al tratamiento de comportamientos no cooperativos y en la aplicación de PACs en SRG para mejorar la satisfacción de los miembros del grupo con respecto a las recomendaciones obtenidas.

A partir de este propósito general se formulan los siguientes objetivos específicos de nuestra investigación:

- *Desarrollar modelos para la gestión de los comportamientos no cooperativos de los expertos participantes en Procesos de Alcance de Consenso a Gran Escala (PACGEs) mediante la ponderación de sus opiniones con operadores* *uninormas y de hipersimilitud, permitiendo que la opinión de un experto penalizado pueda recuperar importancia si éste corrige su comportamiento no cooperativo y vuelve a cooperar con el grupo.*

- *Mejorar la satisfacción de los miembros del grupo en SRG mediante la generación de recomendaciones a grupo basadas en PACs que obtengan recomendaciones con un alto grado de acuerdo en el grupo.*

1.3. Estructura

A fin de cumplir con los objetivos planteados en la sección anterior, y según lo establecido en el artículo 23, punto 3, de la normativa vigente para los Estudios de Doctorado en la Universidad de Jaén, correspondiente al programa establecido en el RD. 99/2011, esta memoria de investigación es presentada como un compendio de artículos publicados por el doctorando, que recogen la investigación realizada durante los años de doctorado.

Estas publicaciones forman la parte central de la tesis y corresponden a dos artículos científicos publicados en Revistas Internacionales indexadas por la base de datos *JCR* (Journal Citation Reports), producida por *ISI* (Institute for Scientific Information), junto a un capítulo de libro publicado y otro aceptado que está pendiente de publicación. Así, la memoria está formada por un total de cuatro publicaciones, dos artículos publicados en revistas de reconocido prestigio y dos capítulos de libro.

A continuación, se describe la estructura de esta memoria de manera sucinta:

- **Capítulo 1**: Presenta una introducción general a la problemática de investigación que se trata en esta memoria de tesis, así como la motivación y objetivos perseguidos en la investigación realizada.

- **Capítulo 2**: Revisa los conceptos teóricos empleados en las propuestas formuladas, teniendo en cuenta los antecedentes en los distintos ámbitos sobre los que se sustenta la investigación recogida en esta memoria de tesis. En primer lugar, se abordan los problemas de TD y TDG al ser la base para poder profundizar en los PACs en TDGGE y los SRG, tratados a continuación.

- **Capítulo 3**: Resume las propuestas sobre las que se fundamenta la memoria de investigación, recorriendo los principales resultados obtenidos en nuestra investigación. Para cada uno, se realiza una breve descripción resaltando las características principales de cada propuesta en cuestión, que serán detalladas posteriormente en el capítulo 4 en sus correspondientes publicaciones.

- **Capítulo 4**: Constituye el núcleo de la tesis doctoral, conteniendo un compendio de las publicaciones obtenidas como resultado de la investigación realizada. Para cada una de las publicaciones se indican sus índices de calidad.

- **Capítulo 5**: Expone las conclusiones finales extraídas de esta investigación y los posibles trabajos futuros.

- **Apéndice A**: Recoge un resumen en inglés de la memoria de investigación. En primer lugar, se especifican la motivación y los objetivos que justifican la investigación. Posteriormente, se describen las propuestas planteadas y los resultados obtenidos. Por último, se exponen las conclusiones finales, posibles trabajos futuros, y las publicaciones que recogen la investigación realizada.

- **Apéndice B**: Describe los conceptos fundamentales del paradigma de la Computación con Palabras (CCP). Este paradigma es utilizado en las propuestas planteadas en el capítulo 3.

- **Apéndice C**: Introduce las nociones básicas de los operadores de agregación *uninorma*, los cuales son usados en la propuesta descrita en la sección 3.1.1.

- **Apéndice D**: Expone las propiedades de la medida de hipersimilitud, que es usada en la propuesta presentada en la sección 3.1.2.

- **Apéndice E**: Recoge las definiciones de algunos de los términos más importantes presentes en la memoria de tesis.

- **Apéndice F**: Contiene una lista con los acrónimos utilizados a lo largo de la memoria.

Capítulo 2

Conceptos Teóricos y Antecedentes

En este capítulo se recogen los antecedentes y los conceptos teóricos necesarios para comprender las propuestas presentadas en la tesis. En primer lugar, se introducen los problemas de TD, haciendo especial hincapié en las particularidades de los problemas de TDG. Seguidamente, nos centramos en los PACs y los retos que presenta la participación de grandes grupos. Por último, se describen las características de los SR y SRG, así como las similitudes de estos últimos con los problemas de TDG.

2.1. Problemas de Toma de Decisión en Grupo

La TD se basa en la elección de la mejor opción de entre un conjunto de posibles alternativas [10,59,111,140]. Este proceso no es una tarea trivial ya que normalmente dependiendo del escenario en el que nos encontremos podría resultar más conveniente la elección de una alternativa u otra. Aunque a priori podamos pensar en infinidad de situaciones en las que se llevan a cabo procesos de TD, lo cierto es que todos los problemas clásicos de decisión comparten los siguientes elementos [25]:

- Uno o varios objetivos por resolver.

- Un conjunto de alternativas o decisiones posibles para alcanzar dichos objetivos.
- Un conjunto de factores o estados de la naturaleza que definen el contexto en el que se plantea el problema de decisión.
- Un conjunto de valores de utilidad o consecuencias asociadas a los pares formados por cada alternativa y estado de la naturaleza.

A pesar de estos elementos comunes, también existen una serie de particularidades que caracterizan a cada problema. En las publicaciones referentes a la Teoría de la Decisión se pueden distinguir los siguientes criterios que permiten clasificar a estos problemas atendiendo a diferentes enfoques:

1. *Según el número de criterios o atributos que se han de valorar para cada alternativa.* Podemos tener problemas de TD de un solo criterio o multicriterio [14, 67, 140, 156]. Un ejemplo del primer caso podría ser la compra de un coche atendiendo exclusivamente al *precio*, mientras que en el segundo caso, el comprador además tendría en cuenta más criterios como pueden ser la *calidad*, el *carburante*, el *consumo* o el *color*.
2. *Según el número de expertos.* Tenemos problemas de TD individual y de TDG [63, 77]. El segundo caso añade complejidad al proceso de TD al coexistir expertos con distintos perfiles y conocimiento.
3. *Según el ambiente de decisión en el que se han de tomar las decisiones.* Se pueden diferenciar problemas de TD en ambiente de *certidumbre* (se tiene la certeza de lo que ocurrirá en el futuro), *riesgo* (el decisor asigna probabilidades a los estados de la naturaleza) e *incertidumbre* (no es posible asignar probabilidades a los estados de la naturaleza y la decisión se ve orientada por la orientación psicológica del decisor) [77, 130].

Concretamente, la investigación presentada en esta tesis se centra en problemas de TDG bajo incertidumbre.

Los problemas de TDG conllevan la participación de múltiples expertos que tienen que tomar una decisión de manera colectiva para encontrar la solución a un problema determinado. La principal justificación para llevar a cabo este tipo de procesos radica en que en los procesos de TD donde intervienen expertos con diferentes perfiles (experiencia y conocimiento experto) normalmente dan lugar a mejores decisiones que en aquellos casos en los que la TD recae sobre un único experto [83].

En términos generales, un problema de TDG está formado por los siguientes elementos [59]:

1. Un problema común al que hay que darle una solución.

2. Un conjunto $X = \{x_1, \ldots, x_n\}$, $(n \geq 2)$, de *alternativas* o posibles soluciones al problema.

3. Un conjunto $E = \{e_1, \ldots, e_m\}$, $(m \geq 2)$, de *expertos* que expresan sus opiniones o preferencias sobre el conjunto de alternativas X. En el caso de problemas de TDGGG, $m >> 2$.

Cada experto expresa su opinión sobre las distintas alternativas mediante una estructura en la que se representan sus preferencias. En la literatura podemos encontrar numerosos estudios en los que se asume que todos los expertos usan el mismo formato para expresar sus preferencias, aunque en la práctica esto no suele suceder [26, 38, 52]. En esta investigación usamos Relaciones de Preferencia Difusas (RPDs) además de por ser una de las estructuras más utilizadas en procesos de TDG [94], también por su capacidad para modelar opiniones de los expertos mediante distintas representaciones, y por tanto facilitar la heterogeneidad de formatos de representación [120]. Una Relación de Preferencia (RP) P_i asociada al experto e_i, puede ser representada para un conjunto finito de alternativas X mediante una matriz $n \times n$:

$$P_i = \begin{pmatrix} - & \dots & p_i^{1n} \\ \vdots & \ddots & \vdots \\ p_i^{n1} & \dots & - \end{pmatrix}$$

donde cada valoración $p_i^{lk} = \mu_{P_i}(x_l, x_k) \in [0,1]$, representa el grado de preferencia de la alternativa x_l sobre x_k, $l,k \in \{1,\dots,n\}$, $l \neq k$, según el experto e_i, de forma que:

- $p_i^{lk} > 0.5$ indica preferencia de x_l sobre x_k.
- $p_i^{lk} < 0.5$ indica preferencia de x_k sobre x_l.
- $p_i^{lk} = 0.5$ indica indiferencia entre x_l y x_k.

Las RPDs pueden cumplir las siguientes propiedades [27, 41, 42]:

- Reciprocidad: $p_i^{lk} + p_i^{kl} = 1, \forall l,k = 1,\dots,n$.
- Completitud: $p_i^{lk} + p_i^{kl} \geq 1, \forall l,k = 1,\dots,n$.
- Transitividad max-min: $p_i^{lk} \geq min(p_i^{lj}, p_i^{jk}), \forall l,j,k = 1,\dots,n$.
- Transitividad max-max: $p_i^{lk} \geq max(p_i^{lj}, p_i^{jk}), \forall l,j,k = 1,\dots,n$.
- Transitividad max-min restrictiva:
  $p_i^{lj} \geq 0.5, p_i^{jk} \geq 0.5 \Rightarrow p_i^{jk} \geq min(p_i^{lj}, p_i^{jk}) \forall l,j,k = 1,\dots,n$.
- Transitividad max-max restrictiva:
  $p_i^{lj} \geq 0.5, p_i^{jk} \geq 0.5 \Rightarrow p_i^{jk} \geq max(p_i^{lj}, p_i^{jk}) \forall l,j,k = 1,\dots,n$.
- Transitividad aditiva: $p_i^{lj} + p_i^{jk} - 0.5 = p_i^{lk}, \forall l,j,k = 1,\dots,n$.

Estas propiedades han sido estudiadas en profundidad aplicándose a modelos de consenso para TDG con RPDs [103, 133]. Por consiguiente, a fin de facilitar la construcción de RPs consistentes, en los trabajos de esta tesis se ha asumido la propiedad de reciprocidad en RPDs. Esto significa que si el experto e_i da una valoración $p_i^{lk} = x$, estando $x \in [0, 1]$, $l \neq k$, entonces $p_i^{kl} = 1 - x$.

Para obtener la solución a un problema de TDG se puede emplear tanto el *enfoque directo* como el *enfoque indirecto* [47], tal y como se muestra en la Figura 2.1.

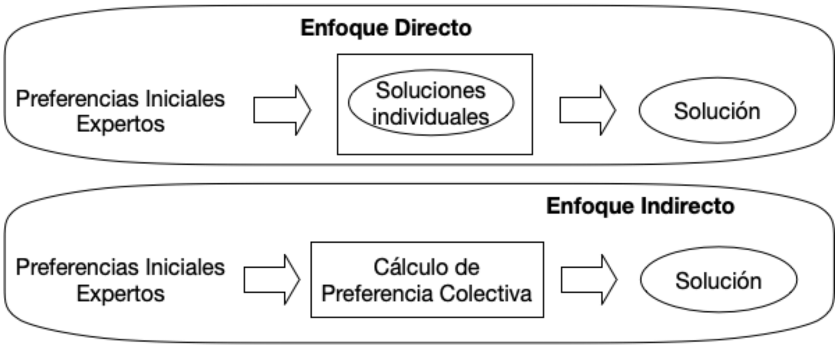

Figura 2.1: Enfoques para la obtención de una solución en problemas de toma de decisión en grupo.

En el primer enfoque, la solución es obtenida directamente a partir de las preferencias individuales de cada experto sin construir previamente una opinión de grupo [48]. Por contra, en el segundo caso es necesario calcular una opinión de grupo o *Preferencia Colectiva (PC)* a partir de las opiniones individuales de los expertos para, posteriormente, concluir la solución del problema.

Independientemente del enfoque utilizado, el clásico proceso de selección de alternativas para resolver un problema de TDG se compone de dos fases [116], tal y como muestra la Figura 2.2:

(I) *Fase de agregación*, en la que las preferencias de los expertos se combinan mediante un operador de agregación.

(II) *Fase de explotación*, en la que se aplica un criterio de selección específico para obtener la alternativa o conjunto de alternativas que dan solución al problema.

Cuando un problema de TDG es resuelto aplicando sólo el proceso de selección de alternativas, pueden darse casos en los que ciertos expertos tengan la sensación

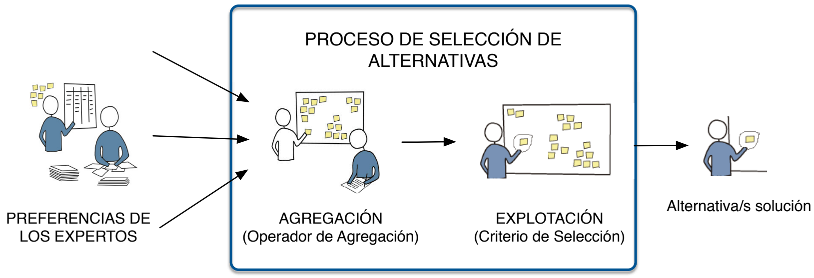

Figura 2.2: Clásico proceso de resolución para problemas de toma de decisión en grupo.

de que sus opiniones no hayan sido tenidas en cuenta para alcanzar la solución final [2]. Este hecho podría desembocar en que los expertos en cuestión no acepten la solución final, considerando que el proceso de TDG no ha tenido éxito, lo que puede conllevar a que sean reticentes a participar en futuros procesos de TDG. Debido a que existen numerosos casos en la vida real en los que es absolutamente crucial tener un alto grado de acuerdo colectivo entre los expertos participantes para alcanzar una determinada solución, resulta necesario aplicar un PAC, añadiéndose así una nueva fase al proceso de TDG con el objetivo de que los expertos lleguen a un alto nivel de acuerdo antes de tomar la decisión [17, 96].

2.2. Procesos de Alcance de Consenso

En la literatura podemos encontrar diversas definiciones del término *consenso*. La Real Academia Española (RAE) lo define como el "*acuerdo producido por consentimiento entre todos los miembros de un grupo o entre varios grupos*". Para Saint y Lawson se trata de un "*estado de acuerdo mutuo entre los miembros de un grupo, donde todas las opiniones e inquietudes de cada individuo han sido tenidas en cuenta para conseguir la satisfacción del grupo*" [118]. En ambas definiciones se asume la idea de que ningún experto participante en un problema de TDG pueda estar en desacuerdo sobre la solución adoptada, aunque ellos puedan tener el convencimiento de que la solución que hubieran tomado individualmente fuera mejor que la tomada de manera colectiva. Para conseguir el acuerdo necesario resulta imprescindible que *todos* los expertos modifiquen sus opiniones iniciales tratando de acercarse a una opinión grupal que satisfaga a todos los miembros del grupo.

El concepto de *consenso* ha ido evolucionando en la literatura de TDG. En un principio se consideraba como sinónimo de "*unanimidad*" [68], de manera que se alcanzase un acuerdo total entre todos los expertos. Sin embargo, es complicado que se den este tipo de situaciones en la práctica debido a la diversidad de perfiles y objetivos de los expertos participantes. Hay casos en los que la *unanimidad* se consigue aplicando métodos de presión sobre los participantes del PAC, los cuales no actúan libremente y alcanzan un acuerdo a causa de la coacción a la que son sometidos, *consenso normativo* [30]. Ante estos escenarios resulta necesario aclarar que no debemos interpretar el *consenso* como un acuerdo unánime, ya que sería más correcto concebirlo como la consecuencia de llevar a cabo un proceso de negociación en el que la solución final podría diferir de las opiniones iniciales de los participantes. Es lo que se conoce como "*consenso cognitivo*", el cual implica que los expertos cambien sus preferencias iniciales a lo largo de las rondas del proceso de negociación [87]. Siguiendo este enfoque, encontramos propuestas de consenso más flexibles, en las que se tienen en cuenta acuerdos parciales entre los miembros del grupo a distintos niveles [17, 49, 60]. En esta línea, Kacprzyk presenta el concepto de "*soft consensus*" [59], que basándose en la idea de mayoría difusa, considera que se ha llegado al consenso en un grupo cuando *la mayor parte de los expertos están de acuerdo en las alternativas importantes* [60].

El principal objetivo de los PACs reside en que los expertos alcancen un mínimo nivel de acuerdo antes de llevar a cabo el proceso de selección de alternativas, mediante la modificación de sus preferencias a lo largo de las rondas de un proceso de negociación. Este hecho implica una total predisposición por parte de los participantes para alcanzar un acuerdo previo a la elección de la solución, de manera que ésta sea escogida buscando el máximo beneficio del grupo. Para ello, todos los expertos se comprometen a cumplir un *contrato de colaboración*, que normalmente implica la modificación de sus preferencias iniciales [87]. Se trata de un proceso dinámico e iterativo que en la mayoría de los casos conlleva varias rondas de discusión entre los participantes. Habitualmente está coordinado por la figura del moderador [87], que es el encargado de supervisar y guiar a los expertos a lo largo del proceso de discusión [118].

Uno de los puntos esenciales de este proceso es la definición de medidas de consenso adecuadas para cuantificar el nivel de acuerdo en el grupo a partir de las preferencias de los expertos. En la literatura existen numerosas propuestas de medidas de consenso que pueden ser clasificadas en:

- *Medidas de consenso basadas en distancia a la preferencia colectiva* [9,49,125]. La opinión del grupo, representada por la preferencia colectiva, P_c, es calculada agregando todas las preferencias individuales de los expertos, P_i. Los grados de consenso se obtienen calculando la distancia que hay entre la preferencia de cada experto y la preferencia colectiva, $d(P_i, P_c)$.

- *Medidas de consenso basadas en distancias entre los expertos* [11, 50, 60, 61]. Para cada par de expertos del grupo, $(e_i, e_j), i < j$, los grados de similitud entre sus opiniones se calculan según las métricas de distancia. Posteriormente, los valores de similitud, $L(P_i, P_j)$ son agregados para obtener el grado de consenso.

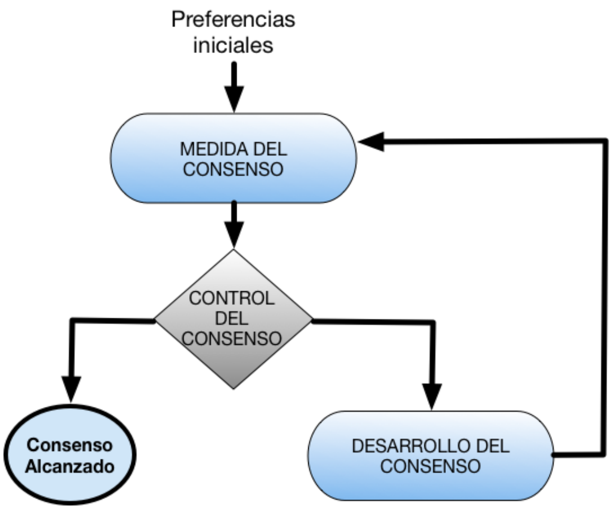

Figura 2.3: Esquema general de los procesos de alcance de consenso.

Son muchos los modelos de consenso que se han propuesto a lo largo de los últimos años, cada uno centrándose en distintos criterios [53, 96, 129]. La Figura 2.3 muestra el esquema general común a la mayoría de los modelos propuestos, los cuales comparten las siguientes fases:

1. *Medida del consenso.* Todas las preferencias de los expertos, $P_i, i \in \{1, \ldots, m\}$ son recogidas para calcular el nivel de acuerdo que hay en el grupo. Para ello se utilizan las medidas de consenso.

2. *Control del consenso.* Se compara el Grado de Consenso (GC) actual con el Umbral de Consenso (UC) definido al inicio del PAC. Si el GC es mayor que el UC entonces el PAC finaliza habiendo alcanzado el consenso. En cambio, si el GC es menor que el UC, el PAC continua otra ronda hasta que se alcance el consenso o bien se llegue al número máximo de rondas permitidas.

3. *Desarrollo del consenso.* Para incrementar el nivel de acuerdo en futuras rondas, existen distintos métodos que pueden ser aplicados dependiendo de si el modelo de consenso permite a los expertos modificar sus preferencias a partir de recomendaciones o si las preferencias son actualizadas automáticamente:

   - *Generación de Recomendaciones*: En los modelos de consenso que implementan este mecanismo [13, 20, 55, 91], el moderador sugiere a los expertos por medio de recomendaciones, las modificaciones que éstos deben hacer para acercarse a la opinión del grupo. Cada recomendación de cambio generada consiste en una terna $(e_i, (x_l, x_k), \text{Dirección})$, que indica que el experto e_i, debe modificar la preferencia p_i^{lk}, en la dirección dada por Dirección $\in \{Aumentar, Disminuir\}$

   - *Actualizaciones automáticas.* Estos modelos de consenso [9, 134, 157] no incorporan ningún mecanismo de recomendaciones, sino que a partir de las preferencias iniciales de los expertos, realizan cambios automáticos a lo largo del PAC para aumentar el nivel de acuerdo.

En la investigación recogida en esta tesis nos hemos centrado en los PACs con recomendaciones. La Figura 2.3 muestra un esquema general de estos tipos de PACs.

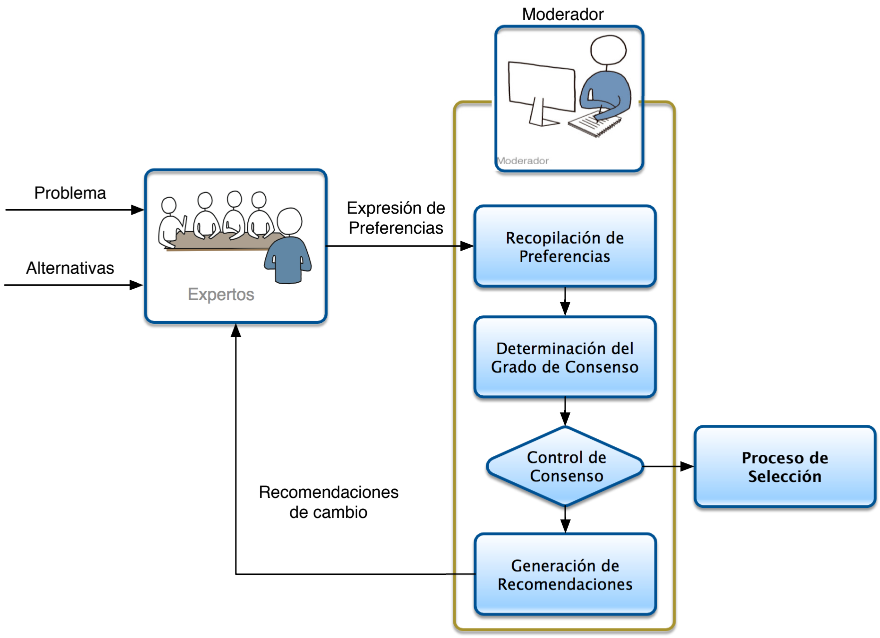

Figura 2.4: Esquema general de procesos de alcance de consenso con recomendaciones.

El PAC con recomendaciones, normalmente conlleva la necesidad de que los expertos revisen y modifiquen sus opiniones en base a las recomendaciones recibidas, con el objeto de acercarlas a las del resto del grupo. Este proceso se repite hasta que se alcance o el número máximo de rondas, o un nivel de acuerdo igual o superior al umbral de consenso. En el primer caso, el proceso terminaría sin haberse llegado a un acuerdo por lo que para llegar a una solución es necesario aplicar estrategias de TDG tales como [118]:

(I) Delegar la decisión a un subgrupo.

(II) Aplicar el voto de la mayoría.

(III) Excluir a los expertos que no quieren contribuir a que se alcance el consenso.

En el segundo caso, se pasaría al proceso de selección tal y como se detalla en la fase 2, *Control del consenso*.

En los últimos años los distintos modelos usados en PACs [96] se han visto notablemente afectados por los avances tecnológicos y sociales, que han traído consigo nuevos retos a los que hacer frente, como es la participación de grandes grupos en los PACs [75, 98]. En los Procesos de Alcance de Consenso con Grandes Grupos (PACGGs), el alto número de participantes hace que sea más probable la diversificación de perfiles entre los expertos, por lo que a priori, las opiniones de los participantes estarían alejadas y por tanto el esfuerzo para alcanzar un grado de consenso aceptable sería mayor. Además, la función del moderador se complica exponencialmente, siendo casi imposible poder supervisar de manera personalizada a los participantes durante el PAC. Este hecho puede dar lugar a que algunos expertos se puedan aprovechar del gran número de participantes e incumplan el contrato de colaboración, yendo en contra de lo que se les indica en las recomendaciones, dando prioridad a sus intereses personales sobre los del grupo. Este tipo de comportamientos no cooperativos pueden derivar en que el PAC pueda verse afectado:

(i) Necesitándose un mayor número de rondas para alcanzar el consenso.

(ii) Bloqueándose el proceso, de modo que se llegue al número máximo de rondas sin haber alcanzado el nivel de acuerdo mínimo.

(iii) Desviando la solución hacia sus intereses personales.

En estos problemas, el incremento del número de expertos participantes añade complejidad a la hora de controlar y supervisar el proceso de TD, a diferencia de los problemas de TDG clásicos donde esta gestión resulta más sencilla. Labella y otros realizan una comparativa entre modelos clásicos de consenso para pocos expertos aplicándolos a problemas de TDGGG en [75] mediante el software AFRYCA 2.0 [74], concluyendo que en general el funcionamiento de estos modelos se ve directamente afectado cuando el PAC es aplicado en problemas con un gran número de expertos. Entre los retos más importantes que presentan los problemas de TDGGG destacan:

- *Comportamientos no cooperativos*: En problemas con un gran número de expertos es posible que algunos participantes no cooperen, bien rechazando las recomendaciones recibidas o incluso moviéndolas en dirección contraria.

- *Comportamientos de subgrupos*: En PACs en problemas de TDGGG pueden existir subgrupos de expertos que compartan los mismos intereses. Esto hace que si su opinión dista de la recomendación recibida, éstos puedan colaborar rompiendo el contrato de colaboración [87] con el objetivo de desviar la solución hacia los intereses del subgrupo.

- *Opiniones minoritarias*: Xiong y otros destacaron en [136] la importancia de las opiniones minoritarias en los PACs, proponiendo para ello un mecanismo para proteger dichas opiniones. Sin embargo, en problemas de TDGGG, considerar la opinión de la minoría resulta altamente complicado.

- *Supervisión*: La necesidad de supervisión humana durante el PAC hace que con el incremento del número de expertos, esta tarea se complique de manera notable [56, 64, 97, 139].

Teniendo en cuenta estos retos, hay que destacar que uno de los pilares fundamentales de la investigación realizada en esta tesis se centra en la gestión de comportamientos no cooperativos en PACs.

Actualmente, no existen muchas propuestas que traten la gestión de expertos con comportamientos no cooperativos. La primera propuesta que encontramos fue introducida por R. Yager en [142], en la que se insta a penalizar a los expertos que se nieguen a cooperar, eliminándolos del proceso de TD. Esta propuesta es bastante drástica y por este motivo Palomares y otros presentan un nuevo enfoque en el que se penaliza a los expertos que no obedecen a las recomendaciones del moderador del PAC, reduciéndoles el peso de su opinión a la hora de calcular el grado de consenso [98]. Sin embargo, esta propuesta presenta como inconveniente que los expertos que han sido penalizados, nunca pueden recuperar el peso de su opinión aunque adopten un comportamiento cooperativo en futuras rondas. Esta situación puede dar lugar a que el experto no se sienta satisfecho con la solución pese a haber corregido su comportamiento.

En el capítulo 3 se proponen distintas soluciones con el objetivo de gestionar a los expertos con comportamientos no cooperativos en PACs.

El principal objetivo de gestionar los expertos con este tipo de comportamientos radica en asegurar el buen funcionamiento del PACGG, de modo que todos los expertos se sientan partícipes de la solución final y la consideren como suya. Esto da como resultado una mejora en la satisfacción de los participantes. Otro escenario en el que la satisfacción de los integrantes de un grupo se puede ver comprometida por las opiniones individuales de los miembros del grupo lo encontramos en los SRG.

2.3. Sistemas de Recomendación y Sistemas de Recomendación a Grupos

Los SR se definen como "*sistemas que producen recomendaciones personalizadas como salida o tienen el efecto de guiar al usuario de una forma personalizada a productos interesantes o útiles entre un espacio sobrecargado de productos disponibles*" [16]. Así, partiendo de los gustos y necesidades del usuario, los SR le proporcionan un abanico de productos, servicios o contenidos que pueden ser de su interés, filtrando aquellos que le son irrelevantes.

Normalmente, la información que el sistema guarda de cada usuario, puede ser obtenida de manera:

(i) *Implícita*, mediante la interacción del usuario con el sistema [70, 79, 93].

(ii) *Explícita*, método en el que el usuario proporciona sus preferencias de manera activa, valorando los productos según su satisfacción [31].

Por regla general la información obtenida de manera explícita suele ser más precisa que la información implícita, aunque en la práctica es más difícil de obtener, ya que requiere que los usuarios dediquen parte de su tiempo a introducir sus datos [155]. No obstante, independientemente del tipo de información, los SR tendrán información de cada uno de los usuarios $U = \{u_1, ..., u_m\}$, sobre los productos $I = \{i_1, ..., i_n\}$. De esta manera, el conjunto de valoraciones de preferencia (R) se puede representar como:

$$R : U \times I \rightarrow D, \tag{2.1}$$

donde R es el conjunto de valoraciones de preferencia que los usuarios U han dado sobre los productos I en un dominio D. Uno de los dominios más comunes a la hora de representar las valoraciones de preferencia es $D = \{1, 2, 3, 4, 5\}$, siendo 1 el valor de preferencia más bajo (no se ajusta para nada al gusto del usuario) y 5 el valor de preferencia más alto (se ajusta totalmente al gusto del usuario).

En términos matemáticos, podemos definir el problema recomendación, como un problema de predicción [1], en el que a partir de un conjunto de valoraciones de preferencia tomadas como una función incompleta, el sistema trata de aproximarse utilizado distintas técnicas. Esto es, los SR tratan de encontrar el producto o conjunto de productos que maximizan la predicción para el usuario (ver Fórmula 2.2).

$$\text{Recomendación}(I, u) = \arg\,\text{máx}_{i_k \in I}[\text{Predicción}(i_k, u)] \tag{2.2}$$

Básicamente los SR realizan las siguientes tareas [44]:

- *Cálculo del perfil de usuario.* El SR calcula un perfil de usuario a partir de la información que se tiene sobre él. Para esta tarea se han aplicado distintos enfoques, como el modelado del comportamiento, modelado de intereses o de intenciones.
- *Modelado de productos.* Se calcula un perfil de producto partiendo de la información que se tiene de cada producto.

- *Filtrado.* Tras calcular los perfiles de usuarios y de productos se intenta predecir qué valoración daría un usuario a un producto atendiendo a dichos perfiles.

Estas tareas se realizan de forma distinta según el tipo de SR. En la literatura podemos encontrar numerosas propuestas que dan solución al problema de recomendación, las cuales pueden ser clasificadas en los siguientes tipos [1]:

- *Sistemas de Recomendación Demográficos (SRD).* Están basados en el pensamiento de que aquellas personas que comparten características demográficas tales como la edad, el sexo, o el nivel de educación, tendrán gustos similares [72, 105, 110].

- *Sistemas de Recomendación Basados en Contenido (SRBCont).* A partir de la información sobre las características de cada producto se tratan de extraer relaciones entre éstas y las valoraciones de preferencia de un experto. Para ello, se construye un perfil de usuario a partir de los productos que ha valorado y se utiliza para evaluar los productos sin valorar. Este tipo de SR no trata de predecir la valoración que un usuario daría, sino que asigna una puntuación a los productos, que indica la idoneidad del producto para el usuario. Se utilizan dos tipos de información del producto, su (i) *descripción textual*, y sus (ii) *atributos*.

- *Sistemas de Recomendación Colaborativos (SRC).* Estudian las valoraciones de los usuarios, buscando relaciones entre ellos para generar predicciones [45, 71, 108]. Las recomendaciones se calculan completando las siguientes fases:

    1. *Cálculo de vecinos*: Se calculan los usuarios que presentan los gustos o las necesidades más parecidas al usuario activo.

    2. *Predicción de la valoración de preferencias*: Tras la primera fase se realiza una predicción que estima el valor de preferencia que el usuario activo daría a cada uno de los productos que no ha valorado.

3. *Recomendación de los n-primeros productos*: Se ordena la lista de los productos a partir de los valores calculados en la fase 2 y se recomiendan los n primeros productos de la lista.

- *Sistemas de Recomendación Basados en Conocimiento (SRBCono).* Estos sistemas utilizan una base de conocimiento para describir cómo y en qué medida los productos satisfacen las demandas del usuario, utilizando para ello procesos de inferencia [15]. Entre las propuestas más destacadas dentro de este tipo de SR se encuentran:

  (i) *Razonamiento basado en casos.*

  (ii) *Conocimiento mediante ontologías.*

  (iii) *SR basados en restricciones.*

  (iv) *SR que utilizan relaciones de preferencia.*

- *Sistemas de Recomendación Basados en Utilidad (SRBU)* La información del usuario se obtiene mediante una función de utilidad que éste define para valorar los productos [122] y que sigue el esquema de la teoría de la Utilidad Multi-Atributo [119].

- *Sistemas de Recomendación Híbridos (SRH).* Estos sistemas aparecen como resultado de combinar distintas técnicas de recomendación, con el objetivo de beneficiarse de las fortalezas que cada sistema presenta por separado [16].

En la investigación recogida en esta tesis se utilizan los SRC, al ser los más extendidos en el mundo real [71].

Los SR tradicionales están centrados en la recomendación de productos a individuos. Sin embargo, hay ciertos casos en los que resulta de mayor interés la recomendación de productos a grupos de usuarios, principalmente por su carácter social. Algunos ejemplos de estos productos los podemos encontrar en los ámbitos de la hostelería (restaurantes, hoteles...) o cultural (música, atracciones turísticas, eventos culturales...). Por este motivo surgen los SRG. Estos SR extienden a los

SRC recomendando productos a grupos de usuarios $G = \{g_1, ..., g_r\}$, en vez de a individuos (véase ecuación 2.3).

Formalmente, los SRG, al igual que los SR, tratan de encontrar el producto o conjunto de productos que maximizan la predicción para el grupo:

$$\text{RecomendaciónGrupo}(I, G) = \arg\,\text{máx}_{i_k \in I}[\text{Predicción}(i_k, G)] \tag{2.3}$$

En la recomendación a grupos existen dos técnicas para extender los SR individuales:

- *Agregación de valoraciones*: se genera un perfil asociado al grupo, o pseudo-usuario, a partir de las valoraciones de los miembros del grupo.
- *Agregación de las recomendaciones*: se calculan las predicciones para cada miembro del grupo y se agregan para generar las recomendaciones.

En la figura 2.5 se pueden observar los esquemas de funcionamiento de estos tipos de agregación. En la tesis se utilizará un SRG con agregación de recomendaciones. Como se puede apreciar, en primer lugar, el sistema calcula una lista de recomendaciones para cada miembro del grupo, las cuales se agregan posteriormente para generar la recomendación a grupo.

En la literatura, podemos encontrar estudios que destacan que los enfoques anteriores serán más idóneos dependiendo del escenario sobre el que se implemente el SRG [89, 95]. De hecho, los modelos actuales de SRG basados en agregación presentan los siguientes problemas [23]:

(i) *Pérdida de información.* El proceso de agregación implica resumir información original, dando lugar a la pérdida de información de acuerdo a su distribución, diversidad y forma.

(ii) *Minimización de la disatisfacción.* Se trata de minimizar la miseria de los miembros del grupo para satisfacer a la mayoría de los usuarios del grupo. Esto

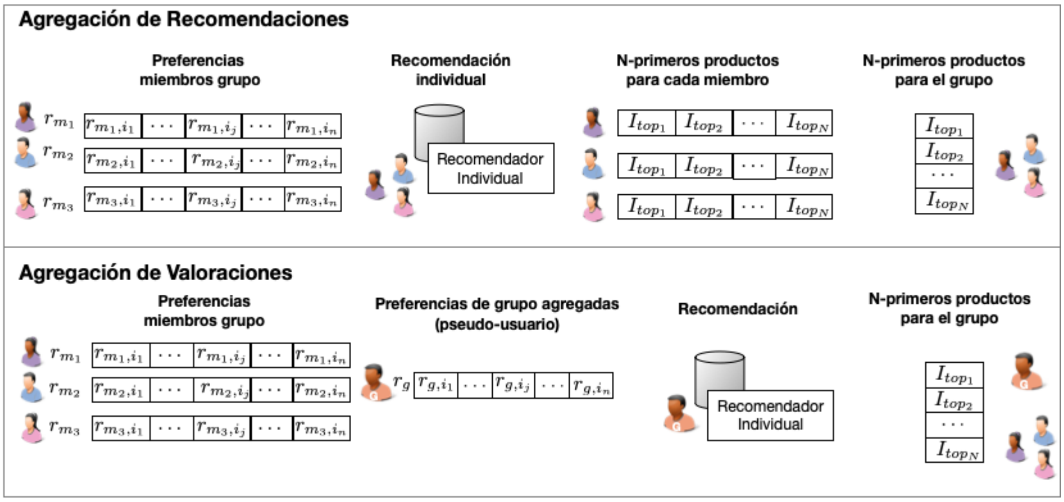

Figura 2.5: Técnicas de agregación en sistemas de recomendación a grupos.

resulta problemático principalmente con grandes grupos ya que una valoración baja es suficiente para penalizar a un producto al ser la valoración del grupo el mínimo valor del conjunto de valoraciones.

(III) *Promedio de la satisfacción.* Al promediar las valoraciones de los usuarios, cuando dichos valores son cercanos, la satisfacción de los usuarios cuyas valoraciones han sido agregadas será elevada. Sin embargo, en casos en los que las valoraciones de los usuarios son extremas, la satisfacción se verá afectada de manera drástica.

Analizando estos problemas observamos cómo resulta necesaria la aplicación de mecanismos que traten de mejorar la satisfacción de los usuarios del grupo.

Por otro lado, resulta llamativo el paralelismo que existe entre los elementos y los objetivos de los problemas de TDG y los SRG. La tabla 2.1 muestra una comparativa entre los problemas de TDG y la recomendación a grupo. Mientras que los primeros están compuestos por un grupo de expertos y varias alternativas sobre las que los expertos expresan sus preferencias, los segundos están formados por un grupo de usuarios y de productos sobre los que los usuarios expresan sus preferencias. En lo relativo al principal objetivo de cada caso, el de los problemas de TDG radica en la selección de la mejor alternativa, mientras que en los SRG se trata de seleccionar

Toma de Decisión en Grupo	**Recomendación a Grupos**
Grupo de expertos	Grupo de usuarios
Varias alternativas	Varios productos
Preferencias de cada experto sobre las alternativas	Preferencias de los usuarios sobre los productos
Objetivo: Seleccionar la mejor alternativa	**Objetivo**: Seleccionar el mejor producto

Cuadro 2.1: Comparación entre características de la toma de decisión en grupo y la recomendación a grupos.

el mejor producto. Como se ha visto previamente, en los problemas de TDG los expertos pueden no estar satisfechos con la alternativa seleccionada, en cambio en los SRG se necesita mejorar la satisfacción individual hacia la recomendación de grupo.

Estas similitudes entre los problemas de TDG y los SRG posibilitan que podamos aplicar técnicas usadas para la mejora de la satisfacción en problemas de TDG, como los PACs, e importarlas en SRG.

Por lo tanto, nuestro objetivo será mejorar la recomendación a grupos mediante la aplicación de técnicas de alcance de consenso. Esto se realizará integrando el PAC en la recomendación a grupos. En la propuesta aplicaremos dicho proceso sobre las recomendaciones individuales, que serán modificadas hasta que se alcance el nivel de consenso establecido. De este modo, para aplicar técnicas de consenso, la propuesta realiza, en primer lugar, una fase de recomendación individual, en la que se obtienen los mejores productos para cada miembro y posteriormente, se aplica un PAC que acerca dichas recomendaciones antes de generar las recomendaciones consensuadas para el grupo. Esta propuesta aparece detallada en el apartado 3.2.

Capítulo 3

Discusión de los Resultados

El presente capítulo resume las propuestas llevadas a cabo para satisfacer los objetivos de investigación planteados en el capítulo 1. Para cada propuesta se realiza una breve descripción de los conceptos fundamentales y un análisis de los resultados obtenidos, que serán mostrados en mayor detalle en las publicaciones respectivas del capítulo 4.

Las propuestas resumidas en este capítulo serán las siguientes:

1. *Gestión de Expertos con Comportamientos no Cooperativos en PACGEs*
   - *a)* *Propuesta basada en Uninormas.*
   - *b)* *Propuesta basada en la medida de Hipersimilitud.*
2. *Sistema de Recomendación en Grupo basado en Consenso.*

3.1. Gestión de Expertos con Comportamientos no Cooperativos en Procesos de Alcance de Consenso a Gran Escala

El primer objetivo en el desarrollo de este doctorado ha sido la mejora de la gestión de expertos con comportamientos no cooperativos en PACGEs. Como se ha visto en los capítulos previos, las propuestas para la gestión de los expertos con este tipo de comportamientos, confluían en la idea de penalizar a dichos expertos. Esto se llevaba a cabo quitándoles peso a sus opiniones sin permitirles recuperar el peso perdido [98], llegando en algunos casos, a eliminar completamente a los expertos del PAC [142]. Nuestra investigación se centra en gestionar los comportamientos que desarrollan los expertos a lo largo del PACGE, penalizando a aquellos que desobedecen a las recomendaciones, pero permitiéndoles recuperar el peso de sus opiniones si corrigen su comportamiento y deciden cooperar. Para ello se propone añadir una nueva fase al PACGE para la *gestión de los comportamientos de los expertos* entre la fase de *recopilación de las preferencias de los expertos* y la de *cálculo del grado de consenso*, tal y como se muestra en la figura 3.1.

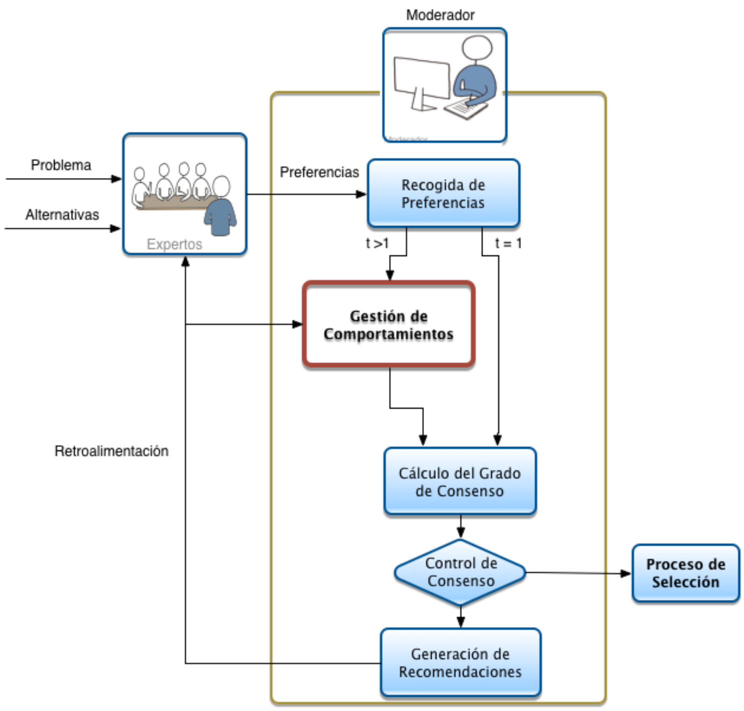

Figura 3.1: Esquema general de gestión de comportamientos en procesos de alcance de consenso a gran escala con recomendaciones.

Hay que reseñar que la fase de *gestión de los comportamientos de los expertos* tiene lugar a partir de la segunda ronda, una vez que éstos han cambiado sus preferencias tras conocer las recomendaciones dadas por el moderador. Esta fase está compuesta a su vez por dos subfases, como se puede apreciar en la figura 3.2:

- *Detección de comportamientos.* Se determina el tipo de comportamiento de los expertos basándose en si han cambiado o no sus preferencias conforme a las recomendaciones.

- *Gestión de comportamientos en PACGE mediante la ponderación de expertos.* Se actualiza el peso de las opiniones de los expertos antes de llevar a cabo la fase del *cálculo del consenso*, atendiendo tanto al tipo de comportamiento de los expertos, como a los pesos de sus opiniones actuales.

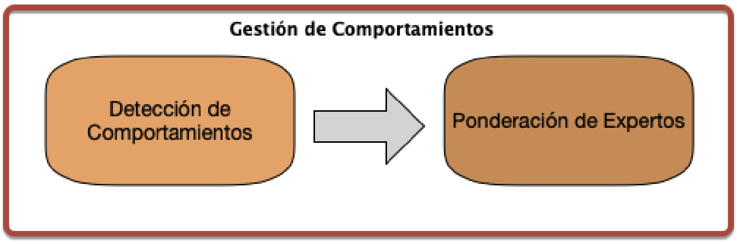

Figura 3.2: Subfases de la gestión de comportamientos.

A continuación se resumen las dos propuestas que hemos desarrollado en nuestra investigación para la gestión de expertos con comportamientos no cooperativos en PACGEs, que son explicadas en profundidad en el capítulo 4. En la primera de ellas se utilizan los operadores uninormas [146], detallados en el anexo C, mientras que la segunda propuesta tiene como principal peculiaridad el uso de la medida de hipersimilitud [145], descrita en el anexo D. Ambas propuestas hacen uso del paradigma de la computación con palabras, detallada en el apéndice B. Concretamente, se usará este paradigma para definir el término lingüístico *cooperativo*, cuya función de pertenencia se utilizará para determinar el tipo de comportamiento adoptado por cada experto.

3.1.1. Propuesta basada en el uso de uninormas

Las uninormas [146] son operadores que se utilizan para llevar a cabo procesos de agregación. Entre sus propiedades, detalladas en el apéndice C, destaca el *refuerzo* [109], que permite reforzar tendencias teniendo en cuenta los valores de una variable a lo largo del tiempo. Los operadores uninormas poseen un *refuerzo completo*, al beneficiarse tanto del *refuerzo negativo* de las t-normas como del *refuerzo positivo* de las t-conormas. En la propuesta se utiliza esta característica para reforzar el comportamiento de los expertos, de manera que dependiendo de la tendencia en el comportamiento del experto (cooperativo o no cooperativo), su opinión ganará o perderá peso respectivamente.

La propuesta se compone de las siguientes fases, tal y como se muestra en la figura 3.3:

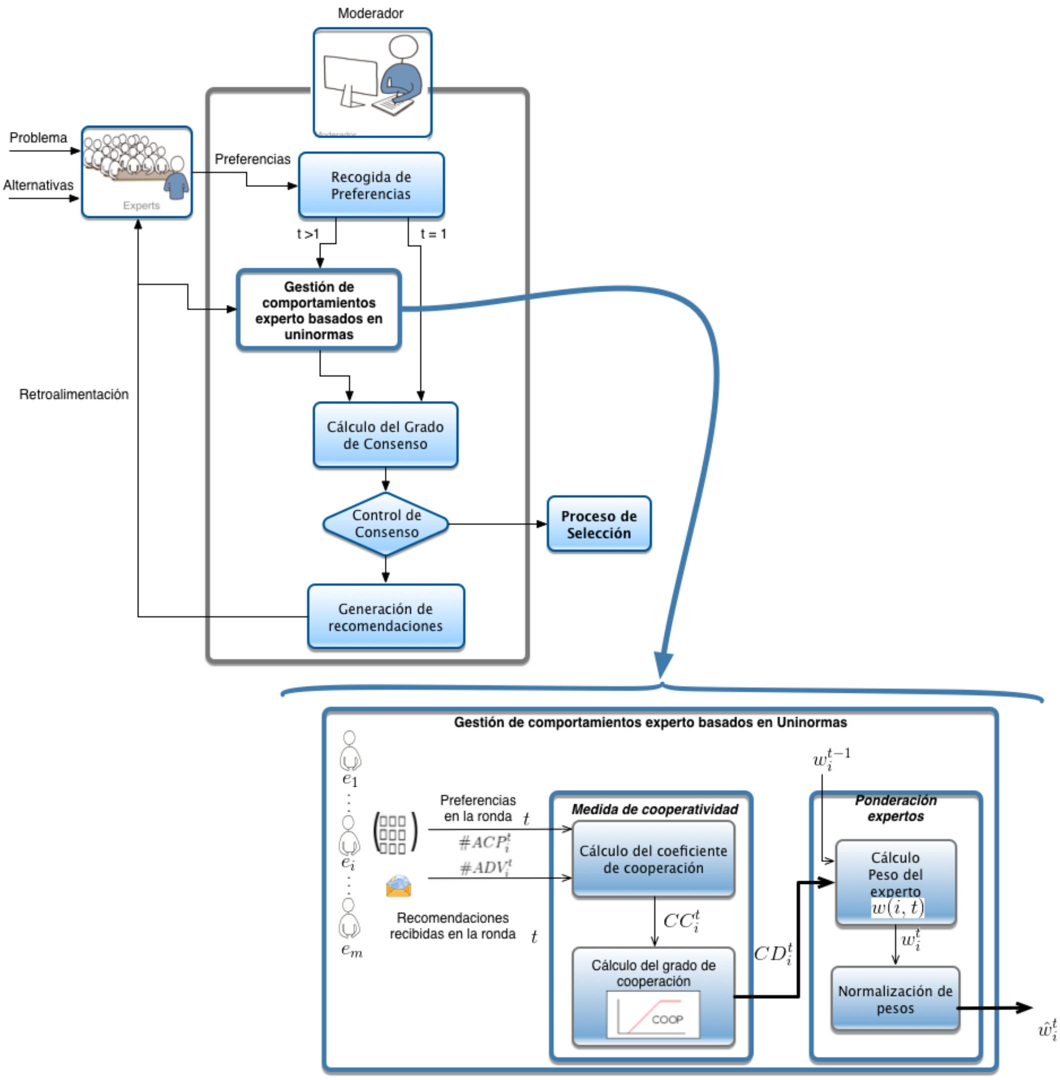

Figura 3.3: Esquema de gestión de comportamientos basados en uninormas para procesos de alcance de consenso a gran escala.

- *Medición de la* cooperatividad. Se utiliza la teoría de los conjuntos difusos y la computación con palabras para evaluar el grado de cooperación de cada experto, $e_i \in E$, en la ronda actual, $t \in \mathbb{N}$, del PACGE. Esta fase se divide a su vez en dos subfases:

  1. *Cálculo del coeficiente de cooperación.* El coeficiente de cooperación evalúa el comportamiento del experto en base a la cantidad de recomendaciones recibidas para cambiar sus preferencias, así como al número de cambios realizados atendiendo a dichas recomendaciones. El comportamiento será más cooperativo cuanto más obedezca el experto a las recomendaciones y realice el cambio de sus preferencias conforme éstas le indican, o cuanto menor sea el número de recomendaciones de cambio recibidas, lo que significará que la opinión del experto está cerca del consenso.

     Definición 1 *Siendo $\#ADV_i^t$ el número total de recomendaciones de cambio recibidas y $\#ACP_i^t$ el número de valoraciones que el experto e_i cambia conforme a las recomendaciones recibidas en la ronda t, el coeficiente de cooperación de e_i en la ronda t, $CC_i^t \in [0,1]$, se define como:*

     $$CC_i^t = \begin{cases} 1 & si\ \#ADV_i^t = 0, \\ \eta \frac{\#ACP_i^t}{\#ADV_i^t} + (1-\eta)\left(1 - \frac{\#ADV_i^t - \#ACP_i^t}{n(n-1)}\right) & en\ otro\ caso. \end{cases} \tag{3.1}$$

     siendo $n(n-1)$ el número total de valoraciones en P_i. El parámetro $\eta \in [0,1]$ controla el grado de penalización aplicado para valores altos de $\#ADV_i^t$, de manera que cuanto mayor es el valor de η, mayor penalización se aplicará cuando $\#ADV_i^t$ es alto.

  2. *Cálculo del grado de cooperación.* A partir del coeficiente de cooperación calculamos el grado de cooperación en lo que se ha denominado *cooperatividad.* Para ello utilizamos el paradigma de la computación con palabras para definir el término lingüístico *cooperativo.*

Definición 2 *Sea "cooperativo" un término lingüístico cuya semántica viene dada por el conjunto difuso $COOP \in [0,1]$, con la siguiente función de pertenencia no decreciente:*

$$\mu_{COOP}(y) = \begin{cases} 0 & si\ y < \alpha, \\ \frac{y-\alpha}{\beta-\alpha} & si\ \alpha \leq y < \beta, \\ 1 & si\ y \geq \beta. \end{cases} \tag{3.2}$$

con $\alpha, \beta, y \in [0,1], \alpha < \beta$. Así el grado de cooperación de e_i en la ronda t, definido como, CD_i^t, corresponde al grado de pertenencia de CC_i^t en el conjunto difuso COOP. Por ejemplo, $CD_i^t = \mu_{COOP}(CC_i^t) \in [0,1]$. La figura 3.4 muestra la función de pertenencia del término "cooperativo" a lo largo del PAC.

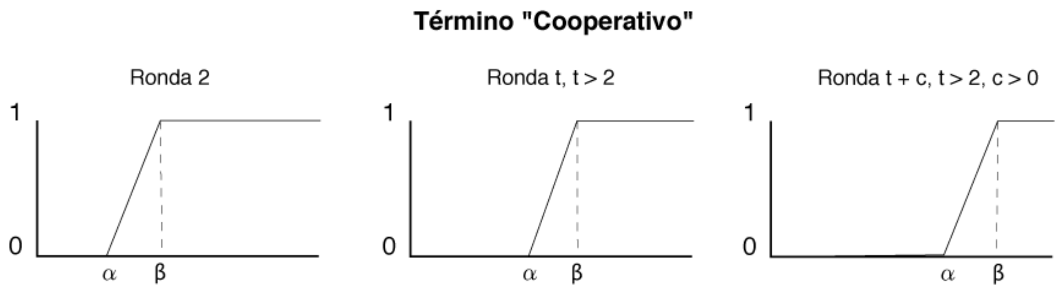

Figura 3.4: Función de pertenencia del término "*cooperativo*".

- *Ponderación de expertos.* Una vez calculados los grados de cooperación de cada uno de los expertos, el siguiente paso será aplicar a cada experto el peso de sus preferencias para la siguiente ronda, de acuerdo a su grado de cooperación. Esta fase está formada por las siguientes subfases:

  1. *Cálculo del peso de los expertos.* Para esta subfase utilizamos los operadores uninormas para calcular el peso de cada experto en cada ronda. La principal razón para utilizar estos operadores es aprovechar su propiedad de refuerzo completo para tener en cuenta el comportamiento acumulado en rondas previas y el comportamiento de la ronda actual. El operador uninorma hará que la importancia de la opinión del experto se

vea reforzada *positivamente* o *negativamente* dependiendo de si el experto coopera o no.

Definición 3 *La función $w(i,t)$ devuelve el peso de la opinión del experto e_i en la ronda t, denotado como $w_i^t \in [0,1]$ y calculado como sigue:*

$$w_i^t = w(i,t) = \begin{cases} g & si\ t = 1, \\ U(CD_i^t, w_i^{t-1}) & si\ t > 1. \end{cases} \quad (3.3)$$

siendo $CD_i^t, w_i^{t-1} \in [0,1]$, U un operador uninorma [19,109] y $g \in]0,1[$ su elemento neutro, de manera que un valor de entrada que esté por encima de g es entendido como un buen comportamiento y viceversa.

La figura 3.5 muestra un ejemplo de los dos tipos de refuerzo. En la imagen *(a)* podemos ver cómo los valores están por encima del elemento neutro, g, por lo que el peso se va reforzando positivamente. Por contra, en el gráfico *(b)* podemos observar cómo cuando el valor del operador está por debajo de g, éste se refuerza negativamente, sin embargo, tras dos rondas en las que el valor está por encima del elemento neutro, podemos apreciar cómo se refuerza de manera positiva.

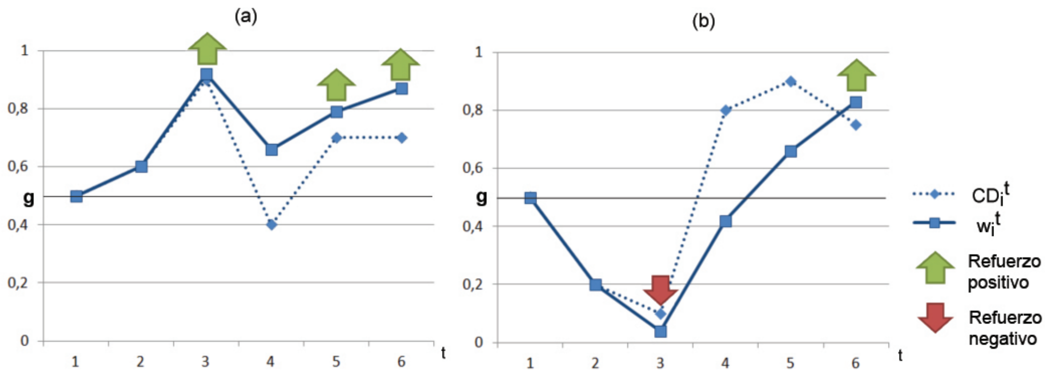

Figura 3.5: Ejemplo de refuerzo positivo y refuerzo negativo.

2. *Normalización de pesos.* A partir de los pesos de cada experto calculados en la fase anterior, en esta fase se procede a la normalización de estos pesos. Esto resulta esencial ya que será lo que permita que aquellos

expertos que hayan sido penalizados puedan recuperar el peso de sus opiniones. Para ello se utiliza la siguiente fórmula:

$$\hat{w}_i^t = \frac{w_i^t}{\sum_{i=1}^{m} w_i^t} \tag{3.4}$$

con $\hat{w}_i^t \in [0,1]$ y $\sum_i \hat{w}_i^t = 1$. Una vez que los pesos han sido normalizados, éstos son tenidos en cuenta en la ronda actual tanto para calcular la preferencia colectiva P_c mediante la agregación de las preferencias de los expertos, como en el cálculo del grado de consenso.

Los pesos normalizados se usarán en cada ronda para calcular el grado de consenso, de manera que las opiniones de los expertos con comportamiento cooperativo tengan más peso que las de aquellos expertos que no cooperan.

Evaluación

Para evaluar esta propuesta se llevaron a cabo dos experimentos:

- El primero de ellos consistió en la comparación entre (I) un modelo de consenso que no penaliza a los expertos con comportamientos no cooperativos, (II) un modelo que penaliza a este tipo de expertos pero no les permite recuperar el peso perdido, y (III) nuestra propuesta basada en uninormas.

- El segundo experimento estuvo enfocado en la actualización del peso de los expertos a lo largo del PACGE. En este caso se seleccionaron 3 tipos de expertos con los siguientes perfiles de comportamientos: (I) un tipo de experto que siempre tiene un comportamiento cooperativo, (II) un tipo de experto que siempre tiene un comportamiento no cooperativo, y (III) un tipo de experto que en las primeras rondas del PACGE tiene un comportamiento no cooperativo, pero que posteriormente adopta en comportamiento cooperativo.

Para realizar estas evaluaciones, se ejecutó un ejemplo ilustrativo de un PACGE en el que participaban 30 expertos con distintos perfiles de comportamiento definidos a priori.

El análisis entre modelos de consenso, señala a esta propuesta como la mejor a la hora de converger al consenso, llegando al umbral de consenso una ronda antes que los demás modelos.

Respecto a la evolución de los pesos de las opiniones de los expertos, se puede observar como el experto que no coopera al principio y decide cooperar posteriormente, recupera el peso de su opinión, llegando a estar al mismo nivel que el experto que coopera siempre.

Analizado de forma visual la figura 3.6, se puede observar cómo la opinión del experto que siempre coopera (experto 22), siempre se acerca a la opinión colectiva. La opinión del experto que nunca coopera (experto 23), se va alejando cada vez más ronda tras ronda. En el caso del experto que no coopera al principio, pero coopera posteriormente, (experto 24) se observa como en un principio se aleja de la opinión de grupo, pero en cuanto comienza a cooperar, su opinión se vuelve a acercar.

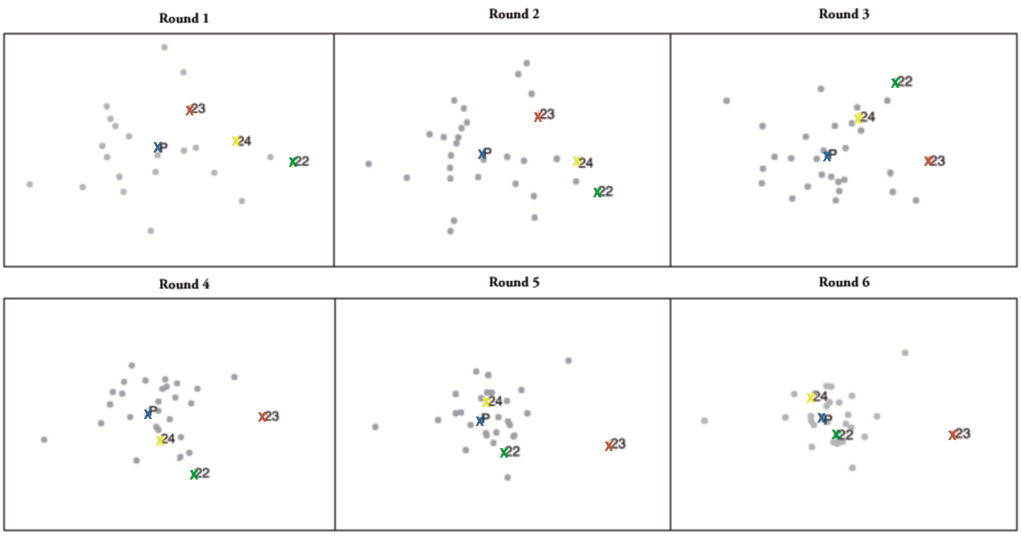

Figura 3.6: Evolución de la opinión de los diferentes tipos de expertos a lo largo del proceso de alcance de consenso.

Esta propuesta está recogida en el artículo (ver Sección 4.1):

F. J. Quesada, I. Palomares and L. Martínez. *Managing experts behavior in large-scale consensus reaching processes with uninorm aggregation operators.* Applied Soft Computing, vol. 35, pp. 873 - 887, 2015.

3.1.2. Propuesta basada en el uso de la medida de hipersimilitud

La principal característica de esta propuesta es el uso de la medida de hipersimilitud, descrita en el apéndice D. En esta propuesta se aprovecha la propiedad de amplificación de valores extremos a la hora de agregar los valores de un atributo para amplificar los comportamientos de los expertos teniendo en cuenta la trayectoria del comportamiento del experto. De manera que el comportamiento de rondas previas (trayectoria de comportamiento) pueda ser de valor para el cálculo del peso de la opinión de los expertos. A diferencia de la propuesta anterior en la que se usaban las uninormas, en esta propuesta la trayectoria del comportamiento del experto a lo largo de todas las rondas del PACGE, tiene una mayor influencia en el cálculo del peso de la opinión de cada experto. La Figura 3.7 muestra el esquema del PACGE con la fase de gestión de comportamientos que utiliza la medida de hipersimilitud. Como se puede observar, el esquema es muy similar al de la propuesta anterior (ver sección 3.1.1), excepto en lo que respecta a la subfase de la *ponderación de expertos*. A continuación se describen las particularidades de esta propuesta:

- *Ponderación de expertos.*

    1. *Cálculo de la función de amplificación.* La función de amplificación se utilizará para reforzar el peso de la opinión de cada experto teniendo en cuenta la cercanía de sus preferencias, P_i, respecto a la opinión del grupo, P_c. Así, si la opinión de un experto se mantiene cerca de la opinión del grupo a lo largo del PACGE, esto indicará que el experto ha tenido un comportamiento cooperativo durante el proceso. En cambio, si la opinión del experto se va alejando conforme avanza el

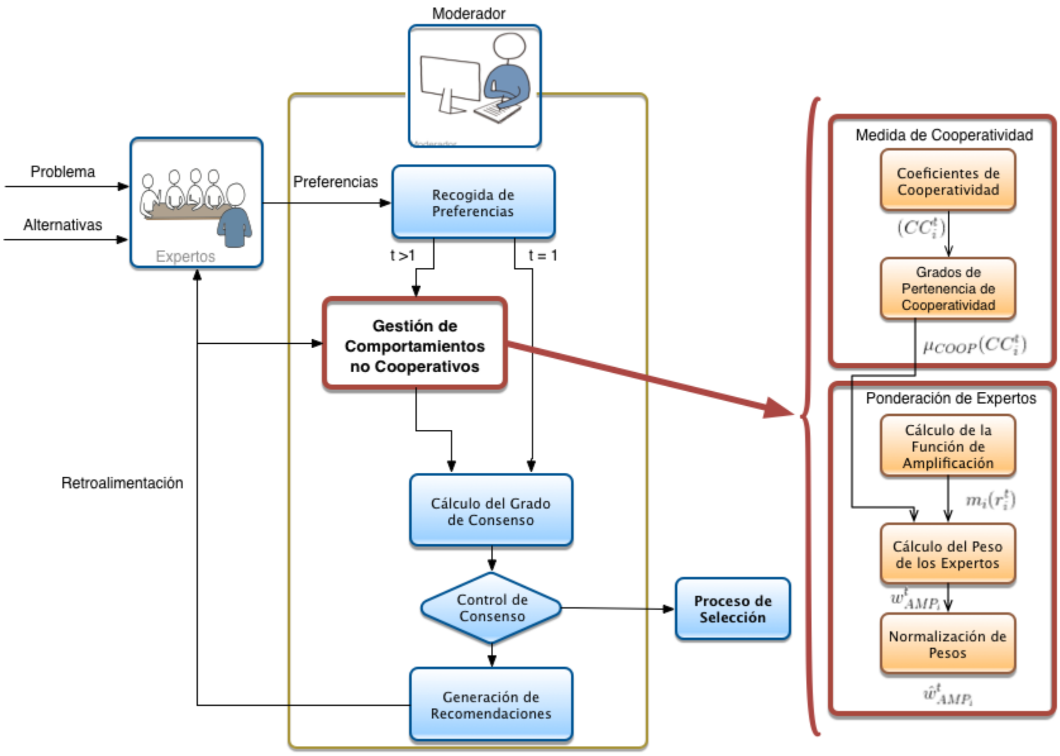

Figura 3.7: Esquema de gestión de comportamientos basados en hipersimilitud para procesos de alcance de consenso.

PACGE, significará que el experto está adoptando un comportamiento no cooperativo. Basándonos en esta idea, calculamos la tasa de cooperación a partir de las recomendaciones recibidas en una ronda concreta.

Definición 4 *Sea r_i^t el número de valoraciones p_i^{lk} del experto e_i que están cerca del consenso en la ronda t. $r_i^t \in [0,1]$ se define como:*

$$r_i^t = 1 - \frac{\sharp ADV_i^t}{n(n-1)} \quad (3.5)$$

Cuanto menor sea el valor de las recomendaciones de cambio recibidas, ADV_i^t, esto indicará que la opinión del experto está más cerca del consenso, P_c, y viceversa.

Definición 5 *Una vez calculado r_i^t y basándonos en las ideas de Yager para la hipersimilitud [145], la función de amplificación $m(r_i^t)$ se define como:*

$$m(r_i^t) = r_i^t + 1, m(r_i^t) \in [1, 2] \tag{3.6}$$

2. *Cálculo del peso de los expertos.* Para calcular el peso de los expertos se utilizará el valor del amplificador, $m(r_i^t)$ y el valor de la medida de similitud $\mu_{COOP}(CC_i^t)$ calculada en la fase en la que se calcula la *medida de la cooperatividad*[1]. Así, los pesos de las opiniones de los expertos se calcularán mediante la siguiente fórmula:

$$w_{AMP_i}^t = \frac{m(r_i^t)\mu_{COOP}(CC_i^t)}{2}, \tag{3.8}$$

   resulta necesario dividir entre 2 para que el resultado de $w_{AMP_i}^t$ esté dentro del intervalo unidad, siendo $w_{AMP_i}^t \in [0, 1]$.

3. *Normalización de pesos.* Para permitir que las opiniones de los expertos puedan recuperar su peso, es necesario normalizar los pesos amplificados:

$$\hat{w}_{AMP_i}^t = \frac{w_{AMP_i}^t}{\sum_1^n w_{AMP_i}^t} \tag{3.9}$$

   estos pesos serán los que se utilicen a la hora de calcular el grado de consenso.

Evaluación

La evaluación de esta propuesta tiene como objetivo observar la evolución del peso de la opinión de los expertos a lo largo del PACGE en distintos modelos. Para ello se definió un problema de consenso en el que participaban 30 expertos con distintos perfiles de comportamiento:

[1]La fase para el cálculo de la medida de cooperatividad es similar a la descrita en la sección 3.1.1, compartiendo el mismo término lingüístico *cooperativo*, pero utilizando la siguiente fórmula:

$$CC_i^t = \begin{cases} 1 & \text{si } \#ADV_i^t = 0, \\ \frac{\#ACP_i^t}{\#ADV_i^t} & \text{en otro caso.} \end{cases} \tag{3.7}$$

- *Cooperativo* ($e_1 - e_{21}$), los expertos con este comportamiento siempre aplican los cambios sugeridos a lo largo de todo el PAC.
- *Manipulador* (e_{22}), expertos que tratan de manipular la solución alternando obediencia (se aplican los cambios recibidos) y desobediencia total (se aplican cambios totalmente opuestos a los recibidos).
- *No cooperativo* (e_{23}), comportamiento en el que nunca se aplican los cambios recibidos.
- *Indefinido* ($e_{24} - e_{30}$), estos expertos alternan entre aplicar o ignorar los cambios recibidos, pero nunca aplican cambios diametralmente opuestos a las recomendaciones.

Este problema se ejecutó en los siguientes modelos: (i) un modelo que penaliza a los expertos con comportamientos no cooperativos, pero no permite que sus opiniones recuperen importancia ni tiene en cuenta la trayectoria de comportamiento del experto [98], (ii) un modelo en el que las opiniones de los expertos pueden recuperar peso, pero no se considera la tendencia de comportamiento de los expertos [100], y (iii) esta propuesta, que permite la recuperación de peso en las opiniones de los expertos y que tiene en cuenta la trayectoria de sus comportamientos.

El análisis de los resultados de esta propuesta, tal y como se pueden observar en la figura 3.8, muestran cómo la trayectoria de comportamiento de un experto a lo largo del PACGE es fundamental, ya que permite premiar a aquellos expertos que mantienen un comportamiento cooperativo continuado, mientras que se penaliza a aquellos que alternan entre cooperación y no cooperación. Este hecho influye positivamente en el PACGE, ya que impide la manipulación del proceso por parte de expertos con comportamientos no cooperativos. Esta propuesta está recogida en el capítulo de libro (ver Sección 4.2):

F. J. Quesada, I. Palomares and L. Martínez. *Using Computing with Words for Managing Non-cooperative Behaviors in Large Scale Group Decision Making.* Granular Computing and Decision-Making, vol. 10: Springer International Publishing, pp. 97-121, 2015.

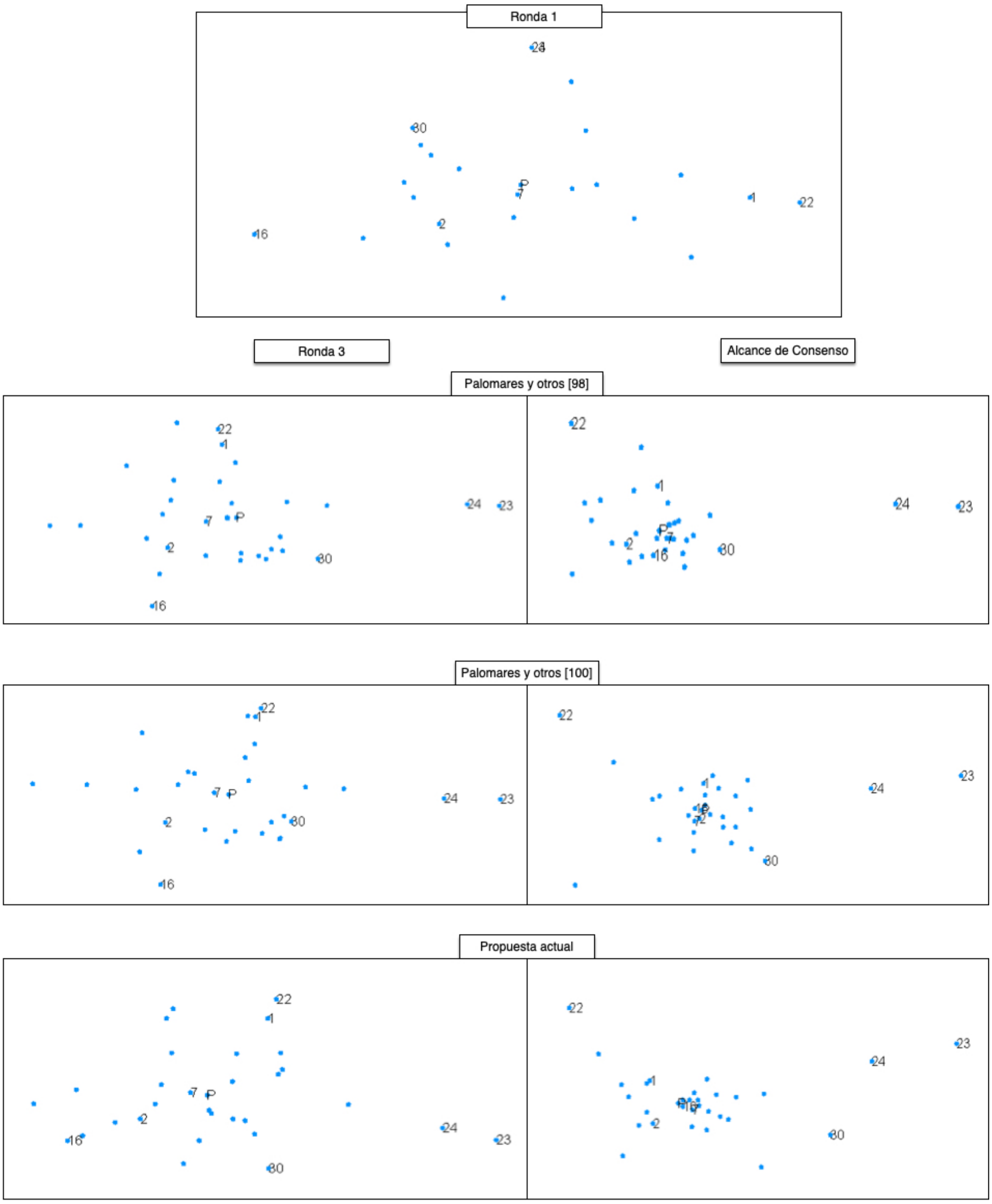

Figura 3.8: Comparativa de la evolución de las opiniones de los expertos a lo largo del proceso de consenso

3.2. Sistema de Recomendación a Grupo basado en Consenso

El segundo objetivo de la investigación relativa a este doctorado, se centra en la mejora de la satisfacción de los usuarios de SRGs mediante la generación de recomendaciones consensuadas.

Para ello, se propone modificar el esquema general de los SRGs añadiendo una fase de consenso. En la sección 2.3 se identifican dos métodos de agregación para los SR, estos son: la *agregación de valoraciones* y la *agregación de recomendaciones*. Dado que nuestro principal objetivo radica en la mejora de la satisfacción de los usuarios, el método utilizado es la *agregación de recomendaciones*. En esta nueva fase, se generarán recomendaciones con alto grado de acuerdo entre los miembros del grupo. Así, el sistema estaría formado por dos fases tal y como se ilustra en la figura 3.9:

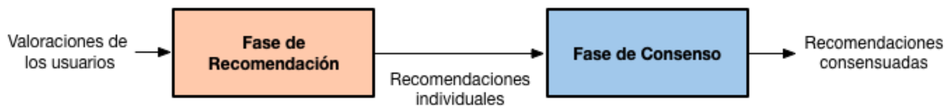

Figura 3.9: Esquema general de los sistemas de recomendación en grupo basados en consenso.

1. *Fase de recomendación*. Se generan las recomendaciones para cada miembro del grupo a partir de las valoraciones de cada usuario.

2. *Fase de consenso*. A partir de las recomendaciones de cada miembro, generadas en la fase anterior, se lleva a cabo un PAC automático con el objetivo de generar recomendaciones con un alto nivel de consenso para el grupo.

Fase de Recomendación

En esta fase se realizan los cálculos necesarios para generar un conjunto de recomendaciones para cada miembro del grupo $g_i \in G$. Este proceso está formado por las siguientes etapas, tal y como se muestra en la figura 3.10:

1. *Valoración de productos.* En esta etapa cada usuario u_i expresa para cada producto i_l su valoración r_{il}. Una vez terminado el proceso, obtenemos una matriz de usuarios U y productos I valorados.

2. *Predicción de productos para los miembros del grupo.* A partir de la matriz obtenida en la etapa anterior, en esta etapa se generan, para los miembros del grupo G, las predicciones de productos $i_l \in I$ que todavía no han sido valorados por los usuarios. Para ello aplicamos el algoritmo de filtrado colaborativo vecinos cercanos basado en usuarios [121]. Este algoritmo generará los valores predictivos, $\tilde{r}_{il}$, de estos productos para todos los usuarios. Una vez generados estos valores, sólo se tendrán en cuenta en futuras etapas, las predicciones $\tilde{r}_{il}$ correspondientes a los miembros del grupo $g_i \in G$.

3. *Filtrado de los n-primeros productos comunes.* Tras generar las predicciones para los miembros del grupo $g_i \in G$, es necesario realizar un filtrado para eliminar aquellos productos que no sean comunes a todos los miembros del grupo. El principal motivo para realizar esta tarea reside en que es posible que distintos miembros del grupo G puedan recibir recomendaciones sobre diferentes subconjuntos de productos de I; por lo que resulta necesario extraer un subconjunto de productos que hayan sido recomendados simultáneamente a todos los miembros del grupo. Debido a que puede haber casos en los que el número de recomendaciones comunes sea excesivamente alto, vamos a extraer sólo los n-primeros productos comunes a los miembros del grupo, $X = \{x_1, \ldots, x_n\}, X \subset I$, siendo $n << t$. Para la generación de este subconjunto utilizamos técnicas de agregación de ranking tales como Borda o el algoritmo de voto acumulativo [6].

4. *Orden de preferencia de los n-primeros productos comunes.* A partir de las predicciones de cada usuario, $\tilde{r}_i = [\tilde{r}_{i1}, \tilde{r}_{i2}, \ldots, \tilde{r}_{in}]$, del subconjunto de los n-primeros productos, se obtiene el orden de preferencia de cada usuario $O_i = \{o_i(1), \ldots, o_i(n)\}$. De esta manera, los productos en X estarán ordenados de mayor a menor valor de predicción.

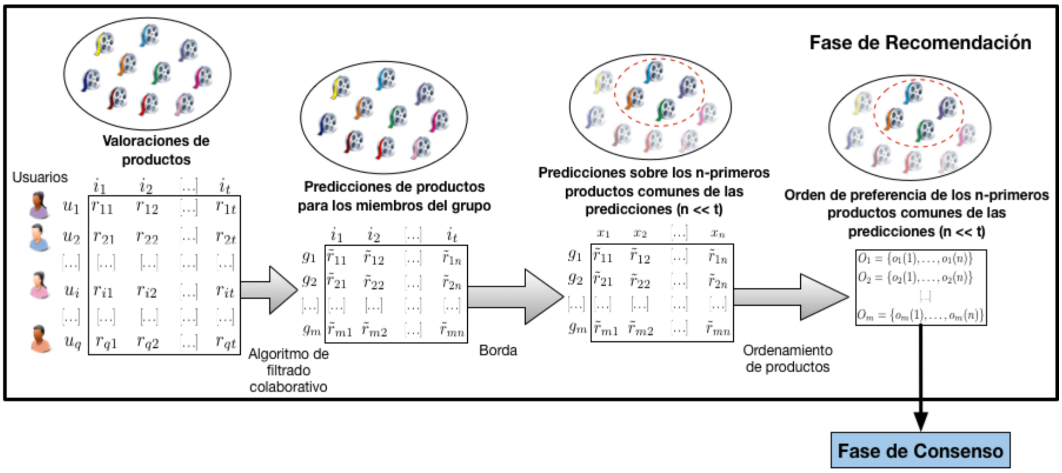

Figura 3.10: Esquema de la fase de recomendación.

Fase de Consenso

Tras la fase de recomendación, en esta fase se realizará un PAC con el objetivo de obtener un orden de recomendaciones con alto nivel de acuerdo.

Esta fase se compone de las siguientes subfases tal y como se ilustra en la figura 3.11:

1. *Generación de relaciones de preferencia difusas.* A partir de los órdenes de preferencia de los usuarios sobre los n-primeros productos comunes generados en la fase de recomendación, en esta subfase se transformará para cada usuario su orden de preferencia, $O_i = \{o_i(1), \ldots, o_i(n)\}$, en una relación de preferencia difusa, $P_i = (p_i^{lk})_{n \times n}$. Para ello utilizaremos la función de transformación

propuesta por Chiclana y otros en [28]:

$$p_i^{lk} = \frac{1}{2}\left(1 + \frac{o_i(k) - o_i(l)}{n-1}\right) \tag{3.10}$$

2. *Proceso de Alcance de Consenso.* La relaciones de preferencia difusas obtenidas en el paso anterior, serán la entrada para el PAC, junto con el umbral de consenso, $\mu \in [0,1]$, y el número máximo de rondas $maxRounds \in \mathbb{N}$. El PAC aplicado es un proceso automático en el que las preferencias serán modificadas de acuerdo a las recomendaciones [102]. El proceso finalizará cuando se haya alcanzado el umbral de consenso (se alcanza el consenso) o cuando se llega al número máximo de rondas (no se alcanza el consenso). Una vez que se ha llegado al mínimo nivel de acuerdo determinado por el umbral de consenso, podemos afirmar que todas las relaciones de preferencia están bajo consenso. Llegados a este punto, el siguiente paso sería agregar todas las relaciones de preferencia, formando así la relación de preferencia de consenso o preferencia colectiva, $P_c = (p_c^{lk})_{n \times n}$.

3. *Proceso de Selección.* Por último, se lleva a cabo el proceso de selección de alternativas. Para ello se transformará la preferencia colectiva en un orden de preferencia, $O_c = \{o_c(1), \ldots, o_c(n)\}$, que indicará un ranking de recomendaciones consensuadas para el grupo .

Evaluación de la propuesta

Para evaluar la propuesta se ha creado un caso de estudio en el que se generan recomendaciones de películas a 30 grupos de 5 usuarios de acuerdo a distintos grados mínimos de consenso (0,80, 0,85 y 0,90). El dataset utilizado ha sido *MovieLens 100k*[2]. Para ello se ha estudiado el valor del área bajo la curva y la precisión [46].

Respecto al primer parámetro, el análisis de los resultados muestra cómo la aplicación de un PAC para generar una recomendación a grupo mejora

[2]Recopilado por el GroupLens Research Project, en la Universidad de Minnesota. Accesible en http://grouplens.org/datasets/movielens/

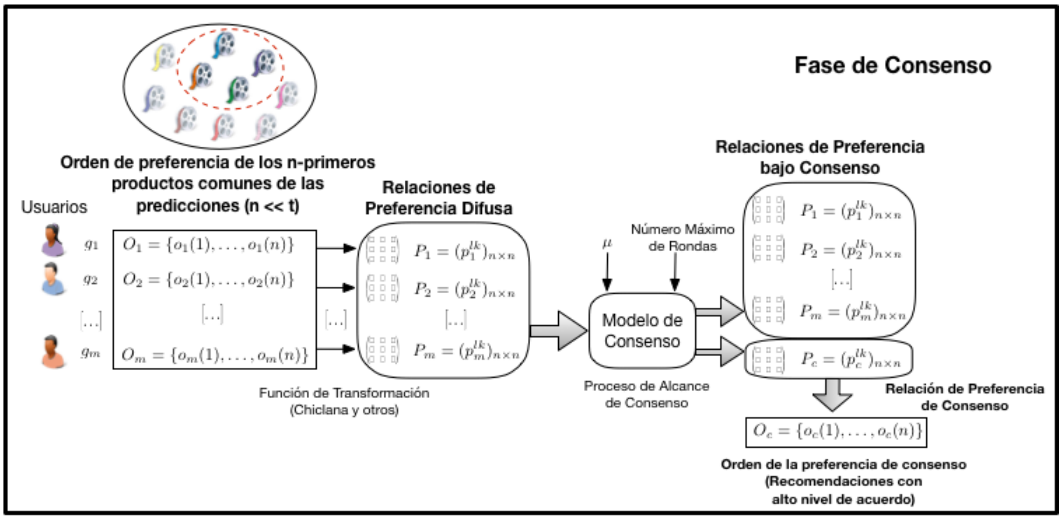

Figura 3.11: Esquema de la fase de consenso.

significativamente los resultados con respecto a la línea base. Aunque todas las configuraciones con consenso tienen una mejor actuación que la referencia, el sistema alcanza su funcionamiento óptimo cuando la configuración del umbral de consenso es igual a 0,80.

En cuanto a la precisión, la configuración con un umbral de consenso igual a 0,80 vuelve a ser la que obtiene un mejor rendimiento. En concreto, para listas de 4 o menos recomendaciones, esta configuración es la mejor de las estudiadas, estando en similar rendimiento al resto de configuraciones de consenso para listas con mayor número de recomendaciones.

En vista a los resultado obtenidos, podemos concluir que la inclusión de un PAC en la generación de recomendaciones a grupo supone mejoras significativas, conllevando a un incremento en la satisfacción de los miembros del grupo.

Esta propuesta está recogida en el artículo (ver Sección 4.3):

J. Castro, F. J. Quesada, I. Palomares and L. Martínez. *A Consensus-Driven Group Recommender System.* International Journal of Intelligent Systems, vol. 30, no. 8, pp. 887–906, 2015.

Y en el capítulo de libro (ver Sección 4.4):

F. Moya, F. J. Quesada, J. Castro, R. M. Rodríguez, I. Palomares and L. Martínez. *Improving group recommendations with consensus reaching processes.* Soft computing and Hybrid Systems for Knowledge Discovery and Decision-making, Atlantis Computational Intelligence Series (ACIS), Aceptado.

Capítulo 4

Publicaciones

En virtud de lo establecido en el artículo 25, punto 2, de la normativa vigente para los Estudios de Doctorado de la Universidad de Jaén, correspondiente al programa establecido en el RD. 99/2011, en este capítulo se presentan las publicaciones que componen el núcleo de la presente tesis doctoral.

Dichas publicaciones se corresponden a dos artículos científicos publicados en Revistas Internacionales indexadas por la base de datos JCR (*Journal Citation Reports*), producida por ISI (*Institute for Scientific Information*), un capítulo de libro publicado y un capítulo de libro aceptado pendiente de publicación.

4.1. Managing Experts Behavior in Large-scale Consensus Reaching Processes with Uninorm Aggregation Operators

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Managing experts behavior in large-scale consensus reaching processes with uninorm aggregation operators

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ABSTRACT

In many real-life large scale group decision making problems, it can be necessary and convenient a consensus reaching process, which is an iterative procedure aimed at seeking a high degree of agreement amongst experts' preferences before making a group decision. Although a wide variety of models and approaches have been proposed and developed to support consensus reaching processes, in large groups there are some important aspects that still require further study, such as the treatment of experts' behaviors that could hamper reaching the wanted agreement. More specifically, it would be necessary an approach to deal with experts properly, based on the overall behavior they present during the discussion process, as well as reinforcing repeated patterns of cooperative (or uncooperative) behavior adopted by experts. This paper presents an expert weighting methodology for consensus reaching processes in large-scale group decision making, that incorporates the use of uninorm aggregation operators. Such operators, which are characterized by their property of full reinforcement, are used in the proposed methodology to allow the experts' weighting based on their overall behavior during the consensus process and the behavior evolution across the time. This proposal is integrated in a consensus model for large-scale group decision making problems under uncertainty, and it is put in practice to show an illustrative example of its effectiveness and improvements with respect to other approaches.

1. Introduction

Decision making is a frequent process in human daily lives, in which there exist several alternatives and the best one/s shall be chosen. Group decision making (GDM) problems, characterized by the participation of multiple individuals or experts in such a process, have been subject of an extensive research in the last decades [1,2].

In the traditional resolution process for GDM problems [3], each expert provides his/her preferences over alternatives and the best alternative or subset of them is selected, disregarding the degree of agreement between experts' preferences. This often leads to the drawback that some experts may not accept the decision made [4], because they might consider that their opinions have not been heard. For this reason, the study of consensus reaching processes (CRPs), in which experts aim at reaching a collective agreement before making a decision [5], has become a prominent research topic in GDM [6–8]. CRPs are iterative discussion processes in which experts *must* accept a priori to collaborate bringing their opinions closer to each other in order to achieve an agreement [9].

Classically, GDM problems taking place in most organizations occur at a strategic level, in which a small number of people are responsible for making the decision. However, the expansion of technological paradigms such as social media and e-marketplaces, is causing that the so-called large-scale GDM problems [10–12], in which a larger number of experts can take part, attain a greater importance. In CRPs carried out in these contexts, it may occur that some experts or coalitions of them, who may have established a collaboration contract [9], try to break it at some stage in such processes. These experts might refuse to cooperate with the rest of the group to reach an agreement [13] and try to strategically bias the solution for the GDM problem [14], hence it is necessary to identify and deal with these non-cooperative experts' behaviors to ensure a normal CRP development.

Some early proposals to deal with strategic manipulation of preferences in classical GDM problems were proposed by Yager [14,15], where experts' preferences may be penalized before applying the alternatives selection process, by analyzing how drastic

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1568-4946/

and biased their opinions are. Later on, an approach focused on consensus-based GDM problems was proposed in [13], where a consensus model for dealing with non-cooperative behaviors of experts in CRPs was presented. Such a model defines a methodology based on fuzzy clustering to identify non cooperating experts and subgroups, and applies a weight-based scheme to penalize them, according to the behavior they presented. In this penalizing scheme, importance weights of experts are updated if they show a non-cooperative behavior only, by penalizing their current value. Nevertheless, the values of experts' weights cannot be increased again, even though they change their mind and decide to adopt a more cooperating attitude from a specific discussion round onwards. Moreover, in [13] the weight updating applied on each expert's preferences at a given discussion round is based on his/her behavior at such a round only, not taking into account neither how his/her behavior was previously nor how it has evolved since the beginning of the CRP. Considering for instance a situation in which two experts present a currently cooperative behavior after four rounds of discussion, they should not be assigned the same importance weight if only one of them has kept cooperating since the beginning of the CRP, and the other one has not cooperated until now.

Regarding the previous cases, making use of the available historical information about experts' behavior in CRPs is, as far as we know, a challenge not properly addressed in this research field yet. If tackled properly, this aspect would allow a more accurate and appropriate management of such behaviors. Nevertheless, there exist several proposals for dynamic multi-criteria decision making in recent literature [16,17], in whose framework the set of alternatives varies over time and each alternative is assessed at multiple time instants (similarly to CRPs for GDM problems, in which experts must provide and revise their assessments over alternatives across several discussion rounds). In these frameworks, a global (dynamic) assessment value for each alternative is computed, so that historical information about previous assessments on that alternative is also considered. For instance, in [16] Campanella and Ribeiro proposed the use of associative aggregation operators to compute global assessments in dynamic multi-criteria decision making scenarios. More specifically, they illustrated the usefulness of uninorm aggregation operators [18,19], due to the interesting properties that they present to reinforce both positive and negative assessments on alternatives at successive time instants.

Inspired by the reinforcement-based aggregation techniques mentioned above to integrate historical information in dynamic decision making approaches, in this paper we propose a uninorm-based methodology for managing non-cooperative behaviors based on the overall behavior of each expert over the course of CRPs in large-scale GDM problems. To do this, we present a weighting scheme based on fuzzy set theory and the methodology of computing with words (CW) [20], that incorporates the use of uninorm aggregation operators, aiming at three goals:

(i) Assigning importance weights to experts based not only on their current behavior, but also on their patterns of behavior presented at previous consensus rounds.
(ii) Such weights are computed based on a linguistic modeling to represent the uncertainty related to the experts' behavior.
(iii) Exploiting the full reinforcement property of uninorm operators to reinforce repeated patterns of cooperative (or non-cooperative) behaviors by an expert at successive rounds.

A consensus model for large-scale GDM under uncertainty that incorporates the proposed weighting scheme is also introduced. Finally, an illustrative example is presented to show the properties of the weighting scheme in practice, as well as its advantages with respect to other consensus approaches.

This paper is set out as follows: in Section 2, some basic concepts about CRPs in GDM, uninorm aggregation operators and CW methodology for reasoning processes are reviewed. Straightaway, Section 3 presents in further detail the uninorm-based weighting scheme for experts in CRPs. A consensus model for managing experts' behaviors that integrates the proposed scheme is then presented in Section 4. Section 5 presents the illustrative example conducted, and some concluding remarks are finally drawn in Section 6.

2. Preliminaries

This section firstly revises some basic concepts about GDM problems and CRPs, followed by an overview of uninorm aggregation operators and the methodology of CW for reasoning processes, both of which are taken into account in the proposal presented in this paper.

2.1. Consensus reaching processes in GDM

GDM entails the participation of multiple experts who must make a collective decision to find a common solution to a problem. Decision processes in which several experts with different knowledge and experience take part, may usually lead to better decisions than those made by just one expert [2].

A GDM problem is formally characterized by the following elements [1]:

- The existence of a common problem to be solved.
- A set $X = \{x_1, \ldots, x_n\}$ $(n \geq 2)$, of *alternatives* or possible solutions to the problem.
- A set $E = \{e_1, \ldots, e_m\}$ $(m \geq 2)$, of individuals or *experts*, who express their opinions or preferences over alternatives X. As previously indicated, this paper focuses on large-scale GDM problems, such that $m \gg 2$.

In order to express their opinions over alternatives, each expert utilizes a preference structure. Fuzzy preference relations are one of the most widely utilized preference structures in many GDM approaches found in [21]. A fuzzy preference relation P_i associated to expert e_i, can be represented for X finite as a $n \times n$ matrix, as follows:

$$P_i = \begin{pmatrix} - & \ldots & p_i^{1n} \\ \vdots & \ddots & \vdots \\ p_i^{n1} & \ldots & - \end{pmatrix}$$

being each numerical *assessment* $p_i^{lk} = \mu_{P_i}(x_l, x_k) \in [0, 1]$, the degree of preference of the alternative x_l over x_k, $l, k \in \{1, \ldots, n\}$, $l \neq k$, according to e_i, such that:

- $p_i^{lk} > 0.5$ indicates e_i's preference of x_l over x_k.
- $p_i^{lk} < 0.5$ indicates e_i's preference of x_k over x_l.
- $p_i^{lk} = 0.5$ indicates e_i's indifference between x_l and x_k.

Fuzzy preference relations can accomplish diverse properties [22–24]. In [25,26], some of these properties have been studied and considered in a consensus model for GDM with fuzzy preference relations. Accordingly, in order to provide or facilitate the construction of consistent preference relations, in this work we assume the reciprocity property in fuzzy preference relations, i.e. assessments accomplish that if $p_i^{lk} = x$, $x \in [0, 1]$, $l \neq k$, then $p_i^{kl} = 1 - x$.

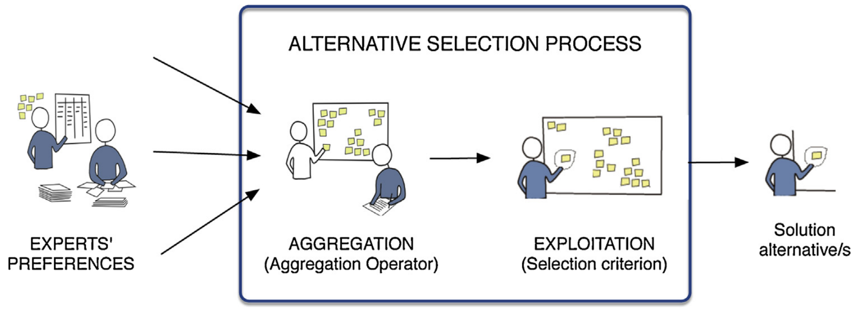

Fig. 1. Classical resolution process for GDM problems.

The solution for a GDM problem can be obtained by applying either a *direct approach* or an *indirect approach* [3]. In the former, a solution is directly obtained from the individual preferences of each expert, without constructing a social opinion first [27], whereas in the latter, a social opinion or *collective preference* is firstly computed from individual opinions and then used to obtain the solution for the problem. Regardless of the approach considered, the traditional selection process for reaching a solution to GDM problems is made up by two phases [28] (see Fig. 1): (i) an *aggregation phase*, in which preferences of experts are combined by using an aggregation operator, and (ii) an *exploitation phase*, where a selection criterion is applied to obtain an alternative or subset of them as the solution for the problem.

When a GDM problem is solved by applying the alternatives selection process only, it may occur that some experts feel that their opinions have not been taken into consideration to find the solution, therefore they would not accept it. Since a sufficient level of collective agreement is crucial in many real-life situations, it becomes necessary to apply a CRP, introducing an additional phase in the resolution process for GDM problems. CRPs aim at obtaining a high level of group agreement before making a decision [4,6].

The term *consensus* can be defined as the agreement produced by mutual consent between all members in a group or between several groups [5]. The process to reach consensus is a dynamic and iterative process, consisting of several rounds of discussion, and frequently coordinated by a human figure: the moderator. The moderator is a key figure in CRPs, being in charge of supervising and guiding experts across the discussion process [5]. Reaching consensus implies that experts *must* modify their initial opinions throughout the CRP, bringing them closer to the rest of the group's opinions. In this sense, they must comply with a collaboration contract [9], according to which they accept to collaborate with each other to search for a common agreed solution.

Fig. 2 shows a general CRP scheme followed in a large number of GDM approaches [6]. In the following, its main phases are briefly described:

1 *Gathering preferences*: Each expert e_i provides his/her preferences over alternatives to the moderator, e.g. by means of a fuzzy preference relation.
2 *Determine degree of consensus*: The current degree of consensus in the group, *cr*, is computed. Such a consensus degree is usually measured as a value in the [0,1] interval (where a value of 1 indicates full or unanimous agreement between all experts on all the alternatives). To do so, different consensus measures can be utilized, based on the use of similarity/distance metrics to calculate

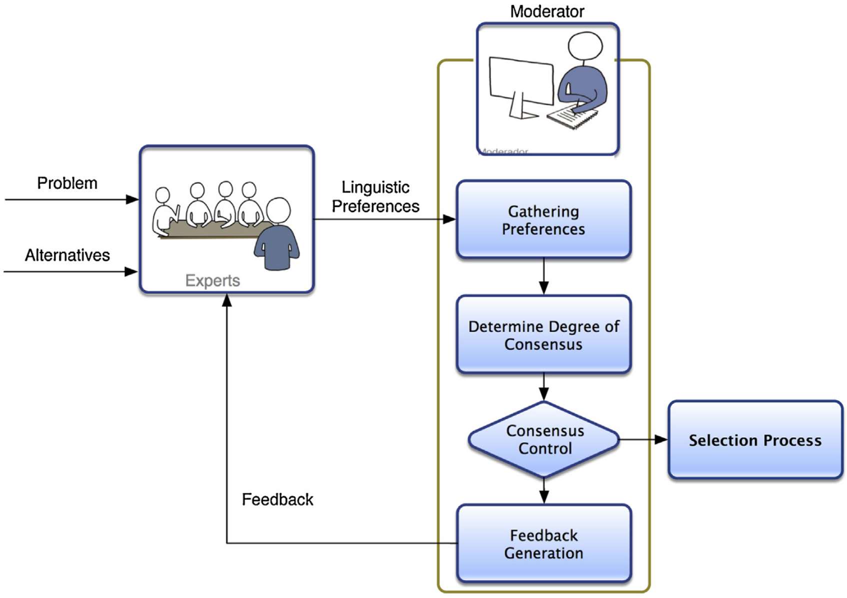

Fig. 2. General scheme for CRPs.

degrees of similarity between preferences of experts, and aggregation operators that obtain the degree of consensus in the group from such similarity values [6].

3 *Consensus control*: In this phase, the consensus degree *cr* previously obtained, is compared with a consensus threshold $\mu \in]0, 1]$ fixed a priori, which indicates the minimum level of agreement required. If $cr \geq \mu$ then consensus has been achieved and the group proceeds to the selection process; otherwise, it is necessary another discussion round. In order to limit the number of discussion rounds allowed, a parameter $Maxround \in \mathbb{N}$ can be introduced. If the number of consensus rounds applied exceeds *Maxround*, a different GDM strategy might be adopted. Some examples of such strategies have been provided in the literature [5], such as: (i) delegating the decision to a subgroup, (ii) conducting a community building session, (iii) applying a simple majority vote, or (iv) excluding experts who do not contribute to achieve consensus.

4 *Feedback generation*: The moderator computes a collective preference, P_c, by aggregating the individual preferences of all experts. Based on P_c, the moderator identifies the farthest experts' assessments from consensus, and advises them to modify such assessments with the aim of increasing the consensus degree in the following round. Each expert is responsible for modifying his/her own assessments (and, consequently, committing with the collaboration contract established), by increasing/decreasing assessment values and bringing them closer to P_c. Each piece of advice generated consists in a triplet $(e_i, (x_l, x_k), Direction)$ which indicates that the expert e_i *must* modify his/her assessment p_i^{lk} in the direction given by $Direction \in \{increase, decrease\}$.

2.2. Uninorm aggregation operators

Uninorm aggregation operators were introduced by Yager and Rybalov [18], and they provide an unification of *t*-norm and *t*-conorm operators. For this reason, before reviewing uninorm operators the formal definitions of *t*-norm and *t*-conorm are revised.

Definition 1. [18]. A triangular norm or *t*-norm *T* is a mapping,

$T : [0, 1] \times [0, 1] \rightarrow [0, 1]$

having the following properties for all $a, b, c, d \in [0, 1]$:

i) Commutativity: $T(a, b) = T(b, a)$.
ii) Monotonicity: $T(a, b) \geq T(c, d)$ if $a \geq c$ and $b \geq d$.
iii) Associativity: $T(a, T(b, c)) = T(T(a, b), c)$.
iv) Neutral element: $T(a, 1) = a$.

T-norms are conjunctive aggregation operators, therefore they exhibit the following property,

$T(a_1, \ldots, a_n) \leq \min_i [a_i]$

From this property, we can see that the aggregated value is never greater than the lowest a_i. Moreover, if all a_i's values are low, then such values shall reinforce each other so that the resulting aggregated value is even lower. This property is known as *downward reinforcement* [29].

Some well-known examples of *t*-norms are:

- Minimum: $T_{\min}(a, b) = \min(a, b)$.
- Product: $T_{prod}(a, b) = ab$.
- Łukasiewicz *t*-norm: $T_{Luk}(a, b) = \max\{0, a + b - 1\}$.

Definition 2. [18]. A triangular conorm or *t*-conorm *S* is a mapping,

$S : [0, 1] \times [0, 1] \rightarrow [0, 1]$

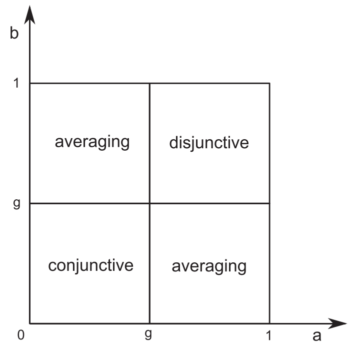

Fig. 3. Behavior of uninorms for different input values. Adapted from [16].

having the following properties for all $a, b, c, d \in [0, 1]$:

i) Commutativity: $S(a, b) = S(b, a)$.
ii) Monotonicity: $S(a, b) \geq S(c, d)$ if $a \geq c$ and $b \geq d$.
iii) Associativity: $S(a, S(b, c)) = S(S(a, b), c)$.
iv) Neutral element: $S(a, 0) = a$.

T-conorms are disjunctive aggregation operators, therefore they exhibit the following property,

$S(a_1, \ldots, a_n) \geq \max_i [a_i]$

From this property, we can see that the aggregated value is always at least as high as the largest a_i. Additionally, if all a_i's values are high, then such values shall reinforce each other, thus leading to an even higher aggregated value. This property is known as *upward reinforcement* [29].

Some well-known examples of *t*-conorms are:

- Maximum: $S_{\max}(a, b) = \max(a, b)$.
- Probabilistic sum: $S_{prob}(a, b) = a + b - ab$.
- Łukasiewicz *t*-conorm: $S_{Luk}(a, b) = \min\{a + b, 1\}$.

Uninorm operators were proposed by Yager and Rybalov to provide a generalization of the *t*-norm and the *t*-conorm, such that the neutral element can lie anywhere in the unit interval, and whose behavior varies depending on the values to aggregate being higher or lower than such a neutral element [18,19]. They are defined as follows:

Definition 3. [18] A uninorm is a mapping,

$U : [0, 1] \times [0, 1] \rightarrow [0, 1]$

having the following properties for all $a, b, c, d \in [0, 1]$:

i) Commutativity: $U(a, b) = U(b, a)$.
ii) Monotonicity: $U(a, b) \geq U(c, d)$ if $a \geq c$ and $b \geq d$.
iii) Associativity: $U(a, U(b, c)) = U(U(a, b), c)$.
iv) Neutral element: $\exists g \in [0, 1] : U(a, g) = a$.

Unlike *t*-norms and *t*-conorms, in which the neutral elements are 1 and 0 respectively, uninorms can take any value in the unit interval as the neutral element. Uninorms may have conjunctive, disjunctive or averaging behavior, depending on input values *a*, *b* being greater or lower than *g*, as illustrated in Fig. 3 [16].

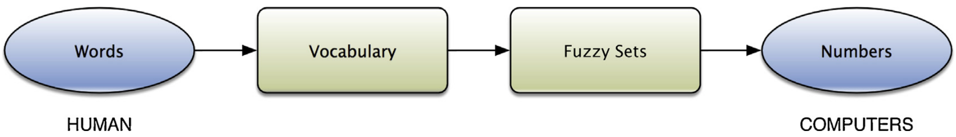

Fig. 4. Paradigm of man-machine understanding.

Taken from [34].

Two general families of uninorm operators with neutral element g were introduced by Fodor et al. [30]:

$$U(a,b)=\begin{cases} (a) \quad gT_U\left(\frac{x}{g},\frac{y}{g}\right) & \text{if } 0\le a,b\le g,\\ (b) \quad g+(1-g)S_U\left(\frac{x-g}{1-g},\frac{y-g}{1-g}\right) & \text{if } g\le a,b\le 1,\\ (c1) \quad \max(a,b) & \text{if } \min(a,b)\le g\le \max(a,b),\\ (c2) \quad \min(a,b) & \text{if } \min(a,b)\le g\le \max(a,b).\end{cases} \tag{1}$$

with T_U and S_U being any t-norm and any t-conorm operator, respectively. The difference between both families of uninorm operators are in the use of either item (c1), which defines the so-called $\mathcal{U}_{\max}$ family of uninorm operators, or item (c2), which defines the $\mathcal{U}_{\min}$ family of uninorm operators [31].

Some later works generalized the case in which $\min(a, b) \le g \le \max(a, b)$, by considering the use of any averaging aggregation operator M_U (see Fig. 3), such that $\min(a, b) \le M_U(a, b) \le \max(a, b)$ [16]. For instance, in [32] Ribeiro et al. is proposed an adaptation of uninorm operators that applies the OWA (*ordered weighted averaging*) operator [33] in such a case.

For any $g \in]0, 1[$, we can see that uninorms consider an upward reinforcement when aggregating high input values (above g), and a downward reinforcement when aggregating low input values (below g), hence they can be used as *full reinforcement* operators, as it will be shown in this paper to reinforce cooperative or non cooperative behaviors of experts over the course of a CRP.

Some examples of uninorms which shall be utilized in our proposal, are shown below:

Example 1. Consider the product t-norm and probabilistic sum t-conorm reviewed above. Then, the following uninorm is defined based on the general families introduced by Fodor et al. (Eq.(1))[30]:

$$U(a,b)=\begin{cases} \dfrac{ab}{g} & \text{if } 0\le a,b\le g,\\ \dfrac{a+b-ab-g}{1-g} & \text{if } g\le a,b\le 1,\\ M_U(a,b) & \text{if } \min(a,b)\le g\le \max(a,b).\end{cases} \tag{2}$$

with $M_U(a, b)$ being an averaging operator.

Example 2. The cross-ratio uninorm [22] is an example of continuous uninorm in $[0, 1]^2 \setminus \{(0, 1), (1, 0)\}$, with neutral element $g = 0.5$:

$$U(a,b)=\begin{cases} 0 & \text{if } (a,b)\in\{(0,1),(1,0)\},\\ \dfrac{ab}{ab+(1-a)(1-b)} & \text{otherwise.}\end{cases} \tag{3}$$

2.3. Computing with words methodology for reasoning processes

In many real situations, human beings utilize natural language consisting of words and expressions, to communicate, reason and understand their environment. Computers, on the other hand, use much more formal symbols [34]. The paradigm of *Computing with Words* (CW) was proposed by Zadeh [20] to establish a comprehensive link of communication between human beings and computer systems, and to increase the use of natural language in communication, reasoning and decision making processes carried out by such systems. This methodology facilitates human–computer cooperation to a high degree, since it provides a framework in which concepts are modeled in an amenable way to both sides.

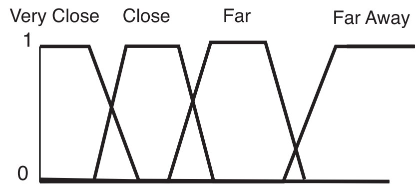

Fig. 5. Different linguistic terms for the attribute *distance*.

The methodology of CW is based on fuzzy sets theory [35], so that concepts belonging to a vocabulary can be modeled by means of fuzzy sets, thus being easily understood by human beings and computers (see Fig. 4). Some key elements in CW are the concepts of linguistic variable and linguistic term, formulated by Zadeh.

Definition 4. [36–38] A linguistic variable is characterized by a 5-tuple $(H, T(H), U, G, M)$, where H is the name of the variable; $T(H)$ symbolizes the set of linguistic terms or linguistic values of H, with each value being a fuzzy variable generically denoted as X that ranges over a universe of discourse U; G is a syntactic rule (normally given by a grammar) to generate the names of linguistic terms in H; and M is a semantic rule to associate each element in H with its meaning, $M(X)$, given by a fuzzy set in U.

Based on Zadeh's definition of linguistic variable, we can see that a linguistic term is a word or phrase, utilized to express the value of the variable. Aided by linguistic terms, human beings can better understand and reason about the different features of their environment. For example, considering the linguistic variable *distance*, some possible linguistic terms to express the value of such an attribute could be: "*very close*","*close*","*far*" and "*far away*".

Given the inherent vagueness and imprecision that the values of linguistic terms present, fuzzy sets [35] constitute a useful tool to formalize the concepts associated to them, as shown in Fig. 5, where the semantics of linguistic terms belonging to the variable *distance* are represented as fuzzy sets with trapezoidal membership functions [39]. Thus, by using fuzzy set theory computers are capable of understanding and carrying out computational and reasoning processes over such concepts. Let $\tau \in T(H)$ be a linguistic term (e.g. "close") belonging to a vocabulary associated to a linguistic variable H (e.g. *distance*). We can then express τ as a fuzzy subset in the domain $Y \in U$ of H. Given a value $y \in Y$, its membership degree to τ, $\mu_\tau(y) \in [0, 1]$ indicates the compatibility degree of the value y with the linguistic term τ. These ideas will be used in our proposal to determine how compatible an expert's behavior is with the concept of cooperativeness, which is given by a linguistic term.

3. Uninorm-based management of experts' behavior in CRPs

This section presents a novel expert weighting methodology to deal with non-cooperative behaviors of experts in CRPs carried out in large-scale GDM problems, aimed at overcoming the drawbacks

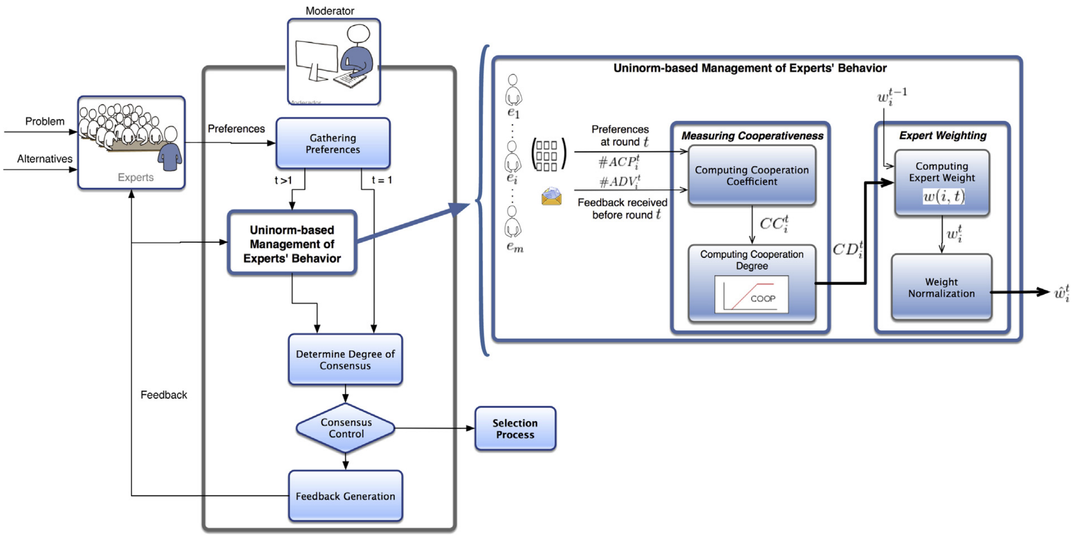

Fig. 6. Scheme of the uninorm-based method to weight experts based on their behavior in CRPs.

such behaviors might cause in these processes. Unlike classical strategic GDM problems with a few number of decision makers (less than ten generally), large-scale GDM problems with a larger number of them (from dozens to several thousands) present more difficulties to detect and deal with experts who try to manipulate the CRP, because such manipulations can be hidden in an easier way. The methodology consists of two main phases, depicted in Fig. 6:

- *Measuring cooperativeness*: A measure to evaluate the degree of cooperation of each expert at current CRP round, based on CW and uninorm operators, is defined (see Section 3.1).
- *Experts weighting*: Experts' importance weights are computed based on their behavior at the current and previous CRP rounds, i.e., taking into account their overall behavior since the beginning of the CRP and how such a behavior evolved at each round (see Section 3.2).

The management of non-cooperative behaviors of experts is applied from the second consensus round onwards, since its computations require information about the feedback received by experts at the end of the previous round.

Eventually, in Section 4 this methodology will be integrated in a consensus model as it is shown in Fig. 6

3.1. Measuring cooperativeness

In this phase, fuzzy set theory and the methodology of CW are utilized to measure the degree of cooperation that each expert presents at a given consensus round. To do this, the two following steps are conducted: (i) compute a cooperation coefficient for each expert, and (ii) compute a cooperation degree based on each cooperation coefficient value.

3.1.1. Computing cooperation coefficient

The objective in this step is to evaluate the behavior adopted by each expert, $e_i \in E$, in the current consensus round, $t \in \mathbb{N}$, by defining a coefficient that indicates how cooperative his/her behavior is. Thus, we introduce the so-called *cooperation coefficient*, which evaluates an expert's behavior based on the amount of feedback received and the amount of assessments he/she modified according to such feedback. This coefficient is defined as follows:

Definition 5. Let $\#ADV_i^t$ be the total number of preference degrees or assessments p_i^{lk} that e_i has been advised to modify and $\#ACP_i^t$ be the amount of assessments that he/she accepted to modify according to the feedback received at round t. The cooperation coefficient of e_i at round t, $CC_i^t \in [0,1]$, is defined as:

$$CC_i^t = \begin{cases} 1 & \text{if } \#ADV_i^t = 0, \\ \eta \dfrac{\#ACP_i^t}{\#ADV_i^t} + (1-\eta)\left(1 - \dfrac{\#ADV_i^t - \#ACP_i^t}{n(n-1)}\right) & \text{otherwise.} \end{cases} \quad (4)$$

Remark 1. $n(n-1)$ is the total number of assessments in P_i. The higher the value of CC_i^t, the more cooperative e_i's behavior is at round t. Notice that the lower $\#ACP_i^t$, the more penalizing is applied on CC_i^t. On the other hand, for values of $\#ADV_i^t$ closer to zero, the resulting CC_i^t will be less penalized, even if $\#ACP_i^t$ is low, since a low $\#ADV_i^t$ means that most of e_i's assessments are close to consensus. Parameter $\eta \in [0,1]$ is utilized to control the penalizing on CC_i^t attending to different criteria, e.g. the amount of advices accepted by the experts and the total number of advices received. If e_i does not receive any advice at a given round, then $CC_i^t = 1$ since in such a case all of his/her assessments values are close to consensus.

As will be later shown in the consensus model presented in Section 4, a parameter $\varepsilon \geq 0$ called acceptability threshold is utilized to identify which assessments should be modified by an expert. This parameter is also used here to decide whether the degree of change applied by an expert on an identified assessment, is enough or not to consider that the advice has been accepted by him/her. Let p_i^{lkt} and $p_i^{lk(t+1)}$ denote e_i's assessment on (x_l, x_k), before and after revising the changes suggested in the advice generation phase at round t, respectively. On the other hand, let ADV_i^t be the set of assessments that e_i has been advised to modify in such a round, so that $|ADV_i^t| = \#ADV_i^t$. Based on ε, the procedure illustrated in Algorithm 1 is applied for each expert, to determine the number of advices accepted by him/her, $\#ACP_i^t$. Notice here that an advice is considered as accepted if the degree of change applied by the expert exceeds ε.

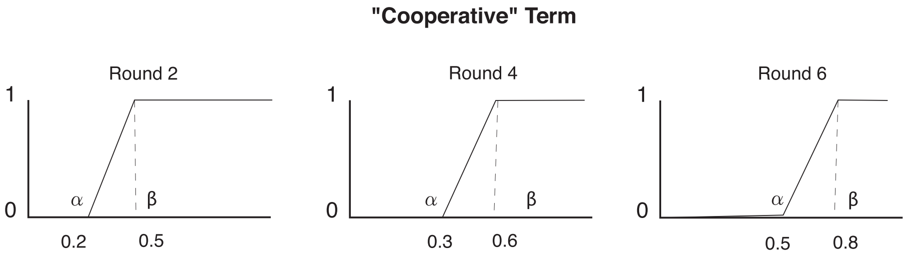

Fig. 7. Evolution of the fuzzy membership function associated to the linguistic term "cooperative" across the CRP.

Algorithm 1. Procedure to compute the number of changes accepted by an expert $e_i \in E$

Assign $\#ACP_i^t \leftarrow 0$.
for each e_i's assessment $p_i^{lkt} \in ADV_i^t$ **do**
  if (e_i modified p_i^{lkt} in the direction advised) *AND* ($|p_i^{lkt} - p_i^{lk(t+1)}| > \varepsilon$) **then**
    $\#ACP_i^t \leftarrow \#ACP_i^t + 1$.
  end if
end for

The following example illustrates the computation of the cooperation coefficient.

Example 3. Consider a CRP round t in a GDM problem with four alternatives ($n = 4$). Two experts e_1 and e_2 received feedback to modify one and ten out of their assessments respectively, i.e. $\#ADV_1^t = 1$ and $\#ADV_2^t = 10$. If e_1 refuses to modify the only assessment she was advised to modify, i.e. $\#ACP_1^t = 0$, and e_2 modifies three out of his assessments bringing them closer to consensus, i.e. $\#ACP_2^t = 3$, then for $\eta = 0.5$ we have:

$$CC_1^t = 0.5\left(\frac{0}{1}\right) + 0.5\left(1 - \frac{1-0}{12}\right) = 0.458$$

$$CC_2^t = 0.5\left(\frac{3}{10}\right) + 0.5\left(1 - \frac{10-3}{12}\right) = 0.358$$

As we can see above, the cooperation coefficient regards the fact that, even though e_1 did not modify none of her assessments, she received feedback for one assessment only, which means that most of her preferences were already close to consensus. Therefore, her cooperation coefficient value is slightly higher than that of e_2, who received feedback on almost all of his assessments and did not apply most of them. Keep in mind that the higher η, the more penalized is e_2.

3.1.2. Computing cooperation degree

Based on the cooperation coefficient CC_i^t previously computed, in this phase the degree of cooperation at the current consensus round is determined for each expert. Such a degree of cooperation aims at reflecting the extent to which the value of CC_i^t satisfies the notion of cooperativeness established in the specific GDM problem. To do this, we consider that the concept of cooperativeness can be easily modeled linguistically, by defining it as a linguistic variable. Thus, we propose applying reasoning processes based on the CW methodology (see Section 2.3) to measure the cooperation degrees. The concept of cooperativeness is modeled as follows:

Definition 6. Let "cooperative" be a linguistic term, whose semantics are given by a fuzzy set *COOP* in [0,1], with the following non-decreasing membership function:

$$\mu_{COOP}(y) = \begin{cases} 0 & \text{if } y < \alpha, \\ \dfrac{y-\alpha}{\beta-\alpha} & \text{if } \alpha \le y < \beta, \\ 1 & \text{if } y \ge \beta. \end{cases} \tag{5}$$

with $\alpha, \beta, y \in [0, 1]$, $\alpha < \beta$. The cooperation degree of e_i at round t, denoted by, CD_i^t corresponds to the membership degree of CC_i^t to the fuzzy subset *COOP*, i.e. $CD_i^t = \mu_{COOP}(CC_i^t) \in [0, 1]$.

The fuzzy membership function establishes how restrictive the notion of being cooperative is: the larger α and β, the more restrictive such a notion is, so that only the highest values of CC_i^t are assigned a maximum cooperation degree. In many real-life problems, this notion of cooperativeness may vary across the time, e.g. in a CRP an expert who only cooperates partially should be more penalized if such a process is at an advanced stage, after several discussion rounds; i.e., a given value of CC_i^t may imply distinct degrees of cooperation in two different GDM problems, or even in different phases of a CRP. In order to reflect this, we propose the flexible use of different membership functions for the semantics of the term "cooperative" at each consensus round, by increasing the value of α and β gradually as the CRP goes on, thus reducing the support [35] of the fuzzy subset *COOP* progressively. This changeable meaning of cooperativeness justifies the importance of conducting this step in our proposal, instead of computing experts' weights from CC_i^t directly.

The following example illustrates how to reflect an increasingly restrictive notion of cooperativeness in a CRP.

Example 4. Consider a fuzzy set *COOP*, whose membership function parameters have the initial values $\alpha = 0.2$ and $\beta = 0.5$ at the beginning of the CRP. After the fourth consensus round, $t \ge 4$, the value of both parameters will be increased by 0.1 per round, until α reaches a value of 0.9 and β reaches a value of 1. Thus, the approach is more restrictive with the behavior of experts as the CRP progresses, as shown in Fig. 7.

3.2. Expert weighting

Once the cooperation degrees of experts have been obtained, in this phase we firstly apply uninorm aggregation operators (Section 3.2.1) to assign each expert an importance weight that reflects both his/her current behavior and the behavior previously adopted since the beginning of the CRP. A normalization process is then applied on weights (Section 3.2.2) to allow recovery of importance in experts' preferences along the CRP.

Table 1
Cooperation degrees of two experts at each round.

t	1	2	3	4	5	6
CD_1^t	–	0.6	0.9	0.4	0.7	0.7
CD_2^t	–	0.2	0.1	0.8	0.9	0.75

Table 2
Weights assigned to experts at each round.

t	1	2	3	4	5	6
w_1^t	0.5	0.6	0.92	0.66	0.79	0.87
w_2^t	0.5	0.2	0.04	0.42	0.66	0.83

3.2.1. Computing experts' weights

Uninorm aggregation operators are utilized to compute importance weights of experts due to their full reinforcement property, which allows to reflect that: (i) if both the current and previous expert behaviors are highly cooperative, then his/her importance weight should be reinforced upwards, and (ii) if both the current and previous expert behaviors are highly non-cooperative, then his/her importance weight should be reinforced downwards. The following function is defined to compute importance weights:

Definition 7. The function $w(i, t)$ returns the importance weight of expert e_i at round t, denoted by $w_i^t \in [0, 1]$ and computed as follows:

$$w_i^t = w(i,t) = \begin{cases} g & \text{if}\, t = 1, \\ U(CD_i^t, w_i^{t-1}) & \text{if}\, t > 1. \end{cases} \tag{6}$$

being U an uninorm operator and $g \in]0, 1[$ its neutral element, according to which an input value above g is viewed as a good behavior in the aggregation, and vice versa.

Given any $t > 1$ and $i \in \{1, \ldots, m\}$, the function $w(i, t)$ shows the full reinforcement property, as a direct consequence of U being a uninorm operator.

In Eq. (6), at the beginning of the CRP ($t = 1$), all experts are assigned the same weight, $w_i^1 = g, \forall i$, since there is no available information about their behavior yet. On the other hand, when $t = 2$, experts have already revised their assessments for the first time, based on the feedback received, and $w_i^2 = U(CD_i^2, g) = CD_i^2$. Finally, when $t > 2$, both the current and previous behaviors of each expert are taken into account when computing his/her weight. Due to the full reinforcement property of uninorms, $w(i, t)$ increases or decreases when either a good or a bad behavior is present at successive consensus rounds, respectively. The following example illustrates this property.

Example 5. Consider two experts, $e_1, e_2 \in E$, with the cooperation degrees shown in Table 1, along a CRP consisting of six rounds, and the following operator $U(a, b)$ based on Fodor's general families of uninorms, with $g = 0.5$ [30]:

$$U(a,b) = \begin{cases} 2ab & \text{if}\, 0 \le a, b \le 0.5, \\ 2(a + b - ab - 0.5) & \text{if}\, 0.5 \le a, b \le 1, \\ \dfrac{a+b}{2} & \text{if}\, \min(a,b) < 0.5 < \max(a,b). \end{cases} \tag{7}$$

Table 2 summarizes the importance weights assigned to e_1 and e_2 at each round computed by applying Eq. (6), the values obtained for e_1 are further detailed below:

- $w_1^1 = w(1.1) = g = 0.5$.
- $w_1^2 = w(1, 2) = U(CD_1^2, w_1^1) = U(0.6, 0.5) = 0.6$.
- $w_1^3 = w(1, 3) = U(0.9, 0.6) = 0.92$.
- $w_1^4 = U(0.4, 0.92) = 0.66$.
- $w_1^5 = U(0.7, 0.66) = 0.79$.
- $w_1^6 = U(0.7, 0.79) = 0.87$.

Notice that an upward reinforcement is applied on e_1's weight at $t = 3$, $t = 5$ and $t = 6$, since her current cooperation degree and her previous behavior (given by w_1^{t-1}) are high. Fig. 8 represents and compares such weights with cooperation degrees, CD_i^t, whose values reflect experts' behavior at the current CRP round only, thus disregarding experts' behavior in previous rounds. Both upward and downward reinforcement can be observed at different rounds for e_2's weight, whose behavior varies across the CRP.

The previous example shows that the use of uninorm operators allows to consider information about experts' behavior at previous discussion rounds when computing their importance weight, w_i^t, at a given round t. Thus, Fig. 8(a) shows that when an expert has a good behavior trend, her weight keeps being reinforced upwards, even though her behavior at a particular round is slightly worse than the behavior adopted in the previous round. On the other hand, if an expert has bad behavior trend, his weight suffers a downward reinforcement, but if his behavior improves significantly in the following rounds, he might finally be reinforced upwards (see Fig. 8(b)).

3.2.2. Weight normalization

In order to all experts can recover importance in their opinions along the CRP, in this phase a normalization is applied to weights w_i^t, as follows:

$$\hat{w}_i^t = \frac{w_i^t}{\sum_{i=1}^m w_i^t} \tag{8}$$

with $\hat{w}_i^t \in [0, 1]$ and $\sum_i \hat{w}_i^t = 1$. Once the weights have been normalized, they are taken into account in the current round of discussion to compute the collective preference P_c by aggregating experts' preferences, as well as in the computation of consensus degrees, as described in the consensus model presented below.

4. Consensus model

Keeping in mind our focus on large-scale GDM problems due to its difficulty to manage the strategic manipulations that might carry out the decision makers involved. In this section, we present a consensus model for large-scale GDM under uncertainty, aimed at the management of non-cooperative behaviors in CRPs based on uninorm operators. Therefore, in order to facilitate the treatment of such behaviors, the proposed consensus model incorporates the uninorm-based weighting scheme introduced in Section 3, as previously depicted in Fig. 6.

Remark 2. The consensus model described in this section focuses on the use of fuzzy preference relations by all experts. Nevertheless, the weight-based methodology presented in this paper to deal with experts' behaviors can be utilized in a variety of consensus models with feedback mechanism [6], regardless of the preference structures and information domains considered [40,41].

The weighting scheme computes the weights assigned to experts' preferences following the phases of the consensus model (see Fig. 9):

- The computation of the collective preference by aggregating the individual preferences of experts.
- The computation of consensus degrees by aggregating similarity degrees between all different pairs of experts in the group.

Further detail on the use of weights is given below, in the description of the consensus model, whose phases are based on

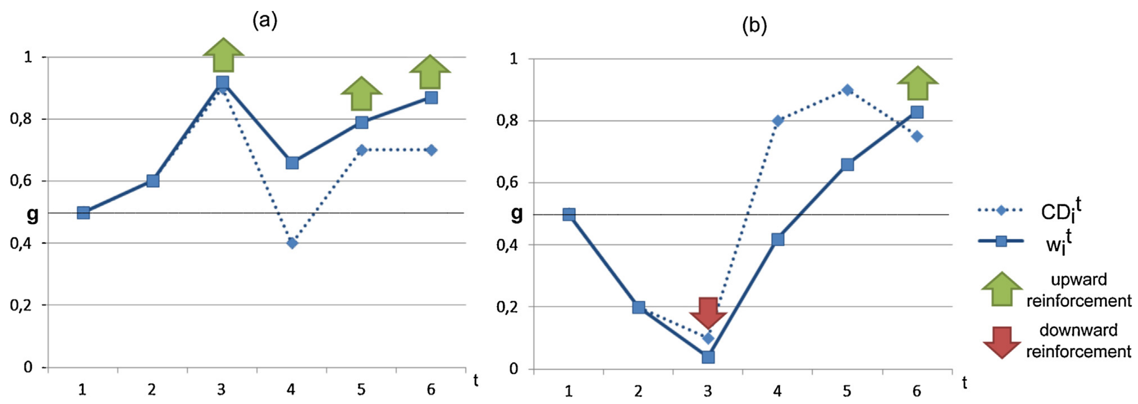

Fig. 8. Cooperation degrees and weights assigned to (a) e_1 and (b) e_2.

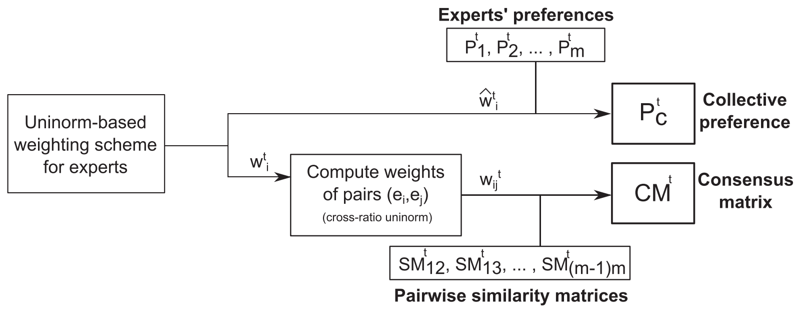

Fig. 9. Use of experts' weights in the consensus model.

the scheme previously shown in Fig. 6. These phases are applied sequentially, once at each CRP round t, until consensus is achieved.

(1) *Gathering preferences*: Each expert provides his/her preferences over alternatives in X, by means of a preference structure (e.g. a reciprocal fuzzy preference relation, $P_i^t = (p_i^{lkt})_{n\times n}$, such that if $p_i^{lkt} = x \in [0,1]$, then $p_i^{klt} = 1 - x$). Notice that preferences are hereinafter denoted by P_i^t, to indicate the round t in which they are utilized.

(2) *Computing consensus degree*: The level of agreement in the group is computed, by means of the following steps:

(a) For each pair of experts e_i, $e_j (i<j)$, compute a similarity matrix $SM_{ij}^t = (sm_{ij}^{lkt})_{n\times n}$,

$$SM_{ij}^t = \begin{pmatrix} - & \ldots & sm_{ij}^{1nt} \\ \vdots & \ddots & \vdots \\ sm_{ij}^{n1t} & \ldots & - \end{pmatrix}$$

with $sm_{ij}^{lkt} \in [0,1]$ being the degree of similarity between e_i and e_j's assessments on the pair of alternatives (x_l, x_k) at round t, computed as follows: [42]:

$$sm_{ij}^{lkt} = 1 - \left| p_i^{lkt} - p_j^{lkt} \right| \tag{9}$$

(b) Compute a consensus matrix $CM^t = (cm^{lkt})_{n\times n}$ by using a weighted averaging aggregation operator. Each element $cm^{lkt} \in [0,1]$, $l \neq k$, is computed as follows:

$$cm^{lkt} = \frac{\sum_{i=1}^{m-1}\sum_{j=i+1}^{m} w_{ij}^t sm_{ij}^{lkt}}{\sum_{i=1}^{m-1}\sum_{j=i+1}^{m} w_{ij}^t} \tag{10}$$

Table 3
Computation of importance weights for pairs of experts.

w_{ij}^t	$w_2^t = 0.75$	$w_3^t = 0.5$	$w_4^t = 0.25$	$w_5^t = 0.6$
$w_1^t = 0.2$	0.43	0.2	0.08	0.27
$w_2^t = 0.75$	–	0.75	0.5	0.82
$w_3^t = 0.5$	–	–	0.25	0.6
$w_4^t = 0.25$	–	–	–	0.33

$w_{ij}^t \in [0,1]$ represents the importance weight associated to a pair of experts (e_i, e_j), and it is obtained from individual weights w_i^t and w_j^t as $w_{ij}^t = U_r(w_i^t, w_j^t)$, being U_r the cross ratio uninorm operator shown in Eq. (3).

Remark 3. The aim of applying weights in Eq. (10) is to assign more importance to similarity values associated to pairs of experts whose degree of cooperation is higher. Thus, we aim at preventing a possible lack of convergence towards agreement due to non cooperating experts who present a strong disagreement with each other.

Example 6. Let $E = \{e_1, e_2, e_3, e_4, e_5\}$ be five experts with the following importance weights at round t: $w_1^t = 0.2$, $w_2^t = 0.75$, $w_3^t = 0.5$, $w_4^t = 0.25$ and $w_5^t = 0.6$. By applying the cross-ratio operator, the weights w_{ij}^t assigned to all different pairs of experts in E are shown in Table 3. From the table it can be observed that, given its neutral element $g = 0.5$, the cross-ratio operator also shows the full reinforcement property, which in our case will be used to reinforce positively the agreement values of those pairs of experts who highly cooperate to achieve consensus, and vice versa.

(a) Once obtained CM^t, consensus degrees are computed at three levels [43]:
  i. *Level of pairs of alternatives* (cp^{lkt}): Obtained from CM^t as $cp^{lkt} = cm^{lkt}$, $l, k \in \{1, \ldots, n\}$, $l \neq k$.
  ii. *Level of alternatives* (ca^{lt}): Degrees of consensus on each alternative $x_l \in X$ are computed as:

$$ca^{lt} = \frac{\sum_{k=1, k \neq l}^{n} cp^{lkt}}{n-1} \tag{11}$$

  iii. *Level of preference relation* (cr^t):

$$cr^t = \frac{\sum_{l=1}^{n} ca^{lt}}{n} \tag{12}$$

(3) *Consensus control*: The consensus degree cr^t previously computed is checked to decide whether it is enough or not. If consensus is enough, the group moves on to the selection process. Otherwise, it is necessary to carry out another round of discussion. Two parameters, whose values are fixed a priori by the group, can be utilized in this phase:
- A consensus threshold $\mu \in]0, 1]$, whose value indicates the minimum level of agreement required amongst members in the group.
- A maximum number of discussion rounds allowed, $Maxround \in \mathbb{N}$. If the number of rounds carried out exceeds this value, then the CRP ends without having reached consensus, in which case a different decision strategy should be adopted by the group (see Section 2.1).

(4) *Advice generation*: If $cr^t < \mu$, the group moves onto this phase, in which farthest experts' assessments from consensus are identified, and a set of change recommendations on such assessments are provided to experts, with the aim of increasing consensus in the following rounds. The following steps are carried out in this phase:
  (i) A collective preference P_c^t is obtained, by aggregating experts' assessments on each pair of alternatives:

$$p_c^{lkt} = \sum_{i=1}^{m} \hat{w}_i^t p_i^{lkt} \tag{13}$$

  where $\hat{w}_i^t \in [0, 1]$ is the normalized importance weight assigned to the expert e_i according to his/her behavior (see Section 3.2.2).

  Remark 4. Experts' weights are utilized in Eq. (13) to obtain a collective preference P_c^t, which better reflects the opinions of experts who contribute positively to achieve consensus, thus making a better group decision (in accordance with the main goal of our proposal).

  (ii) A proximity matrix $PP_i^t = (pp_i^{lkt})_{n \times n}$ between each expert's preference relation and P_c^t, defined by

$$PP_i^t = \begin{pmatrix} - & \cdots & pp_i^{1nt} \\ \vdots & \ddots & \vdots \\ pp_i^{n1t} & \cdots & - \end{pmatrix}$$

  is computed for each expert. Proximity values pp_i^{lkt} are used to identify the farthest preferences from the collective opinion, and they are obtained as follows:

$$pp_i^{lkt} = 1 - \left| p_i^{lkt} - p_c^{lkt} \right| \tag{14}$$

  (iii) Pairs of alternatives (x_l, x_k) whose consensus degrees ca^{lt} and cp^{lkt} are not enough, are then identified:

$$CC^t = \{(x_l, x_k) | ca^{lt} < cr^t \wedge cp^{lkt} < cr^t\} \tag{15}$$

  Afterwards, experts who should change their opinion on each of these pairs are identified, taking into account their proximity degrees to P_c^t. To do this, an average proximity $\bar{pp}^{lkt}$ is calculated:

$$\bar{pp}^{lkt} = \frac{\sum_{i=1}^{m} pp_i^{lkt}}{m} \tag{16}$$

  As a result, experts e_i whose $pp_i^{lkt} < \bar{pp}^{lkt}$ will be advised to modify their assessment on the pair $(x_l, x_k) \in CC$.

  (iv) A set of direction rules are applied to suggest the direction of changes proposed to experts, in order to increase the level of consensus. Such rules are based on the use of an acceptability threshold $\varepsilon \geq 0$ which may take a positive value close to zero and allows a margin of acceptability when p_i^{lkt} and p_c^{lkt} are close enough to each other.
  - DIR.1: If $(p_i^{lkt} - p_c^{lkt}) < -\varepsilon$, then expert e_i should *increase* his/her assessment on the pair of alternatives (x_l, x_k).
  - DIR.2: If $(p_i^{lkt} - p_c^{lkt}) > \varepsilon$, then expert e_i should *decrease* his/her assessment on the pair of alternatives (x_l, x_k).
  - DIR.3: If $-\varepsilon \leq (p_i^{lkt} - p_c^{lkt}) \leq \varepsilon$, then expert e_i does not need to modify his/her assessment on the pair of alternatives (x_l, x_k).

  Notice that, as previously illustrated in Section 3.1, the use of ε in our proposal is twofold: (i) as an acceptability threshold to identify assessments that should be modified, and (ii) in the computation of $\#ACP_i^t$ (see Section 3.1) to control whether the changes applied by the expert are significant enough or not to consider them as accepted.

As we have shown, experts weights are applied in Eqs. (10) and (13) to compute the consensus matrix, CM^t, and collective preference, P_c^t, respectively (see Fig. 9).

5. Illustrative example

In this section, we present an illustrative example based on the resolution of a large-scale GDM problem conducting a CRP. Although the group size in these problems may vary between a few dozens to thousands of experts, this example considers 30 experts for the sake of clarity in the results presented.

The uninorm-based consensus model presented in the previous section, is applied and compared with two other consensus models: (i) a model that utilizes a weighting scheme which only penalizes to manage non-cooperative behaviors [13], not allowing to recover importance in weights despite experts change their behavior, and (ii) a consensus model that does not use any weighting scheme on experts' preferences regarding their behavior.

A GDM problem is firstly formulated, in which experts have different behavioral patterns across the CRP (Section 5.1). A software simulation has been then applied and carried out with a multi-agent system presented in [44] for each of the three consensus models considered, in order to analyze the results obtained by applying the uninorm-based weighting scheme (Section 5.2) and compare them with the results obtained by the other two approaches (Section 5.3).

5.1. GDM problem formulation

Let us suppose a university government panel formed by 30 members, $E = \{e_1, \ldots, e_{30}\}$, who must reach an agreement about choosing a supportive action plan to be launched next year. There are four possible plans to contribute for, $X = \{x_1, x_2, x_3, x_4\}$:

- x_1: Helping typhoon victims.
- x_2: Supporting hospitalized victims.
- x_3: Protecting endangered species.
- x_4: Helping in reforestation tasks.

Table 4
Parameters of membership function for fuzzy set *COOP* at each consensus round.

t	2	3	4	5	6	7	8	9	10
(α, β)	(0.2,0.5)	(0.2,0.5)	(0.3,0.6)	(0.4,0.7)	(0.5,0.8)	(0.6,0.9)	(0.7,1)	(0.8,1)	(0.9,1)

Table 5
Consensus degrees and weight computation for e_{22}–e_{24}.

t Consensus degree		1 0.6714	2 0.7174	3 0.7572	4 0.7973	5 0.8319	6 0.8517
e_{22}	CC_{22}^t		1.0	1.0	1.0	1.0	
	CD_{22}^t		1.0	1.0	1.0	1.0	
	w_{22}^t	0.5	1.0	1.0	1.0	1.0	
	w_{22}^t	0.0333	0.0381	0.0381	0.0432	0.0449	
e_{23}	CC_{23}^t		0.3	0.3	0.3	0.2666	
	CD_{23}^t		0.3333	0.3333	0.0	0.0	
	w_{23}^t	0.5	0.3333	0.2222	0.0	0.0	
	w_{23}^t	0.0333	0.0127	0.0084	0.0	0.0	
e_{24}	CC_{24}^t		0.0	1.0	1.0	1.0	
	CD_{24}^t		0.0	1.0	1.0	1.0	
	w_{24}^t	0.5	0.0	0.5	1.0	1.0	
	w_{24}^t	0.0333	0.0	0.0191	0.0432	0.0449	

Experts elicit their preferences by using reciprocal fuzzy preference relations. The minimum agreement level required is $\mu = 0.85$, and the maximum number of discussion rounds allowed is *Maxround* = 10.

The membership function of the fuzzy set *COOP*, which is used to compute the cooperation degree of experts at each round, takes the following initial parameter values at the beginning of the CRP: $\alpha = 0.2$, $\beta = 0.5$. Then, from the third consensus round onwards, the value of both parameters increases by 0.1 per round, until $\alpha = 0.9$ and $\beta = 1$. Thus, the approach becomes more restrictive with the notion of cooperativeness as the CRP goes on. Table 4 shows the membership function parameters considered at each round.

The uninorm operators utilized in our proposal include:

- The operator based on Fodor general family of uninorm operators shown in Eq. (7) to compute individual weights w_i^t, with $g = 0.5$.
- The cross-ratio operator (see Eq. (3)) to compute pairwise experts' weights w_{ij}^t.

Experts adopted different patterns of behavior, that have been modeled by means of a simulation framework for GDM problems so-called AFRYCA [6]:

- *Cooperative behavior*: 22 out of the 30 panel members, e_1–e_{22} presented a full cooperative behavior throughout the CRP, in the sense that they applied all changes suggested on their assessments accordingly.
- *Mixed behavior*: The remaining eight panel members, e_{23}–e_{30} present a variable behavior across the CRP, i.e. some of them agreed to modify a few of their assessments towards consensus, ignoring some advice received or even modifying their assessments against consensus.

5.2. *Results obtained by applying the uninorm-based approach*

Table 5 summarizes the results of applying our uninorm-based proposal to manage experts' behaviors. Such results include the consensus degree at each round until consensus is achieved, the cooperation coefficients CC_i^t and cooperation degrees CD_i^t, and the weights (before and after normalizing) assigned to a representative subgroup of experts with different behavior, ranging from e_{22} to e_{24}. These three experts have been chosen to be analyzed in further detail in this example, since they adopt different behavior patterns of interests for our proposal throughout the CRP[1]:

- e_{22}: This expert always cooperates, applying all changes on his preferences as indicated in the feedback received.
- e_{23}: This expert presents a highly non-cooperative behavior across the whole CRP.
- e_{24}: This expert does not cooperate at the beginning of the CRP, but she then decides to change her behavior and cooperates in later rounds.

In order to provide a better insight of the CRP state at each round, a visual representation of experts' preferences and the collective preference is shown in Fig. 10. Such a visual representation has been generated with a graphical monitoring tool of preferences so-called MENTOR, presented in [45]. The collective preference is shown by a blue cross and labeled as 'P'. Preferences of e_{22}, e_{23} and e_{24} are also labeled to ease their visualization, and they are depicted by green, red and yellow crosses, respectively. The rest of experts' preferences are depicted by gray points.

The results observed in Table 5 and Fig. 10 are briefly analyzed below:

- e_{22} is always assigned the maximum importance weight for $t \geq 2$, since his behavior is completely cooperative across the CRP. We can observe in Fig. 10 that, despite e_{22}'s preferences are far from consensus at the beginning of the CRP, they quickly become closer to P_c, because his opinions are taken into account to a high degree.
- e_{23} shows a highly non-cooperative behavior over the course of the CRP, therefore his preferences end up quite far from the group preference when consensus is reached (see Fig. 10). Although his cooperation coefficient values are similar at all rounds, we can observe that his weight tends to decrease due to the downward reinforcement property of uninorm operators (see e.g.

[1] A large amount of information is utilized in this example, therefore it has not been included in the paper for space reasons. A supplementary material file including such information and the results of applying the proposed methodology, can be found in the datasets link at: http://sinbad2.ujaen.es/afryca

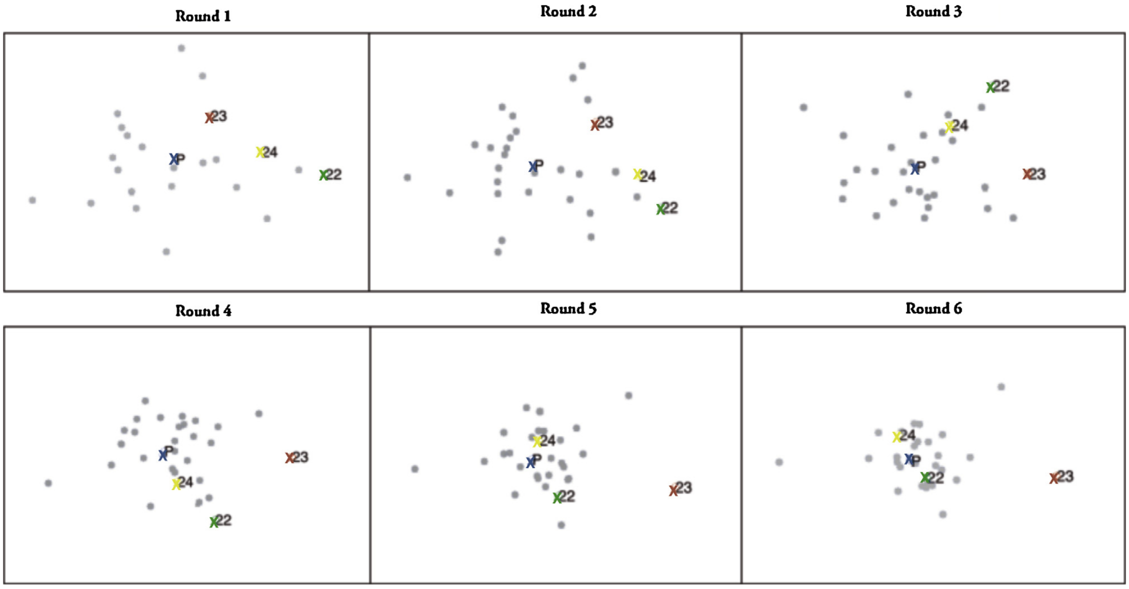

Fig. 10. Visual monitoring of preferences over the course of the CRP. (For interpretation of the references to color in the text, the reader is referred to the web version of this article.)

Table 6
Comparison of non-normalized weights computed with each proposal.

CRP round t	1	2	3	4	5	6	7
Consensus degrees							
No penalizing	0.6714	0.7145	0.7531	0.7831	0.8087	0.8301	0.8515
No importance recovery	0.6714	0.7150	0.7539	0.7859	0.8161	0.8375	**0.8525**
Importance recovery (uninorm-based)	0.6714	0.7174	0.7572	0.7973	0.8319	**0.8517**	

	Weight penalizing scheme	w_i^1	w_i^2	w_i^3	w_i^4	w_i^5	w_i^6
	No penalizing	1.0	1.0	1.0	1.0	1.0	1.0
e_{22}	No importance recovery	1.0	0.1696	0.1000	0.0958	0.0958	0.095
	Importance recovery (uninorm-based)	0.5	1.0	1.0	1.0	1.0	
	No penalizing	1.0	1.0	1.0	1.0	1.0	1.0
e_{23}	No importance recovery	1.0	0.3635	0.1520	0.0033	0.0	0.0
	Importance recovery (uninorm-based)	0.5	0.3333	0.2222	0.0	0.0	
	No penalizing	1.0	1.0	1.0	1.0	1.0	1.0
e_{24}	No importance recovery	1.0	0.0247	0.0111	0.0111	0.0111	0.0111
	Importance recovery (uninorm-based)	0.5	0.0	0.5	1.0	1.0	

$w_{23}^3 = 0.2222$). Furthermore, from the fourth round onwards, the cooperation degree becomes null, due to the change in the fuzzy membership function of *COOP*, therefore the resulting importance weights become null as well.

- e_{24} presents a completely non-cooperative behavior at the second round, therefore her opinions are not taken into account at all in such a round and they initially move further from the group opinion. However, when the expert realizes this, she decides to cooperate obeying all the advice received from the third CRP round onwards. For this reason, her importance weights are gradually increased until they reach the maximum value, and her opinions are finally brought closer to consensus.

These examples allow us to notice that the uninorm-based approach is capable of assigning experts a weight taking into account their overall behavior at previous CRP rounds, not only at the current one.

5.3. *Consensus reaching progress comparison with other proposals*

Once analyzed the behavior managing obtained in the proposal presented in this paper, we have considered adequate to compare the consensus reaching progress in the GDM problem when different consensus models are applied. To do so, we compare the CRP with the behavior managing based on uninorms presented in this paper, and the CRP penalizing model introduced in [13] that penalizes non-cooperative behaviors, but does not allow to improve experts importance, even though they eventually decide to cooperate. The results are also compared with those obtained by applying a CRP that does not manage experts' behavior (as presented in Fig. 2), i.e. all experts have equal importance $w_i^t = 1$ across the CRP. Experts' weights and the consensus degrees obtained with each of the three approaches are summarized in Table 6.

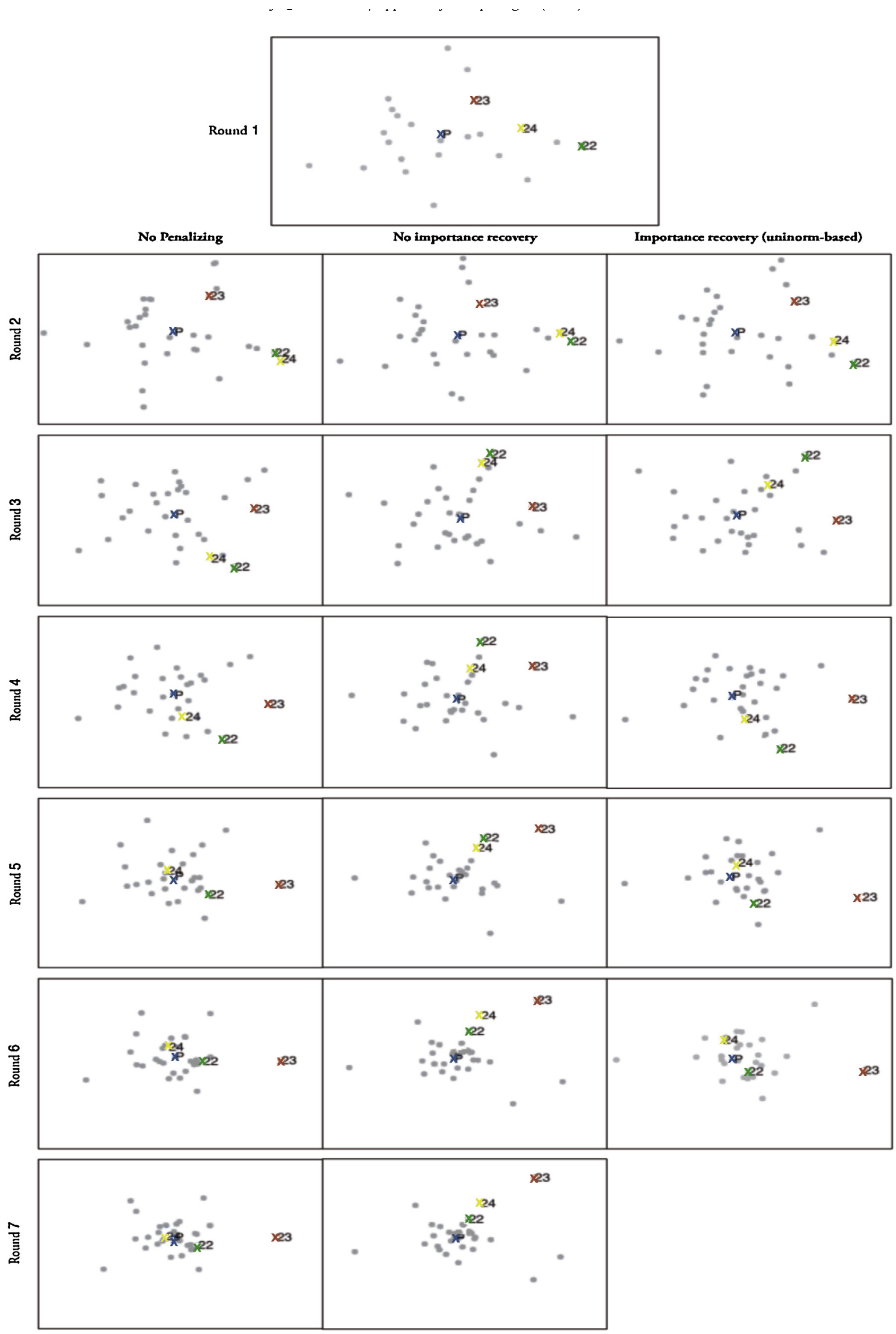

Fig. 11. Visual monitoring of preferences for the three consensus models compared. (For interpretation of the references to color in the text, the reader is referred to the web version of this article.)

Remark 5. In [13], weights are assigned based on the distance between experts' preferences, P_i^t, and the collective preference P_c^t. Moreover, if an expert is penalized due not to being close to P_c^t, then his/her importance weight cannot be recovered at later rounds, regardless of how close his opinions are later brought to P_c^t.

Remark 6. Since this problem involves 30 experts, and the normalized weight values of each expert might be low and difficult to analyze, for the sake of clarity and ease in comparisons Table 6 displays the absolute (non-normalized) weights $w_i^t \in [0, 1]$ obtained for each approach. Nevertheless, weights are internally normalized in the scheme, as shown in Section 3.2.2.

Fig. 11 shows the visual representation of preferences at each round, for the three consensus models compared. The following results can be observed regarding experts e_{22}, e_{23} and e_{24}, for each consensus model:

- Expert e_{22}
  - *No penalizing*: His preferences are equally important as the rest of the group preferences, therefore they are taken into account to obtain the group opinion.
  - *No importance recovery*: Since his initial preferences are far from the group opinion, in this model e_{22} is penalized at the beginning of the CRP. His highly cooperative behavior allows him to approach P_c^t at the end of the CRP, although his opinions are not as close to consensus as the opinions of other cooperating experts whose initial opinions where closer to P_c^t.
  - *Importance recovery*: In this case, preferences are brought much closer to consensus due to the full degree of cooperation presented, which is faithfully reflected in the high importance weights assigned and the closeness to P_c^t.
- Expert e_{23}.
  - *No penalizing*: His preferences are also taken into account to obtain the group opinion, despite his non-cooperative behavior along the CRP. In this model, e_{23}'s final position is slightly closer to P_c^t compared to the other two models, because e_{23} deviates the group solution in his favor.
  - *No importance recovery*: Due to his non-cooperative behavior, his preferences gradually become further from consensus, hence when consensus is achieved his opinions are far from the group opinion.
  - *Importance recovery*: Similarly to the model without importance recovery, his preferences become far from consensus as the CRP progresses.
- Expert e_{24}.
  - *No penalizing*: Her preferences are as important as the rest of the group preferences throughout the CRP, even though she does not cooperate at the first rounds.
  - *No importance recovery*: Since her importance is decreased at the first rounds, this expert's preferences remain far from the group due not to being able to improve her importance despite turning cooperative from the third round onwards.
  - *Importance recovery*: In this case, the uninorm-based weighting scheme allows to reflect e_{24} change of behavior at the third round. As a result, despite her preference becoming further from the rest of the group at the first rounds, they are eventually brought closer to consensus at the last rounds.

Moreover, in the weighting approach without importance recovery, we can see some other unlabeled experts who presented a mixed behavior and could not recover importance in their weight by the end of the CRP: these experts' preferences remain far from P_c^t at the last round.

To sum up, we can conclude that the proposal based on uninorms presented in this paper offers multiple advantages with respect to the other two:

- The weight assigned to an expert is computed based on his/her behavior at the current round of the CRP, as well as the evolution of such a behavior across previous rounds.
- The full reinforcement property of uninorm operators allows to reinforce the weight of experts if their behavior is highly cooperative (or uncooperative) at successive rounds.
- The cooperation coefficient provides a realistic measure of experts' behavior, regarding not only the amount of feedback they accepted to modify, but also the amount of feedback they received, which indicates how close to consensus their preferences are.
- Finally, we can observe that weighting pairs of experts to aggregate similarity values (see consensus model, Section 4), allows to maintain a reasonable convergence towards consensus, which is achieved at the sixth round of discussion with our proposed model, compared to the seven rounds required with the other two models.

6. Concluding remarks and future directions

Large-scale group decision making problems are becoming increasingly common in the last years. When a consensus reaching process must be conducted in these contexts to reach a collective agreement, the presence of individuals or subgroups of them who present a non-cooperative behavior and try to manipulate such a process in their favor is particularly frequent. In this paper, we have presented an approach based on uninorm aggregation operators, fuzzy sets and the methodology of computing with words, to detect and deal with experts' non-cooperative behavior. Such an approach applies a uninorm-based weighting scheme (inspired by previous works on dynamic multi-criteria decision making), to assign experts different importance weights according to their overall behavior across the CRP and the way such a behavior evolves. After integrating the weighting approach in a consensus model for group decision making under uncertainty, an example has been presented to illustrate its advantages with respect to other penalizing methodologies applied to consensus approaches.

Further research in the area is mainly oriented towards the study of new measures of cooperativeness, that facilitate detecting multiple types of behavior associated to the different ways of acting by experts, in order to deal with each type of behavior properly. We also aim at studying the practical application of the proposed methodology in collective intelligence systems, such as e-marketplaces, social networks and group recommender systems.

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4.2. Using Computing with Words for Managing Non-cooperative Behaviors in Large Scale Group Decision Making

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Using Computing with Words for Managing Non-cooperative Behaviors in Large Scale Group Decision Making

Francisco José Quesada, Iván Palomares, and Luis Martínez

Abstract. Normally, in group decision making problems, groups are composed by individuals or experts with different goals and points of view. For these reasons, they may adopt distinct behaviors in order to achieve their own aims. Nonetheless, in such problems in general, specially those demanding a certain degree of consensus, each expert should comply with a collaboration contract in order to find a common solution for the decision problem. When decision groups are small, all experts usually attempt to fulfill the collaboration contract. However, nowadays technologies such as social media allow to make consensus-driven decisions with larger groups, in which many experts are involved, hence the possibility that some of them try to break the collaboration contract might be greater. In order to prevent the group solution from being biased by these experts, it is necessary to detect and manage their non-cooperative behaviors in this kind of problems. Recent proposals in the literature suggest managing non-cooperative behavior by reducing the importance of expert opinions. These proposals present drawbacks such as, the inability of an expert to recover his/her importance if behavior improves; and the lack of expert's behavior measures across the time. This chapter introduces a methodology based on fuzzy sets and computing with words, with the aim of identifying and managing those experts whose behavior does not contribute to reach an agreement in consensus reaching processes. Such a methodology is characterized by allowing the importance recovery of experts and taking into account the evolution of their behavior across the time.

Keywords: Group Decision Making, Computing with Words, Fuzzy sets, Consensus Reaching Processes.

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1 Introduction

In our daily life we can find a myriad of Group Decision Making (GDM) problems, ranging from the choice of a restaurant to have a dinner with friends to the definition of a marketing strategy for a big company. In all of these situations, joining together experiences and knowledge of a group, makes it easier to face complex decision problems and may lead to better decisions. GDM problems are defined as decision situations in which a group of individuals or experts, try to find a common solution to a particular problem made up of a set of alternatives [1, 2]. To do so, experts have to express their opinions over the distinct alternatives that might be a solution to such a problem.

Many real-life GDM problems are often defined under an environment of uncertainty, so that experts should provide the information about their preferences by using an information domain closer to human natural language, which is suitable to deal with such uncertainty [3, 4]. Within Granular Computing [5], there are different approaches to deal with uncertain information, such as fuzzy set theory [6] and the fuzzy linguistic approach [7, 8, 9], which have been some of the most utilized approaches in decision problems under uncertainty [10]. In particular, the Computing with Words (CW) paradigm [4] has been widely considered as a reasoning methodology in decision making problems [11, 12, 13].

Traditionally, the procedure to solve a GDM problem only consists in an alternative selection process, which after gathering experts' opinions and processing them, aims at finding a solution [14, 15, 16]. However, sometimes it is possible that when applying the selection process solely, one or more experts may not feel identified with the decision made and they do not accept it, because they consider that their individual concerns have not been considered sufficiently to reach the solution made. In order to overcome this drawback, Consensus Reaching Processes (CRPs) were introduced as an additional phase in the resolution process for GDM problems [17]. In a CRP, experts try to achieve a high level of agreement before making a decision, by discussing and modifiying their individual preferences, bringing them closer to each other [18]. In the literature, there are many consensus models proposed by different authors to support and guide groups in CRPs conducted in different GDM frameworks [19, 20, 21, 22, 23, 24] attending several criteria [25].

Classically, GDM problems have been carried out by a small number of experts in organizational and enterprise environments. Nevertheless, the appearance of new technological environments and paradigms to make group decisions, such as social networks, e-democracy or group e-marketplaces, have caused that decision problems in which large groups of experts can take part attain greater importance in the last years [24, 26, 27, 28]. In CRPs in which many experts are involved, it may occur that some experts or subgroups of them, seek their own interests rather than the collective interest, which may lead them to break the collaborative contract established amongst participants, in order to achieve a common solution [29]. Therefore, they might not cooperate to bring their opinions closer to the rest of the group [30]. In such situations, it would be convenient to identify and deal with such non-cooperative behaviors of individuals or subgroups, in order to prevent that they deviate the group solution in their favor. This possible deviation of the solution may affect negatively the normal development of the CRP. Currently, there are several approaches that deal with experts who present a non-cooperative behavior in GDM problems [30] and in CRPs [24, 31]. These approaches penalize non-cooperating experts, driving them out of the GDM solution [30], diminishing the importance of their opinions either along the CRP [24] or based on the experts' behavior at the current phase of the CRP [31].

This chapter proposes a novel fuzzy approach based on CW [4] to detect and manage non-cooperative behaviors in CRPs of large scale GDM problems. The CW paradigm facilitates the definition, comprehension and detection of experts' behaviors such as *cooperative*. Additionally to the analysis of experts' current behaviors to manage the manipulation of the CRP performed by experts, this approach also applies a weighting scheme based on hyper-similarity [32] that takes into account the experts behavior across the time. Therefore, the proposed approach provides a mechanism in which cooperative experts outweigh non-cooperative ones in order to achieve a common agreed solution for a GDM problem.

The chapter is set out as follows: Section 2 introduces some preliminaries about CRPs in GDM problems, some related works which deal with non-cooperative behaviors in these problems and the CW paradigm for reasoning processes. Section 3 presents the approach based on CW for managing experts' behaviors in CRPs with large groups and its integration with a consensus model for GDM problems. In Section 4 it is shown an illustrative example which includes a comparison between our proposal and several previous approaches to attempt penalization in CRPs. Finally, in Section 5 some concluding remarks are pointed out.

2 Preliminaries

This section briefly reviews some basic concepts about CRPs in GDM and different works related with the treatment of experts with non-cooperative behaviors in them. Finally, a short conceptual revision of CW, basis of our proposal for managing the non-cooperative behaviors in such processes, is drawn.

2.1 Consensus Reaching Processes in Large Scale GDM

GDM entails the participation of several individuals or experts, who must make a collective decision to find a common solution for a problem. Decision making processes in which several experts take part, who each has his/her own knowledge and experiences, may sometimes lead to better decisions than those made by one expert only [1].

Formally, a GDM problem is composed by [2]:

- The existence of a decision problem to be solved.
- A set $X = \{x_1,...,x_n\} (n \geqslant 2)$, of *alternatives* or posible solutions to the problem.

- A set $E = \{e_1,...,e_m\}(m \geqslant 2)$, of individuals or *experts*, who express their opinions or preferences over the set of alternatives X.

Usually, experts utilize a preference structure to express their opinions over alternatives. One of the most widely used preference structures in GDM problems under uncertainty is the *fuzzy preference relation* [2, 23, 33]. A fuzzy preference relation P_i associated to expert e_i is defined by a membership function $\mu_{P_i} : X \times X \to [0,1]$ and it is represented for X finite as an $n \times n$ matrix:

$$P_i = \begin{pmatrix} - & \dots & p_i^{1n} \\ \vdots & \ddots & \vdots \\ p_i^{n1} & \dots & - \end{pmatrix}$$

where each assessment $p_i^{lk} = \mu_{P_i}(x_l, x_k)$ represents the preference degree of the alternative x_l over x_k according to the expert e_i, so that $p_i^{lk} > 0.5$ indicates that x_l is preferred over x_k. If $p_i^{lk} < 0.5$ then x_k is preferred over x_l, and $p_i^{lk} = 0.5$ indicates indifference between x_l and x_k.

Classically, the process to find a solution for a GDM problem consists of an alternative selection process, which is composed of two phases [15] (Figure 1).

1. *Aggregation*: In this phase experts' preferences are combined by using an aggegation operator [34].
2. *Exploitation*: Here, a selection criterion [14, 16] is applied to obtain an alternative or a subset of alternatives, as the solution for the GDM problem.

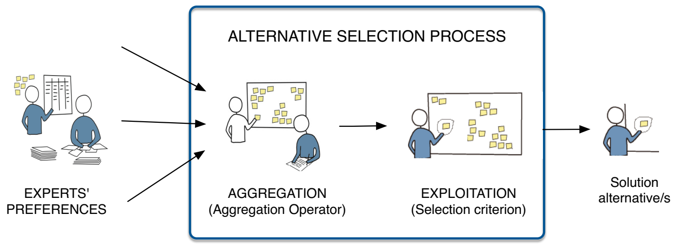

Fig. 1 Selection process in GDM problems

When applying the selection process in a GDM problem solely, it may occur that one or more experts feel that their opinions have not been taken into account sufficiently to reach the solution achieved. This fact could imply that these experts do not feel identified with the solution. There exist some situations in which it is necessary a high agreement level among the participant experts. Therefore, it arises the need of applying a CRP that introduces a new phase in the GDM resolution process with the aim of reaching a high level of agreement between experts before making the decision.

Consensus can be defined as the agreement produced by the mutual consent between all members in a group or several groups [17, 18, 29]. A CRP is a dynamic and iterative process, which is coordinated by a human figure known as *moderator*. The moderator is responsible for supervising and guiding experts over the course of this process [18]. Consensus should be understood as a process in which the final decision may not match the initial position of experts. Thus, experts might change their preferences during several discussion rounds [29]. In the literature, there are many consensus models for a wide variety of GDM frameworks [25]. Figure 2 shows a general CRP scheme followed by many of these models. Its main phases are introduced below [23, 24]:

1. *Gathering Preferences*: Each expert e_i provides his/her preferences on X (e.g. by means of a fuzzy preference relation).
2. *Determine Degree of Consensus*: The moderator calculates the degree of agreement, *cr*, reached in the group. Such a degree can be a value of the interval [0,1] (where the value 1, indicates a full or unanimous agreement between all experts over all alternatives)[1]. Different consensus measures [25] can be utilized in order to calculate the *cr*. Such measures are often based on the use of: (i) metrics to calculate degrees of similarity between preferences of experts, and (ii) aggregation operators that obtain the degree of consensus in the group by aggregating similarity values [23, 24].
3. *Consensus Control*: The consensus degree *cr* is compared with a minimum consensus threshold, $\mu \in [0,1]$. The value of μ is previously established by the group. If $cr > \mu$, then consensus has been reached and after that the group can proceed to the selection phase; otherwise, the CRP must go on with another discussion phase.
4. *Generate Feedback Information*: The moderator calculates the group collective preference, P_c, by aggregating the individual preferences of the experts. On the basis of P_c, he identifies those experts e_i whose assessments p_i^{lk} are farthest to consensus, and advises them how to modify their assessments to increase the consensus degree in the following round (by indicating each expert whether he/she should increase or decrease the value of each assessment). Each recommendation is a triplet with three elements $(e_i,(x_l,x_k),Direction)$, which shows that the expert e_i, should modify the assessment p_i^{lk}, in the direction given by the argument $Direction \in \{Increase, Decrease\}$.

Normally, the consensus process implies the need that experts *accept* to review and modify, in some degree, their opinions on the basis of the recommendations received, with the aim of bringing their opinions closer to the rest of the group. Based on this, it can be assumed that they should accept a priori a collaboration

[1] Consensus degrees are normally based on aggregation of similarity values between experts' assessment. Such values belong to the unit interval, hence the resulting agreement values are computed in [0,1].

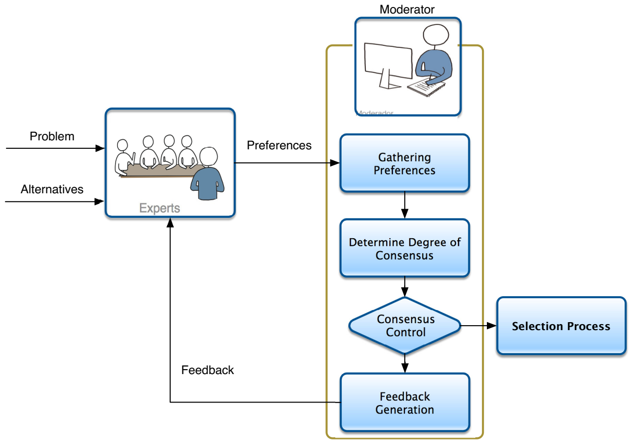

Fig. 2 General CRP scheme

contract [29]. Nevertheless, in large groups, in which normally there are many experts with different aims, it might occur that some experts do not cooperate modifying their assessments as they should do according to the received recommendations, thus breaking the collaboration contract because their own interests outweigh group ones [24]. This chapter considers an expert as cooperative when the group interests outweigh his/her own interests, therefore he/she is willing to change his/her initial opinion. Conversely, an expert is considered as non-cooperative when his/her own goals outweigh group interests, and is not willing to change his/her opinion. This chapter aims at identifying and dealing with the latter type of behaviors. Notwithstanding, there are different proposals, revised in the following subsection, which have already attempted to manage this kind of behaviors. They present important limitations which serve as motivation for our proposal presented in section 3.

2.2 Related Works

In order to solve the shortcoming of experts that break the collaboration contract, trying to strategically deviate the solution for classical GDM problems, Yager proposed in [30] an approach to penalize them. This proposal firstly identifies experts with more drastic opinions (e.g. experts who show a full preference on one alternative and a null preference on the rest of them) as experts with a strategically manipulative pattern of behaviors. After the non-cooperative experts are identified, this approach completely discards the information associated to the preferences of these experts, who are directly excluded from the GDM problem resolution process. This approach might be considered completely drastic, in the sense that it completely eliminates the information associated to the experts whose preferences have been identified as strategically manipulated opinions.

Attempting to solve this issue, Mata et al. [35] extended the previous approach to deal with strategically manipulated preferences in CRPs, assigning experts a weight based not only on drastic opinions, but also on identifying those experts who did not obey the advice received.

More recently, Palomares et al. presented in [24] a methodology, where all expert's opinions have an importance weight $w_i \in [0,1]$. If an expert's opinion is far from consensus and he/she does not cooperate to bring his/her opinion closer to the group opinion at a given round, his/her weight is decreased. Otherwise, his/her weight keeps invariant. These weights are used to calculate the consensus opinion, given by P_c, in the CRP. In this case, all experts keep taking part during the CRP to some degree, although the opinions of those experts who cooperate, are taken into account to a higher degree than the opinions of experts that do not cooperate. However, if an expert's opinion is penalized at a CRP round, by assigning him/her of a low weight, such a weight cannot be increased at later consensus rounds, even though the expert changes his/her behavior and decides to cooperate again in subsequent discussion rounds [24].

Considering that sometimes, non cooperating in a particular consensus round might be a negotiation strategy, it is necessary that opinions of experts with non-cooperative behavior at some specific rounds only can recover their importance if their behavior is significantly improved afterwards. Palomares et al. proposed in [31], a CW-based methodology to assign experts importance weights depending on their behavior at a given consensus round. Thus, if an expert did not cooperate at a previous consensus round, but he/she cooperates in the current one, his/her opinion may recover the importance previously lost to calculate the group opinion. However, the experts' weights are computed based on their behavior at a given consensus round only, not taking into account the evolution of their behavior since the beginning of the CRP.

2.3 Computing with Words for Reasoning Processes

Human beings use linguistic terms to communicate, explain and understand their surroundings. On the other hand, machines such as computer systems require more complex symbols [36]. One of the most extended proposals to establish an understandable communication gateway between humans and computers, is the CW paradigm [4], which was proposed by Zadeh and it is based on fuzzy sets theory [6]. This methodology gives a framework where the concepts can be modelled by means of fuzzy sets, so that they can be easily understood by both machines and humans.

A key concept in computing with words is the linguistic term. We can define a linguistic term as a word or a phrase in natural language which is used to express the value of an attribute [7, 8, 9]. For instance, let us consider an attribute called *distance*, comprehended as the size of the gap between two places[2]. Some possible linguistic terms to express the value of this attribute could be: "very close", "close",

[2] Definition of "Distance" in `http://wordnetweb.princeton.edu/`

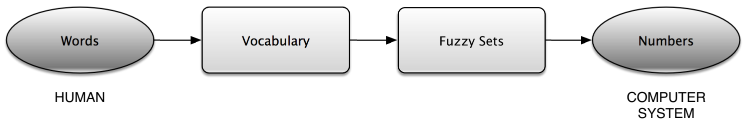

Fig. 3 Paradigm of man-machine understanding. (Taken from [36])

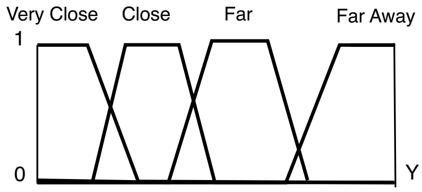

Fig. 4 Different linguistic terms for the attribute *distance*

"far", "far away". Thus, humans can easily understand and reason over their environment with the help of linguistic terms (see Figure 3).

Given the ambiguous and vague nature inherent to the values associated to the linguistic terms, fuzzy sets are a useful tool to formalize the concepts associated to them, thus allowing the comprehension and development of computational processes over these concepts by computers (see Figure 4). If **P** is a linguistic term (i.e. "close") of a vocabulary associated with the attribute **A** (i.e. *distance*), then **P** can be expressed as a fuzzy subset in the domain $Y \subseteq \mathbb{R}$ of **A**. Given a value $y \in Y$, $\mu_P(y)$ shows the compatibility degree of the value y with the linguistic term **P**.

The choice of a linguistic term vocabulary to describe the attributes, as well as definition of the associate meaning of these terms is a human task. For instance, humans should facilitate to the computer the linguistic terms that will be used, as well as, theirs semantics given by fuzzy sets.

3 Managing Non-cooperative Behaviors in Large Scale GDM by Using CW and Hyper-Similarity

In order to prevent the bias and manipulation of CRPs carried out for the resolution of large-scale GDM problems, in which experts' own goals and interests are harder to detect, it is necessary to identify and manage the non-cooperative behaviors they might adopt.

This section presents a novel methodology to deal with the shortcomings caused by different patterns of non-cooperative expert behaviors in CRPs carried out for the resolution of large-scale GDM problems. Such a methodology extends the one presented in [31]. It also deals with some of the drawbacks that arose in the approaches presented in [24, 30, 31] (see Section 2.2). More specifically, our approach is characterized by the following features:

- Unlike Yager's approach, in which penalized experts may completely lose their importance, here we take into account experts' preferences across the CRP to a variable degree, depending on their behavior.
- Opinions of non-cooperative experts in an specific round can recover importance if they adopt a more cooperative behavior in the following rounds.
- The overall expert's behavior since the beginning of the discussion process is taken into account.

The methodology is composed by two phases, which will be developed in the following subsections in further detail: (i) a *Cooperativeness Measurement* phase, and (ii) a *Behavior Management* phase. Straightaway, this methodology will be integrated in the general CRP scheme (see Figure 5).

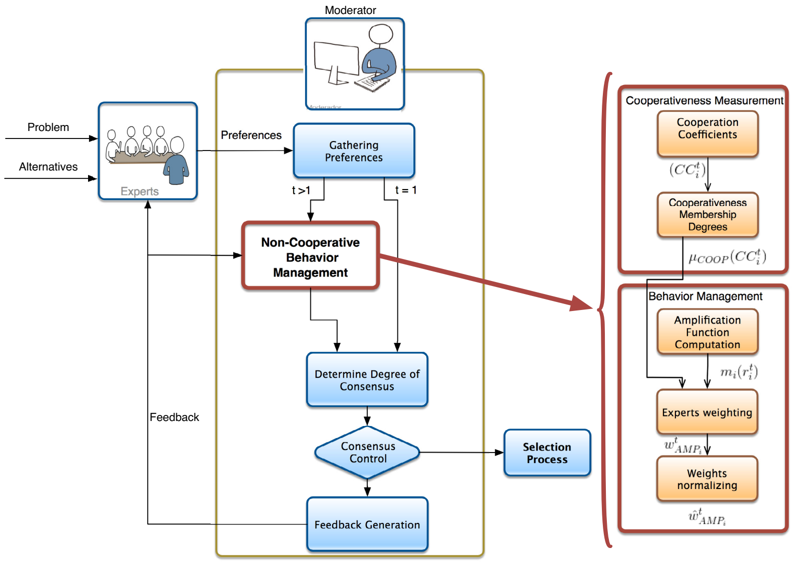

Fig. 5 Integration of the non-cooperative behavior management methodology in the general CRP scheme

3.1 Cooperativeness Measurement Phase

In order to manage experts who present a non-cooperative behavior in large groups, it is firstly necessary to identify them. In this subsection, we define a coefficient that indicates the type of behavior adopted by each expert at a given discussion round. This coefficient is based on the degree of commitment of the collaboration contract amongst experts [29], given by the extent to which they applied changes on preferences based on the feedback received.

Definition 1. Let $\#ADV_i^t$ be the number of advices provided to e_i, advising him/her to modify some of his/her assessments p_i^{lk} at the beginning of the CRP round $t \geq 2$, and let $\#ACP_i^t$ be the number of advice that e_i accepts to modify in accordance with the feedback received. The *Cooperation Coefficient* $CC_i^t \in [0,1]$ of e_i at round t is then defined as follows:

$$CC_i^t = \begin{cases} 1 & \text{if } \sharp ADV_i^t = 0, \\ \frac{\sharp ACP_i^t}{\sharp ADV_i^t} & \text{otherwise.} \end{cases} \quad (1)$$

The value of CC_i^t, represents the degree to which an expert modifies his/her opinions bringing them closer to consensus, as suggested by the advice he/she received. The larger the value of CC_i^t, the more cooperative e_i's behavior regarding this issue. Notice that if an expert does not receive any advice at a given round, this means that all of his/her assessment values are close enough to consensus, therefore we consider that $CC_i^t = 1$ in this case.

The relevance of cooperation varies across the CRP. For example, at the beginning of the CRP, it is usual that experts' opinions might be more distant from each other. Therefore, the level of required cooperation is different than that at the final rounds of the CRP. After several rounds, expert opinions are closer and, consequently, it is necessary to reach consensus before carrying out an excessive number of discussion-rounds. In order to properly model the meaning and the relevance of cooperativeness over the course of the CRP, we use the CW paradigm as follows.

Definition 2. Let *COOP* be a fuzzy set, associated to the linguistic term *cooperative*. This fuzzy set models the meaning of cooperativeness by means of a semi-trapezoidal increasing membership function $\mu_{COOP}(y)$:

$$\mu_{COOP}(y) = \begin{cases} 0 & \text{if } y < \alpha, \\ \frac{y-\alpha}{\beta-\alpha} & \text{if } \alpha \leq y < \beta, \\ 1 & \text{if } y \geq \beta. \end{cases} \quad (2)$$

being $\alpha, \beta, y \in [0,1], \alpha < \beta$. We then define the *Cooperativeness Membership Degree*, $\mu_{COOP}(CC_i^t)$, which indicates the degree of membership of the cooperation coefficient (CC_i^t) to the linguistic term *cooperative*.

The membership function of *COOP* may change at each consensus round to become more restrictive with the concept of cooperativeness as the CRP goes on. The following example illustrates this aspect in detail.

Example 1. Consider a fuzzy set *COOP*, whose membership function parameters have the initial values α=0.2 and β=0.5 at the beginning of the CRP. After the fourth consensus round, the value of both parameters will be increased by 0.1 per round, until each one of them reaches a value of 1. Thus, the approach is more restrictive with the behavior of experts as the CRP progresses. Figure 6 illustrates this process.

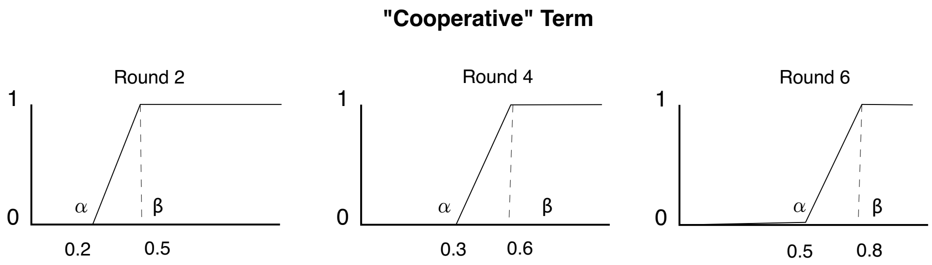

Fig. 6 Evolution of the fuzzy membership function associated to the linguistic term "cooperative" across the CRP (from the 4^{th} round onwards, values of parameters α, β increase by 0.1 per round)

3.2 *Behavior Management Phase*

Once it has been applied the cooperativeness measurement phase, the values of $\mu_{COOP}(CC_i^t)$, for each $e_i \in E$, provide an insight of the experts who have non-cooperative behavior at round t. Therefore, we are now in a position to take the necessary actions on these experts in order to prevent the CRP manipulation by them.

Here, it is proposed a flexible approach to manage experts' opinions, so that their importance weights increase or decrease attending to their behavior across the CRP. This approach extends the work presented in [31], therefore it is necessary to firstly introduce the weight computation methodology utilized in such a work.

Proposition 1. *[31] Once it is computed $\mu_{COOP}(CC_i^t)$, the weight of e_i at round t is calculated as follows:*

$$w_i^t = \begin{cases} 1 & \text{if } t = 1, \\ \mu_{COOP}(CC_i^t) & \text{otherwise.} \end{cases} \tag{3}$$

Such weights are normalized to facilitate computations:

$$\hat{w}_i^t = \frac{w_i^t}{\sum_1^n w_i^t} \tag{4}$$

An example that illustrates this approach to compute each expert's weight is shown below.

Example 2. Let us Suppose a set of three experts, $E = \{e_1, e_2, e_3\}$, who are taking part in a CRP and each expert may adopt one out of these three different behaviors:

- *Full cooperation.* Experts always cooperate along the CRP. This is e_1's behavior.
- *Half cooperation.* In this case, experts with this behavior, cooperate obeying only a half of their feedback advice received. e_2 has this type of behavior.

- *Alternation of null and full cooperation*. Experts with this behavior, alternate null and full cooperation, disobeying all their feedback advice in a specific round and obeying all of them in the following round. e_3 follows this behavior pattern.

At the beginning, all experts' opinions have the same importance because no feedback has been yet generated, therefore it is assigned to them the maximum weight at the first round, $w_i^t = 1$ (see Eq.(3)). Once they receive the feedback advice and modify their assessments, their weights are recalculated according to their behavior by computing the cooperation coefficient, CC_i^t and applying Eq. (3) to calculate $\mu_{COOP}(CC_i^t)$, as explained in the detection phase (Section 3.1). Let us assume that the non-normalized and normalized weights, w_i^t and $\hat{w}_i^t$, of each expert along the CRP, are the ones shown in Table 1.

Table 1 Example of expert opinion weights along the CRP

Experts / Rounds	e_1^t		e_2^t		e_3^t	
t	w_1^t	$\hat{w}_1^t$	w_2^t	$\hat{w}_2^t$	w_3^t	$\hat{w}_3^t$
1	1	0.33	1	0.33	1	0.33
2	1	0.66	0.5	0.33	0	0
3	1	0.4	0.5	0.2	1	0.4
4	1	0.66	0.5	0.33	0	0
5	1	0.4	0.5	0.2	1	0.4

e_1, presents the most cooperative behavior during all the CRP, and $w_1^t = 1$, $\forall t$, hence he/she is assigned the highest $\hat{w}_i^t$ at each round of the CRP. e_2, has always $w_2^t = 0.5$, therefore his/her opinion weight is smaller than that of e_1. The behavior of e_3, is more variable from one round to another, because he/she alternates full and null cooperation along the rounds of the CRP. Here we can see, how at $t = 2$, e_3 does not cooperate at all, therefore $w_3^t = 0$ and $\hat{w}_3^t = 0$. On the other hand, in the third round, $t = 3$, e_3 decides to cooperate bringing all his/her assessments closer to the rest of the group. Thus, $w_3^t = 1$ and $\hat{w}_3^t = 0.4$. At this point, e_3 has completely recovered the importance of his/her opinion. This pattern of varying behavior is repeated again in the fourth and fifth rounds, as we can observe in Table 1.

This proposal allows that experts can recover the importance of their opinions if they improve their behavior between two consecutive CRP rounds. Nevertheless, it may be possible that after several rounds, some experts attempt to use the weight restriction to bias the GDM problem solution. For example, by comparing the weight values of experts e_1 and e_3 in Example 2, we can see that e_1 has full cooperation across the whole CRP, while e_3 alternates null and full cooperation. This kind of behavior entails that the weight of e_3, in those rounds in which he/she cooperates, has the same influence to calculate the consensus opinion as the weight of e_1, who cooperates constantly throughout all the CRP. At certain rounds of the CRP, e_3 might disobey the advice applying contrary changes as he/she has received, moving his position further from the consensus opinion. At the following round, e_3 obeys the advices and changes all the corresponding assessments accordingly. Acting this way, allows e_3 that after several rounds, his/her opinion is brought closer to his/her initial position again and the consensus opinion moves closer to e_3's position, due to the high importance weight given by his/her cooperative behavior at the previous round, like e_1, even though he/she does not fully cooperate during all the CRP like e_1 does. The effect of e_3's varying behavior is illustrated in Figure 7. This illustration has been made using a graphical monitoring tool to represent a group of experts' preferences [37].

For this reason, it seems necessary to take into account not only each expert's behavior at each discussion round, but also how the overall behavior of an expert evolves along the CRP since its beginning. To do so, we will consider the ideas

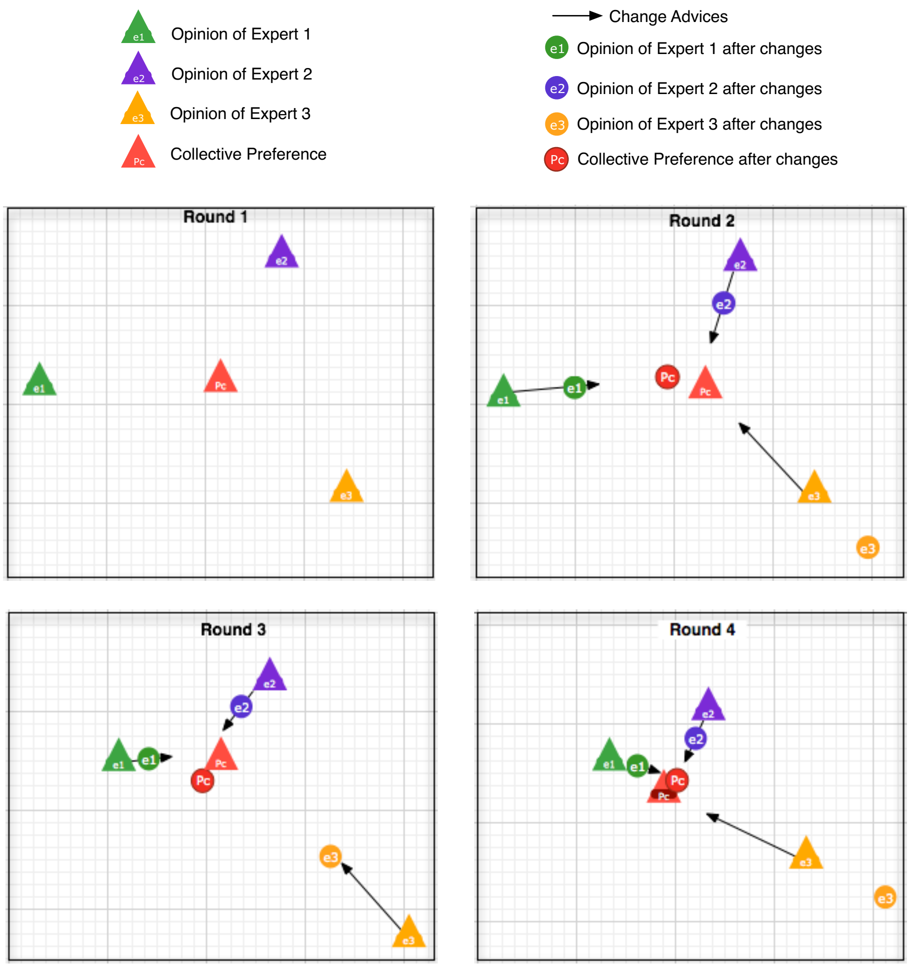

Fig. 7 Example of behavior that attempts to manipulate the CRP, adopted by e_3

expounded by Yager and Petry in [32], where they use hyper-similarity matching to facilitate intuitive decision making.

They suggest that when alternatives are assessed according to certain attributes, those attributes for which an alternative takes an extreme value, should be given a higher importance in the decision making process. For example, for an extremely heavy person, its weight attribute will play an important role in characterizing the weight than in the case of a person with an average weight (i.e. the number of people and the average weight that can be in an elevator). Thus, the focus of their proposal is the effect of these extreme values in amplifiying the effect of the attribute in the characterization of the decision situation.

Definition 3. [32] Let $m_i(S1)$ indicate the amplification effect associated with an attribute A_i for situation S1, denoted as follows:

$$m_i(S1) = f(Dev(A_i(S1))) \tag{5}$$

Yager and Petry use $Dev(A_i(S1))$ to indicate the deviation of $A_i(S1)$ from normal. The function f:$[0,\infty)$ has the following properties

1. f(0)=1
2. f(a) $\geq$ f(b) if a $>$ b; f is monotonic

In our case, inspired by Yager's penalizing proposal to apply an amplification effect, we aim at reinforcing an expert's weight based on the closeness between his/her preferences and the group opinion. Such an amplification effect will be also taken into account to calculate the expert's opinion weight. Thus, in situations when several experts' opinions have the same importance weight at the current round t (e.g. e_1 and e_3 at round $t = 5$, see Example 2), the opinions of an expert which are closer to the group opinion P_c should receive a higher importance, hence the amplification effect should be higher. On the other hand, if the expert's opinion is far from the collective opinion, he/she should have a lower amplification effect, i.e. the resulting importance of his/her opinion should be smaller.

If the expert's opinion position throughout the consensus rounds is close to the group opinion, it means that either this expert is having a cooperative behavior during the CRP or his/her opinions are close enough to consensus, therefore most of his/her assessments do not need to be modified. Otherwise, if an expert's opinion moves further from the group opinion at some stage, it means that this expert is having a non-cooperative behavior.

Definition 4. Let r_i^t be the rate of e_i's assessments p_i^{lk} which are close to consensus at round t. $r_i^t \in [0,1]$ is defined as follows:

$$r_i^t = 1 - \frac{\sharp ADV_i^t}{n(n-1)} \tag{6}$$

Such a rate must be assigned a greater value if e_i has received a lower amount of advice at round t, and vice versa, thus giving a rough insight on how close an expert assessments are to consensus at each round. On the one hand, when $\#ADV_i^t$

is smaller, the opinion of e_i might be close to P_c. On the other hand, if the value of $\#ADV_i^t$ is closer to the total number of assessments provided by e_i, $n(n-1)$, then it means that the opinions of e_i are rather far from consensus, hence most of his/her assessments need to be modified. To sum up, the value of r_i^t gives us an insight on the degree of proximity between an expert's opinion and the collective opinion, P_c.

Definition 5. Once r_i^t is computed, and based on Yager's ideas stated above, an amplification function $m(r_i^t)$ is defined as:

$$m(r_i^t) = r_i^t + 1, m(r_i^t) \in [1,2] \tag{7}$$

After the $m(r_i^t)$ calculation, we compute the weights of experts' opinions as:

$$w_{AMP_i}^t = \frac{m(r_i^t)\mu_{COOP}(CC_i^t)}{2}, \tag{8}$$

here it is necessary divide by 2 to bound $w_{AMP_i}^t$ to the unit interval, being $w_{AMP_i}^t \in [0,1]$.

Finally, we re-normalize experts' weights to allow the recovery of opinion importance (see Proposition 1):

$$\hat{w}_{AMP_i}^t = \frac{w_{AMP_i}^t}{\sum_1^n w_{AMP_i}^t} \tag{9}$$

Let us illustrate this weight computation method in the example shown below.

Table 2 Example of expert opinion weights along the CRP after applying amplification function

	e_1^t				e_2^t				e_3^t			
t	μ_{COOP}	$m(r_i^t)$	$w_{AMP_1}^t$	$\hat{w}_{AMP_1}^t$	μ_{COOP}	$m(r_i^t)$	$w_{AMP_2}^t$	$\hat{w}_{AMP_2}^t$	μ_{COOP}	$m(r_i^t)$	$w_{AMP_3}^t$	$\hat{w}_{AMP_3}^t$
1	1	**1**	1	0.33	1	1	1	0.33	1	**1**	1	0.33
2	1	**1.5**	0.75	0.67	0.5	1.5	0.375	0.33	0	**1.5**	0	0
3	1	**1.75**	0.875	0.46	0.5	1.70	0.425	0.22	1	**1.2**	0.6	0.32
4	1	**1.85**	0.925	0.68	0.5	1.80	0.45	0.32	0	**1.4**	0	0
5	1	**1.95**	0.975	0.49	0.5	1.85	0.4625	0.23	1	**1.1**	0.55	0.28

Example 3. Suppose the same situation as Example 2, but in this case, using our weight management scheme based on Yager's amplification function. Thus, we have three experts who have different behaviors (e_1 cooperates always, e_2 cooperates all the consensus rounds obeying only half of the advices received, and e_3 alternates full and null cooperation along the CRP). Table 2 shows the weights assigned to experts, being in this case computed from $\mu_{COOP}(CC_i^t)$[3] and $m(r_i^t)$ values, thus giving the same importance to the degree of cooperation and the expert opinion position with respect to the consensus opinion.

[3] Being $\mu_{COOP}(CC_i^t)$ expressed in the table as μ_{COOP} for the sake of space.

As we can see in Table 2, e_1 has always the highest weight because he/she co-operates over the course of the CRP, so just as the CRP progresses, his/her opinion will be closer to the group's opinion and therefore his/her amplification value m_i^t will be higher. Regarding e_3's behavior, we can appreciate how its weight is smaller as his/her opinion moves further from the collective opinion. For instance, comparing rounds 3 and 5, we can see how its weight decreases in favour of experts that are closer to the group's opinion, hence its strongly variable behavior is now penalized more accordingly. We can see graphically in Figure 8 how the importance of expert opinions evolves across the CRP rounds.

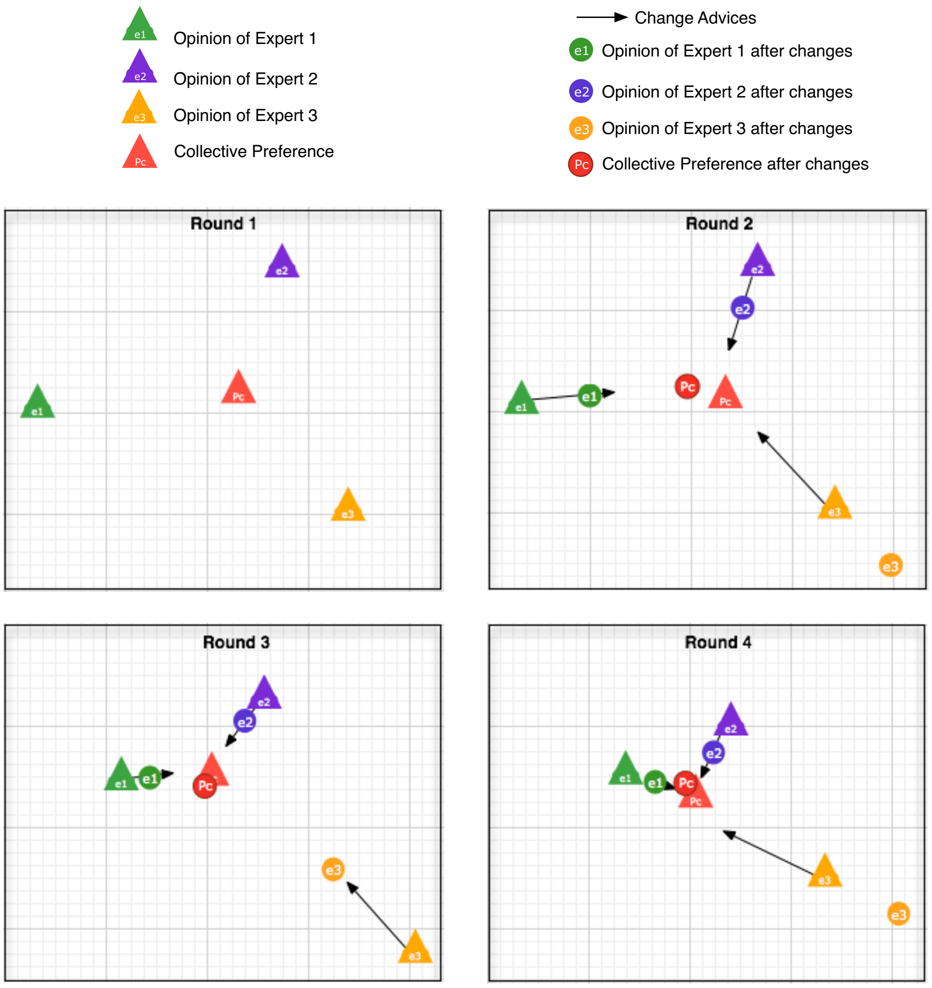

Fig. 8 Example of management non-cooperative behaviors using this proposal

3.3 Integration of the Methodology in the General CRP Scheme

Once it has been defined the cooperativeness measurement and behavior management phases, it is possible to describe in detail the complete scheme of this proposal (see Figure 5).

In the evaluation phase it is computed the cooperation coefficient CC_i^t and the cooperation membership degree $\mu_{COOP}(CC_i^t)$. Once all experts' cooperation coefficients have been computed, we can know the experts' behaviors in the current CRP round, therefore we can start the management phase.

In the management phase, a weight is assigned to each expert depending on his/her behavior. On the one hand it is computed an amplification value which will strengthen or attenuate the expert's importance opinion according to the positions of his/her preferences with respect to the position of the group's opinion. After computing amplification values and taking into account the cooperativeness degrees which have been computed in the detection phase, we can calculate expert's weights $w^t_{AMP_i}$. After that, it is necessary to normalize these weights to allow weight recovery along the consensus rounds. Thus, we obtain the normalized weights of all experts, $\hat{w}^t_{AMP_i}$, which will be used to calculate the collective preference.

In order to integrate the two phases in the general CRP scheme (Figure 2), it is necessary to have, at each round t, the updated expert assessments and change recommendations sent in the feedback at the end of the previous round, $t-1$. For these reasons, this module is firstly applied at the second round of the CRP, because we need the change recommendations of round $t-1$, in this case round 1. Thus, at the first round all the opinions of experts have the same importance because no experts have been assigned any behavior yet. At subsequent consensus rounds, the weights computed will be used to calculate the collective preference, P_c (see Figure 5).

4 Illustrative Example

In this section it is presented an illustrative example which shows the application of the approach presented in this contribution, in order to clarify how it affects in a real CRP for the resolution of a large-scale GDM problem. First of all, the GDM problem and the necessary parameters are introduced and described in detail.

An enterprise committee formed by 30 experts of the different company branches, $E = \{e_1, ..., e_{30}\}$ must reach an agreement about the investment of 100000$ for the company improvement. There are four possible proposals, $X = \{x_1, x_2, x_3, x_4\}$:

- x_1: TV marketing campaign.
- x_2: Research and development.
- x_3: Replace old production machines.
- x_4: Develop a new corporative software.

Experts express their assessments, p_i^{lk}, over pairs of alternatives, by using fuzzy preference relations. The minimum level of agreement required is μ =0.85. Regarding the membership function of the fuzzy set *COOP* to define the meaning of cooperativeness across the CRP, its initial parameter values and the way such values evolve to become more restrictive, we consider the settings previously shown in Example 1 and Figure 6.

Experts present different patterns of behavior, which have been modelled by using a recently developed simulation framework so-called AFRYCA [25]:

- *Cooperative*(experts $e_1 - e_{21}$): These experts always apply all changes suggested on their asssessments, as indicated in the feedback they receive along the whole CRP.
- *Manipulative*(expert e_{22}): this expert alternates null and full cooperation across the CRP in order to manipulate the CRP solution.
- *Non-Cooperative*(experts e_{23}): this expert disobeys always, i.e. he/she does not change his/her assessments at all as it is indicated in the feedback.
- *Undefined*(experts $e_{24} - e_{30}$): the behavior of these experts can change across the CRP, applying or ignoring changes suggested to a variable degree.

This example has been executed on a multi-agent based consensus support system [38] and the results of applying it according to the different approaches will be reviewed in the following subsections. First of all, the results obtained by applying the example on the proposal described in Section 3 are shown. Afterwards, the example will be run by applying proposals [24] and [31]. In section 4.2, a comparison of the results after executing the three proposals is shown. In order to facilitate the results' interpretation, a graphical representation of preferences has been generated for all the cases by using the monitoring tool MENTOR introduced in [37].

4.1 Results Based on the CW and Hyper-Similarity Proposal

Table 3 shows the consensus degree per round and the necessary information to calculate expert's opinion importance, $\hat{w}^t_{AMP_i}$, across the CRP (see Sections 3.1 and 3.2). Notice that this table shows the results associated to a representative set of experts only, over which we can describe what happens attending to the different behaviors previously introduced.

In order to compute $\hat{w}^t_{AMP_i}$ we take into account both the behavior $\mu_{COOP}(CC^t_i)$, and the proximity to the P_c, given by $m(r^t_i)$. For this reason, although several experts have the same behavior, their opinions might have different importance, if their distance to P_c are different.

Once these aspects have been clarified, we start to analyze Table 3.

Experts e_1, e_7 and e_{16} represent experts with *cooperative* behavior. They always accept all their advices, $\sharp ACP^t_i = \sharp ADV^t_i$, so their $\mu_{COOP}(CC^t_i)$ have always the highest value 1. Nevertheless, in order to have the highest $\hat{w}^t_i$ it is also necessary to be close to P_c. e_7 does not receive any advice, $\sharp ADV^t_7 = 0, \forall t$, therefore it means that e_7's opinion is very close to P_c, thus he/she has the highest amplification value, $m(r^t_7) = 2$. He/she always has the highest value $\hat{w}^t_i$ in the group. On the other hand, e_{16} is far from P_c, therefore he/she has to change some of his/her assessments, e.g. $\sharp ADV^2_{16} = 12$,

Table 3 Values of representative experts across the CRP

	t	1	2	3	4	5	6	7
	Conensus Degree	**0.6714**	**0.7153**	**0.7572**	**0.7766**	**0.8099**	**0.8383**	**0.8534**
	α	0.2	0.2	0.2	0.3	0.4	0.5	
	β	0.5	0.5	0.5	0.6	0.7	0.8	
e_1	$\#ADV_1^t$		6	6	5	4	5	
	$\#ACP_1^t$		6	6	5	4	5	
	$\mu_{COOP}(CC_1^t)$		1	1	1	1	1	
	$m(r_1^t)$		1.5	1.5	1.583	1.666	1.583	
	$w_{AMP_1}^t$		0.75	0.75	0.791	0.833	0.791	
	$\hat{w}_{AMP_1}^t$	0.333	0.397	0.339	0.38	0.424	0.410	
e_7	$\#ADV_7^t$		0	0	0	0	0	
	$\#ACP_7^t$		0	0	0	0	0	
	$\mu_{COOP}(CC_7^t)$		1	1	1	1	1	
	$m(r_7^t)$		2	2	2	2	2	
	$w_{AMP_7}^t$		1	1	1	1	1	
	$\hat{w}_{AMP_7}^t$	0.333	0.529	0.453	0.48	0.509	0.518	
e_{16}	$\#ADV_{16}^t$		12	11	8	7	7	
	$\#ACP_{16}^t$		12	11	8	7	7	
	$\mu_{COOP}(CC_{16}^t)$		1	1	1	1	1	
	$m(r_{16}^t)$		1	1.833	1.333	1.416	1.416	
	$w_{AMP_{16}}^t$		0.5	0.541	0.666	0.708	0.708	
	$\hat{w}_{AMP_{16}}^t$	0.333	0.264	0.245	0.320	0.360	0.367	
e_{22}	$\#ADV_{22}^t$		6	6	6	8	9	
	$\#ACP_{22}^t$		0	6	0	8	0	
	$\mu_{COOP}(CC_{22}^t)$		0	1	0	1	0	
	$m(r_{22}^t)$		1.5	1.5	1.5	1.333	1.25	
	$w_{AMP_{22}}^t$		0	0.75	0	0.666	0	
	$\hat{w}_{AMP_{22}}^t$	0.333	0	0.339	0	0.339	0	
e_{23}	$\#ADV_{23}^t$		6	6	6	6	6	
	$\#ACP_{23}^t$		0	0	0	0	0	
	$\mu_{COOP}(CC_{23}^t)$		0	0	0	0	0	
	$m(r_{23}^t)$		1.5	1.5	1.5	1.5	1.5	
	$w_{AMP_{23}}^t$		0	0	0	0	0	
	$\hat{w}_{AMP_{23}}^t$	0.333	0	0	0	0	0	
e_{24}	$\#ADV_{24}^t$		6	6	6	6	6	
	$\#ACP_{24}^t$		3	4	3	1	3	
	$\mu_{COOP}(CC_{24}^t)$		1	1	0.666	0	0	
	$m(r_{24}^t)$		1.5	1.5	1.5	1.5	1.5	
	$w_{AMP_{24}}^t$		0.75	0.75	0.5	0	0	
	$\hat{w}_{AMP_{24}}^t$	0.333	0.397	0.339	0.24	0	0	

hence although $\mu_{COOP}(CC_{16}^2) = 1$, his/her amplification value is very low, $m(r_{16}^2) = 1$. Nevertheless, given her cooperating behavior pattern across the CRP, $\sharp ADV_{16}^t$ will decrease over time (e.g. $\sharp ADV_{16}^4 = 8, \sharp ADV_{16}^5 = 7$) and his/her opinions will take more importance as the CRP goes on ($w_{16}^2 = 0.5, w_{16}^4 = 0.666, w_{16}^5 = 0.708$).

Manipulative behavior pattern is presented by e_{22}. In this case, the expert alternates null and full cooperation ($\sharp ADV_{22}^1 = 6, \sharp ACP_{22}^2 = 0. \sharp ADV_{22}^3 = 6, \sharp ACP_{22}^4 = 6.$) in order to deviate the collective opinion in his favor, P_c, towards opinion position. Nonetheless, with this kind of behavior, at rounds in which he does not cooperate, his opinion is still far to the P_c and his/her amplification value decreases over time ($m(r_{22}^5) = 1.333, m(r_{22}^6) = 1.25$).

The *non-cooperative* is another type of behavior that can be adopted, in this case by (e_{23}). This expert does not cooperate across the CRP (i.e. $\sharp ADV_{23}^2 = 6, \sharp ACP_{23}^2 = 0$). For this reason, he/she always has the minimum possible value ($\hat{w}_{23}^2 = \hat{w}_{23}^3 = \hat{w}_{23}^4 = \hat{w}_{23}^5 = \hat{w}_{23}^6 = 0$). Thus, his/her opinion is not taken into account to compute the P_c.

Finally, the *undefined* behavior (e_{24} and e_{30}) consists in cooperating and non cooperating in an alternative way across the CRP. For instance, e_{24} only modifies some assessments, (e.g. $\sharp ACP_{24}^4 = 3$, $\sharp ADV_{24}^4 = 6$). Thus, he/she will have $\mu_{COOP}(CC_{24}^4) = 0.666$ according the α and β values at $t = 4$.

4.2 *Performance Analysis*

Once we have reviewed the illustrative example data on this contribution's approach, we solve this GDM problem with the other two proposals introduced in [24, 31].

Table 4 Proposals comparison

Non-Cooperative Experts	[24]	[31]	Current Proposal
Participate during all the process	Yes	Yes	Yes
Expert's opinion can recover importance	No	Yes	Yes
Takes into account the behavior across the CRP	No	No	Yes

In [24], it is assigned to experts a weight according to their distance to P_c, being weights values between 0 and 1. In this proposal, experts whose opinion have lost importance can not recover it anymore. On the other hand, [31] allows the importance recovery. In this case, experts with the most cooperative behavior, always have the highest importance, because the weighting of opinions is done attending the cooperation coefficient. It is pointed out again that, in the proposal introduced in this chapter, it is possible for experts to partially recover their importance if their behavior improves. Moreover, the importance of experts' opinions is computed based on the degree of cooperation and the distance to P_c.

In order to analyze our results easily, we have used [37] to extract the associate images to the example application data. Figure 9 shows several representative

Table 5 Weight values of MENTOR comparison

	t	1	2	3	4	5	6	7
e_1	Palomares et al.[24]	1.0	0.3137	0.3137	0.3137	0.3137	0.3137	0.3137
	Palomares et al. [31]	0.0333	0.0379	0.0347	0.0380	0.0396		
	Current proposal	0.333	0.397	0.339	0.38	0.424	0.410	
e_7	Palomares et al.[24]	1.0	0.8594	0.7489	0.7489	0.7206	0.7206	0.7206
	Palomares et al. [31]	0.0333	0.0379	0.0347	0.0380	0.0396		
	Current proposal	0.333	0.529	0.453	0.48	0.509	0.518	
e_{16}	Palomares et al.[24]	1.0	0.0041	0.0041	0.0041	0.0041	0.0041	0.0041
	Palomares et al. [31]	0.0333	0.0379	0.0347	0.0380	0.0396		
	Current proposal	0.333	0.264	0.245	0.320	0.360	0.367	
e_{22}	Palomares et al.[24]	1.0	0.0	0.0	0.0	0.0	0.0	0.0
	Palomares et al. [31]	0.0333	0.0	0.0347	0	0.0396		
	Current proposal	0.333	0	0.339	0	0.339	0	
e_{23}	Palomares et al.[24]	1.0	0.2030	0.0	0.0	0.0	0.0	0.0
	Palomares et al. [31]	0.0333	0.0	0.0	0.0	0.0		
	Current proposal	0.333	0	0	0	0	0	
e_{24}	Palomares et al.[24]	1.0	0.2712	0.1172	0.1172	0.1087	0.1087	0.1087
	Palomares et al. [31]	0.0333	0.0379	0.0347	0.0042	0.0352		
	Current proposal	0.333	0.397	0.339	0.24	0	0	

rounds for the different proposals whose general features are compared in Table 4, and the experts' weights computed throughout the CRP are summarized in Table 5.

- Round 1: the results here are similar for the three proposals. We can see the positions of expert opinions and P_c position (P).
- Round 3: it shows how experts are moving their opinions since the beginning of the CRP.
- Consensus achieved: this is the round when experts reach consensus. In [24] consensus is reached at round 9, in [31] it is reached at round 6 and the proposal of this contribution achieves consensus at round 7.

A further analysis of Figure 9 shows in our proposal that:

- Some experts remain far from the consensus opinion due to their non cooperative behavior at several consecutive rounds.
- The position of consensus strongly reinforces the opinions of highly cooperating experts.

We can conclude that using the proposal issued in this chapter the experts' opinions with an overall cooperative behavior throughout the CRP have more importance than in the other two approaches.

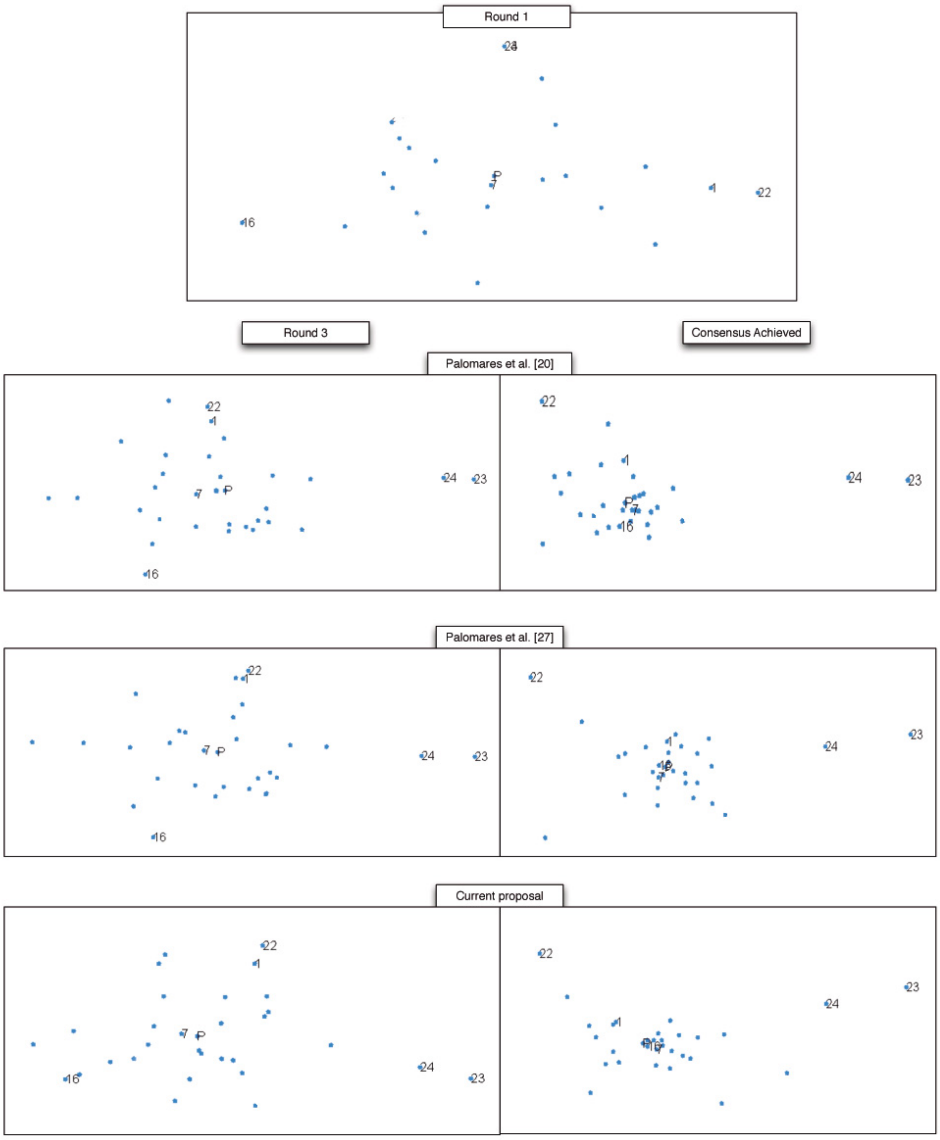

Fig. 9 MENTOR comparison images

5 Concluding Remarks and Future Works

The more experts take part in a CRP for a GDM the more possibility of some of them try to manipulate the CRP to their own interests. Different approaches in the literature were issued to address this problem, however none of them consider the change of behavior of these experts across the CRP. Therefore, in this chapter it has been introduced a new approach to detect and manage non-cooperative behaviors in large-scale GDM problems that use the CW paradigm in order to facilitate the management of changing behaviors across the whole consensus reaching process providing a more flexible and fair negotiation framework. The approach has been integrated in a consensus model and applied to an illustrative example that shows its performance and advantages over other previous approaches to deal with different behaviors of experts.

The proposal presented in this chapter opens the door for future research in decision problems within the field of group recommender systems and social networks among others, because of it will provide a ground to support negotiation processes in large-scale group decision making problems in which biased solutions could be achieved if non-cooperative misbehavior are not properly managed.

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4.3. A Consensus-Driven Group Recommender System

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A Consensus-Driven Group Recommender System

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Recommender systems aim at filtering large amounts of information for users, providing them with those pieces of information which better meet their preferences or needs. Such systems have been traditionally used in diverse areas, such as e-commerce or tourism. Within this context, group recommender systems address the problem of generating recommendations for groups of users who might have different interests. Although different aggregation processes have been extensively utilized in real-life applications to generate group recommendations, such processes do not guarantee that the list of products recommended to the group reflect a high agreement level among its members' individual preferences. Given the need for considering the added value of obtaining group recommendations under a high agreement level, this paper presents a novel group recommender system methodology that attempts to reach a high level of consensus among individual recommendations of group members. To do this, and inspired by existing group decision-making approaches in the literature, a consensus reaching process is carried out to bring such individual recommendations closer to each other before delivering the group recommendations.

1. INTRODUCTION

In the contemporary context, there is an overwhelming amount of information that leads users into the difficult task of filtering information that meets their actual needs. To address this problem, recommender systems were proposed[1] to filter information, thus delivering to users only the information that meets their preferences or needs.

Traditional recommender systems address the problem of providing recommendations targeted to individual users, but there exist certain products or services,

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such as movies,[2] music,[3] and tourist points of interest,[4,5] that have certain social features, therefore they are meant to be enjoyed by a group of users instead of individually. In this context, traditional recommender systems were limited, for this reason it became necessary to extend such systems to overcome this limitation.

Group recommender systems (GRS)[6] are one of the most challenging, yet necessary, aspects of research in the field of recommender systems: the necessity of generating recommendations targeted to a group of users with individual interests that might be different from each other.[7] In group recommendations, as stated by Jameson and Smyth in Ref. 8, there exist four basic recommending subtasks: (i) acquiring member preferences, (ii) generating recommendations, (iii) explaining group recommendations, and (iv) aiding to make the final choice. In this paper, we focus on improving recommendations by applying techniques from group decision making (GDM) and consensus reaching. Regarding the process to generate group recommendations, two extensions of individual recommender systems have been proposed[9] : *rating aggregation* and *recommendation aggregation*.

(i) In rating aggregation, individual ratings are combined to obtain a group profile that represents the group preferences.
(ii) In recommendation aggregation, each member's individual recommendations are obtained, and these recommendation lists are aggregated to obtain a suitable recommendation list for the group.

The recommendation process in GRSs has been explored for both rating and recommendation aggregation. In this paper, our aim is to meet individual users' needs in a direct way; therefore, we focus on recommendation aggregation. A desirable feature of these predictions would be to minimize the misery of members, regarding their possible disagreement with the best recommended products. Thus the minimum operator has been used in some works for the recommendation aggregation process.[2] However, applying this aggregation process solely does not guarantee that there will exist a high level of agreement among the group users over the recommendations received, but rather a minimum level of agreement.

To overcome this situation, our goal consists not only in seeking a group recommendation that mainly suits a satisfied majority of users in the group, but also in giving current GRS the added value of reaching certain agreement level among users regarding such recommendations. To do this, we consider the use of consensus approaches for GDM, by integrating them into the group recommendation process. In a GDM problem, several individuals or experts attempt to find a common solution for a decision problem composed by a set of alternatives or possible solutions to such a problem.[10,11] Thus, each expert expresses his/her preferences on each alternative. Traditional selection processes for the resolution of GDM problems[12] do not regard the fact that some experts might disagree with the decision made; hence, a number of consensus-based approaches[13,14] have been proposed to overcome this limitation by applying consensus reaching processes (CRPs) to achieve a high level of agreement before making group decisions. In a CRP, experts iteratively bring their preferences closer to each other, until a sufficient level of agreement is reached among them.[15]

Based on the previous goals, in this paper we propose a methodology that, given the recommendations for a group of users, attempts to reach a high level of consensus on the recommendations provided to them. Straightaway, a GRS model that implements such a methodology to deliver agreed recommendations to the group is presented.

This paper is set out as follows: in Section, 2 some basic concepts and preliminaries on GRSs and CRPs for the resolution of GDM problems are reviewed. Section 3 presents the consensus-driven GRS for agreed recommendations proposed in this paper, describing in detail its different phases. Section 4 shows a case study to evaluate the proposed GRS technique and compare it with baseline techniques. Finally, some concluding remarks are pointed out in Section 5.

2. PRELIMINARIES

This section first reviews some basic concepts on GRSs, followed by a brief overview of CRP in GDM.

2.1. Group Recommender Systems

In this section, basic concepts in recommender systems (RS) and group recommendation are explained, describing the inputs and basic techniques for group recommendation.

Traditional RSs (single user) use three sources of information:

- Users: $U = \{u_1, \ldots, u_q\}$ is the set of users of the system, which may provide information about themselves such as age, gender, or zip code.
- Items: $I = \{i_1, \ldots, i_t\}$ is the set of items of the system, which may have content information such as metadata or textual description.
- Ratings: $R \subseteq U \times I \rightarrow D$ is the set of users' ratings over the products, to describe how satisfied a user is regarding a particular item in the rating domain D.

RSs attempt to rank and filter items, as well as predicting ratings for unseen items by the user, to perform recommendations using these data sources. Some examples of existing RS techniques include demographic recommendation,[16] content-based recommendation,[17,18] and collaborative filtering approaches,[19,20] which rely on users' data, items, or ratings, respectively. Formally, recommender systems try to recommend the item or set of items that maximizes a given utility function.

$$Recommendation(I, u) = \arg\max_{i_l \in I}[Utility(i_l, u)] \tag{1}$$

Content-based recommender systems use similarity metrics between user and item profiles as the utility function. On the other hand, in collaborative and demographic approaches, the utility function applied consists in rating prediction.

Collaborative filtering techniques rely on rating information, and the more information about all users' preferences is available, the better the performance. A simple and extended technique is collaborative filtering using nearest neighbors

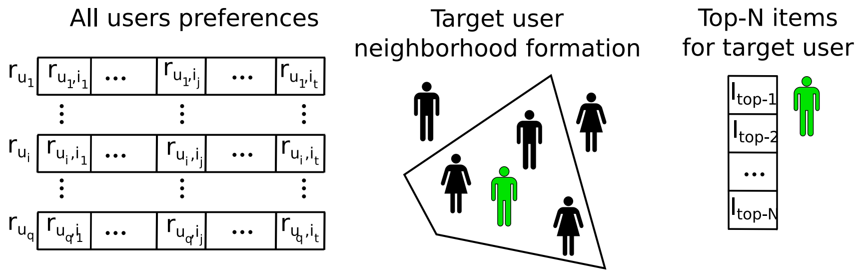

Figure 1. Recommendation aggregation scheme.

approach[21] (see Figure 1). In this technique, users' preferences are represented by a vector with user's ratings, which contains empty cells representing the items that the user has not experienced yet. The idea behind memory-based collaborative filtering is to find the k-most similar users (neighbors) to the target one, computing the similarity between user's ratings. The Pearson correlation coefficient has been proved to be the most suitable similarity measure between users, since it is not affected by the user's bias when rating items,[22] e.g., users who rate items on a consistently high or low basis. Once the neighborhood is selected, it is used to predict the rating for unseen items, by combining the neighborhood ratings on the item. A number of methods to combine neighbors ratings have been proposed.[23] For the sake of simplicity, in this paper, we consider the weighted similarity aggregation.[24] Finally, the recommendation list is constructed as an ordered list of unseen items or *predictions*, ranked by a decreasing order of their prediction value, i.e., an estimated rating value indicating how useful or interesting would the item be for the user.

Basic approaches for GRSs extends RSs so that, instead of recommending to a single user, recommendations are targeted to groups of users ($G = \{g_1, \ldots, g_m\} \subseteq U$). GRSs can operate in different modes, such as finding the most suitable group of users for a target item,[25] or recommending groups to a user for joining them,[26] In this proposal, we focus on item recommendation targeted to groups. Since we apply collaborative approaches, our utility function is rating prediction. Formally, group recommendation consists in finding the item (or set of items) that maximizes the rating prediction for the group of users:

$$GroupRecommendation(I, G) = \arg\max_{i_l \in I}[Prediction(i_l, G)] \tag{2}$$

Taking advantage of previous research on single user RSs, group recommendations are generated by extending them. Thus, there exist two basic approaches:[9]

- *Rating aggregation*,[3] which consists of aggregating individual ratings of each member to compute an aggregated group rating profile or *pseudouser* and perform individual recommendation using this profile as input.
- *Prediction aggregation*,[2,27,28] which aggregates each member's recommendations list into a list targeted to the group (as illustrated in Figure 2).

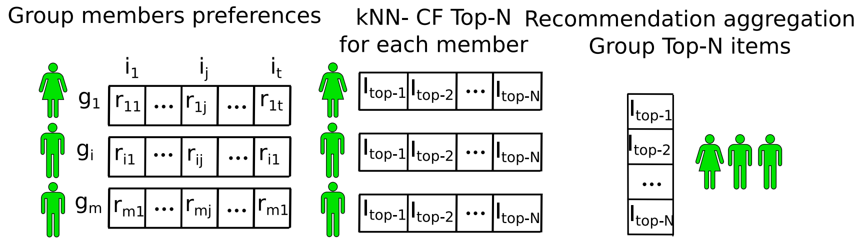

Figure 2. Recommendation aggregation scheme.

A known limitation of GRSs is that recommendations might not meet the individual preferences of all members in the group, e.g., some recommended items to the group might be regarded as satisfactory by some members, and unsatisfactory by several of them. To overcome this, recommendation aggregation minimizing member's misery[2] has been applied, but it does not take into account group dynamics such as group influence on individuals behavior. Our proposal tries to overcome this situation by applying CRPs in recommendation.

2.2. Consensus Reaching Processes in GDM

GDM problems imply the participation of multiple experts, with different knowledge and experience, who have to solve a decision problem making a common decision. A GDM problem is formally characterized by the following elements:[10,11]

- The existence of a decision problem to be solved.
- A set $X = \{x_1, ..., x_n\}(n \geq 2)$, of *alternatives* or possible solutions to the problem.
- A set $E = \{e_1, ..., e_m\}(m \geq 2)$, of participants or *experts*, who express their opinions or preferences over the set of alternatives X.

Decision problems may take place in different environments (certainty, risk, and uncertainty), being most real-life GDM problems usually defined in uncertainty environments. These environments are characterized by existence of vague and imprecise information. To express their opinions in uncertain contexts, experts may express their preferences in different information domains, such as numerical,[29] interval-valued,[30] or linguistic.[31,32]

Usually, experts utilize a preference structure to express their opinions over alternatives. One of the most used preference structures in GDM problems under uncertainty is the *fuzzy preference relation*.[33] A fuzzy preference relation P_i associated with expert e_i is defined by a membership function $\mu_{P_i} : X \times X \to [0, 1]$, and it is represented for X finite as an $n \times n$ matrix:

$$P_i = \begin{pmatrix} - & \cdots & p_i^{1n} \\ \vdots & \ddots & \vdots \\ p_i^{n1} & \cdots & - \end{pmatrix}$$

where each assessment $p_i^{lk} = \mu_{P_i}(x_l, x_k)$ represents the preference degree of the alternative x_l over x_k, $l, k \in \{1, ..., n\}$, $l \neq k$, according to e_i, interpreted as follows:

- $p_i^{lk} > 0.5$ indicates e_i's preference of x_l over x_k.
- $p_i^{lk} < 0.5$ indicates e_i's preference of x_k over x_l.
- $p_i^{lk} = 0.5$ indicates e_i's indifference between x_l over x_k.

Traditionally, the selection process for reaching a solution for a GDM problem only consists of two phases:[34]

1. *Aggregation*: Experts' preferences are combined by using an aggregation operator:[35]
2. *Exploitation*: A selection criterion[12,29] is applied to obtain an alternative or subset of alternatives as the solution to the problem.

The selection process does not guarantee that an agreement level is achieved when making the decision, which would be essential in several real-life situations. To overcome these drawbacks, the so-called CRPs are proposed, in which experts discuss and modify their preferences to bring them closer to each other.

The concept of consensus has been interpreted from different points of view, from a classic view of full agreement (unanimity), usually difficult to achieve in practice, to other softer interpretations. In Ref.15, Saint and Lawson define consensus as *a state of mutual agreement between members of a group, where all legitimate concerns of individuals have been addressed to the satisfaction of the group*. Consensus aims at increasing satisfaction of the group regarding minimizing the misery, which is the main goal pursued in this paper.

The CRP is an iterative and dynamic process aimed at achieving a high degree of agreement before making the decision that solves the GDM problem. Therefore, an essential aspect in such processes is the definition of appropriate *consensus measures* to quantify the level of group agreement from experts' preferences. According to the type of computations and information fusion procedures,[14] the different consensus measures proposed in the literature can be classified as follows (see Figure 3):

- *Consensus measures based on distances to the collective preference*:[36–38] The group opinion, represented by the collective preference, P_c, is calculated by the aggregation of all individual preferences of experts, P_i. Consensus degrees are then obtained by computing the distances between each individual preference and the collective preference, $d(P_i, P_c)$.
- *Consensus measures based on distances between experts*:[39–42] For each different pair of experts in the group, (e_i, e_j), $i < j$, the degrees of similarity between their opinions are computed, based on distance metrics. Similarity values $L(P_i, P_j)$ are then aggregated to obtain consensus degrees.

In the past years, a large number of consensus models have been proposed, having each one different features attending to diverse criteria.[14,43] Figure 4 depicts a general scheme, which encompasses most of these existing approaches, having the following phases:

1. *Consensus Measurement*: All experts' preferences, P_i, $i \in \{1, ..., m\}$, are gathered to calculate the current group agreement level, by means of consensus measures.

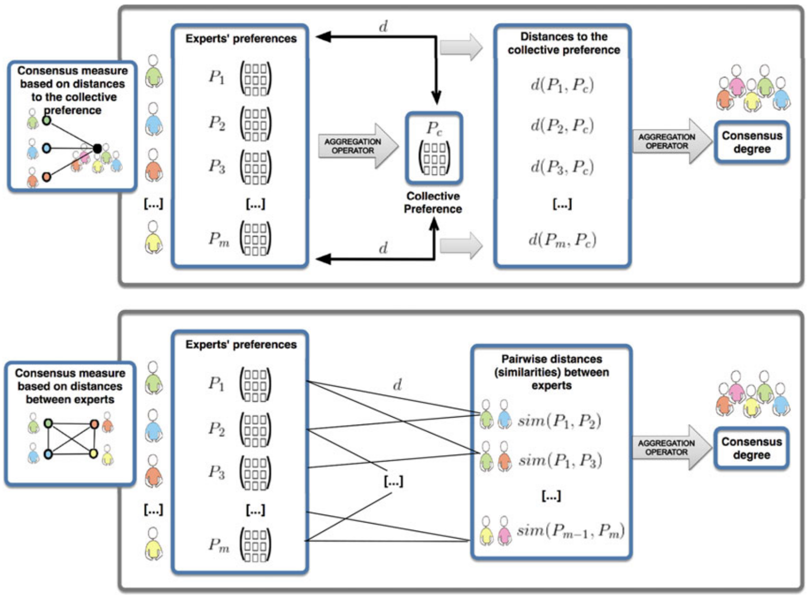

Figure 3. Types of consensus measures.

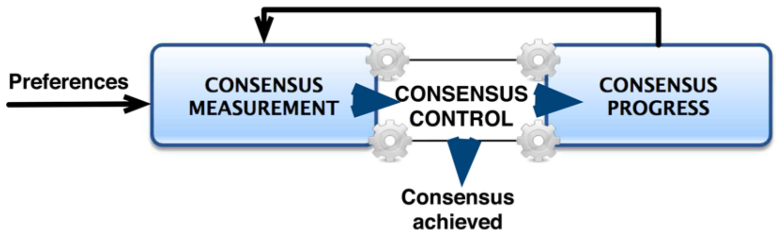

Figure 4. General CRP scheme.

2. *Consensus Control*: Compare the current consensus degree (CCD) with the consensus threshold (CT) defined at the beginning of the CRP. If the CCD is greater than the CT, then the CRP finishes, having achieved consensus. On the other hand, if the CCD is lower than the CT, the CRP continues with another round, until consensus is achieved or the number of consensus rounds exceeds the maximum number of rounds allowed.
3. *Consensus Progress*: To increase the agreement level in the following rounds of the CRP, different procedures can be applied, depending on whether the consensus model considers experts' sovereignty to let them modify their preferences based on feedback received, or such preferences are updated automatically:
   - *Feedback Generation*: In consensus models that implement this mechanism,[44–47] the moderator suggests by means of a feedback mechanism how experts should modify their preferences to bring them closer to the group opinion.
   - *Automatic Updates*: These consensus models[36,48,49] do not incorporate any feedback mechanism. Instead, they implement approaches in which experts provide their initial preferences, and automatic changes on preference values are applied across the CRP to increase the agreement level.

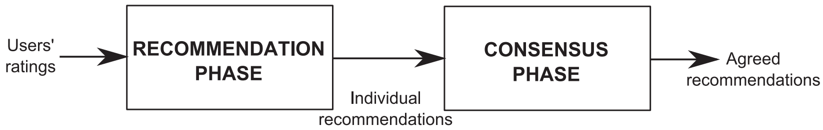

Figure 5. General scheme of the proposal.

REMARK 1. *The consensus model that will be utilized in our proposal of GRS is based on an automatic preferences updating mechanism.*

Further detail on the different types of existing consensus models can be found in Ref.14, where a taxonomy of consensus approaches in a fuzzy context was defined, being models categorized according to the use of feedback generation or automatic updates, as well as the type of consensus measure utilized in each one.

3. CONSENSUS-DRIVEN GROUP RECOMMENDER SYSTEM

This section introduces a novel consensus-based GRS model that delivers recommendations to group of users under a high level of consensus, based on their individual recommendations. In the following subsections, we describe in further details the phases of the underlying methodology in our proposal (see Figure 5):

1. *Recommendation Phase*: In this phase, a collaborative filtering algorithm is applied to obtain individual recommendations for each group member. The resulting recommendations over the top-n commonly predicted items are represented as preference orderings at the end of this phase, and they are used as input for the consensus phase.
2. *Consensus Phase*: The ordered recommendations for group members are transformed into fuzzy preference relations, and a CRP is then applied to bring such preferences closer to each other and obtain a collective preference under a high level of consensus, delivering the recommendations list upon it.

3.1. Notation

Let a GRS be such as

- $U = \{u_1, \ldots, u_q\}$ is the set of all q existing individuals or *users* in the GRS.
- $I = \{i_1, \ldots, i_t\}$ is the set of all t available products or *items* to be recommended.
- $G = \{g_1, \ldots, g_m\}$, $G \subset U$, is a group of m users of the GRS, $m << q$, to whom a list of products shall be recommended.
- D is the rating domain, i.e. the set of possible values that users can utilize to rate items. In this paper we consider $D = \{1, \ldots, 5\}$.
- $r_{il} \in D$ is the rating of user $u_i \in U$, over item $i_l \in I$.
- $\tilde{D}$ is the prediction domain, i.e. the set of possible prediction values that the GRS can assign to the pair formed by a user and an item in I not yet rated by him/her. In our case, $\tilde{D} = [1, 5]$.

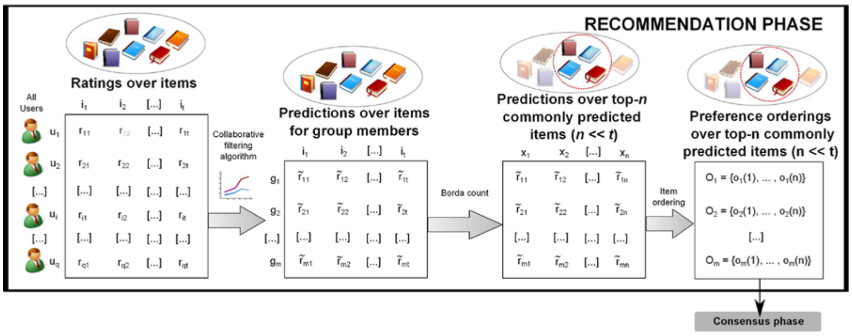

Figure 6. Scheme of computations carried out in the recommendation phase.

- $\tilde{r}_{il} \in \tilde{D}$ represents a prediction for group member $g_i \in G$ over an item i_l that he/she has not rated yet.
- $X = \{x_1, \ldots, x_n\}$, $X \subset I$, is the set of the top-n items commonly predicted to all members in a group G, with $n << t$.

REMARK 2. *Our proposal aims at seeking consensus among individual recommendations of group members over items in X. To do this, a CRP will be applied by using an automatic consensus model, hence our framework must be also modeled as a GDM problem, in which X represents the set of alternatives of such a problem and G represents the group of individuals taking part in it.*

3.2. Recommendation Phase

This phase includes the necessary computations to generate, for each member in the group, $g_i \in G$, a set of recommendations or predictions over items. Such predictions are ordered from the best to the worst one. A ranking or preference ordering, O_i, is generated as a result of this phase for each group member, g_i, over the top-n items $x_l \in X$ that have been commonly predicted to *all* the members in the group. The steps included in this phase are depicted in Figure 6, and they are described in detail below:

REMARK 3. *In Figure 6, all ratings r_{il} are depicted in the scheme for all the existing users and items, but only some of them may have a defined value, i.e. those corresponding to the items that have been rated by each user. The same situation applies with predictions for group members over all the items in I.*

(A) *Applying single user collaborative filtering algorithm*: First, given *all* users' ratings over some of the existing items in the GRS database, it is necessary to generate, for each group member in G, the individual predictions over items $i_l \in I$ not yet rated by him/her. To do this, first a collaborative filtering algorithm is applied to generate prediction values over items, $\tilde{r}_{il}$, for all the existing users. For this step, the collaborative filtering algorithm applied is user-based k-nearest neighbors algorithm (UBCF).[50]
Once applied the UBCF algorithm, only those predictions $\tilde{r}_{il}$ corresponding to group members $g_i \in G$ are taken into account in the following computations carried out in this phase.

(B) *Filtering of group member predictions over top-n commonly predicted items*: Group members' predictions computed after applying the collaborative filtering algorithm do not cover all the existing items in the GRS, i.e. two different members g_i and g_j may receive predictions over different subsets of items from I; hence, only some of the existing items in the RS database would be recommended to both of them simultaneously. In this step, the subset of items that have been commonly predicted for all the members in group G is determined. Moreover, if we consider a small group size and a large item database, the subset of commonly predicted items to group members might be too large; therefore, from this step onwards we will only consider the n items predicted as the best ones for all group members as a whole. Such items will be referred to as the top-n commonly predicted items for the group, $X = \{x_1, \ldots, x_n\}$, $X \subset I$, with $n << t$.
The following computations are carried out to obtain the set X of top-n commonly predicted items:
  (i) Construct a set I^G, such that $X \subseteq I^G \subset I$, of all items commonly predicted to every member in G:

$$I^G = \{i_l \in I : \forall g_i \in G \exists \tilde{r}_{il} \in \tilde{D}\} \tag{3}$$

  with $\tilde{D}$ being the prediction domain, e.g., the continuous interval $\tilde{D} = [1, 5]$ if $D = \{1, \ldots, 5\}$.
  (ii) For each group member $g_i \in G$, rank commonly predicted items in I^G in a descending order of the prediction values assigned to him/her, $\tilde{r}_{il}$.
  (iii) Given $n \in \mathbb{N}$, $n \leq |I^G|$, fixed a priori, a selection over all individual recommendation lists is applied, since GDM and CRP techniques are only capable of working with a small set of items. A number of rank aggregation techniques are applicable to this prefilter such as Borda or cumulative voting.[51] Our proposal applies single transferable voting[52] over the members recommendations to determine X.
  (iv) Once the item set X has been determined, it is necessary to filter predictions, keeping those $\tilde{r}_{il}$ accomplishing $g_i \in G$ and $x_l \in X$, and leaving out those accomplishing $x_l \notin X$ or $u_i \notin G$.

(C) *Constructing preference orderings from top-n common predictions*: Let $\tilde{r}_i = [\tilde{r}_{i1}\tilde{r}_{i2} \ldots \tilde{r}_{in}]$ be each group member's vector of predictions over the top-n commonly predicted items. We then obtain its corresponding preference ordering $O_i = \{o_i(1), \ldots, o_i(n)\}$, being $o_i(\cdot)$ a permutation over the index set $\{1, \ldots, n\}$, such that $o_i(l) < o_i(k) iff \tilde{r}_{il} > \tilde{r}_{ik}$.[53] Therefore, items in X recommended to a group member g_i are ordered from the item having the highest prediction value, to the item having the lowest one.

3.3. Consensus Phase

Once generated, the individual predictions expressed as preference orderings over the top-n items recommended to the group members, this phase aims at using them to obtain a collective list of predictions for the group, which reflects a high level of consensus among individual predictions. For this, a CRP is applied by using an automatic consensus model for GDM problems with fuzzy preference relations.[54] Therefore, individual predictions of group members must be first expressed as fuzzy preference relations so that they can be managed by the consensus model.

The following two steps are carried out in this phase (see Figure 7):

(A) *Representing individual recommendations as fuzzy preference relations*: At the end of the recommendation phase (see Section 3.2), filtered predictions of group members

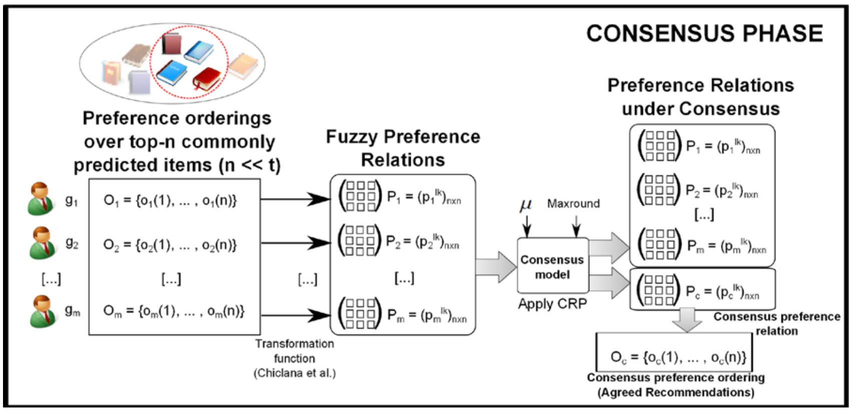

Figure 7. Scheme of computations carried out in the consensus phase.

were conducted into preference orderings in which, for each member, items were ranked from the best to the worst one according to their prediction values. Here, individual recommendations are first expressed as fuzzy preference relations before applying a CRP on them, since a consensus model for GDM with preference relations is utilized. Therefore, a transformation function is used to express each preference ordering O_i as a fuzzy preference relation P_i. In Ref.55, Chiclana et al. developed a number of transformation functions to deal with multiple preference representation formats in decision-making problems under uncertainty. In particular, they proposed the following method to construct the equivalent fuzzy preference relation $P_i = (p_i^{lk})_{n\times n}$ to a preference ordering $O_i = \{o_i(1), \ldots, o_i(n)\}$:

$$p_i^{lk} = \frac{1}{2}\left(1 + \frac{o_i(k) - o_i(l)}{n-1}\right) \quad (4)$$

EXAMPLE 1. *Let $O_1 = \{3, 2, 4, 1\}$ be a set of recommendations for member g_1 over $n = 4$ items expressed as a preference ordering, in which $o(4) = 1$ indicates that item x_4 is the item with the highest prediction value for g_1. Then, its corresponding fuzzy preference relation is obtained as follows:*

$$P_1 = \begin{pmatrix} - & 0.33 & 0.67 & 0.17 \\ 0.67 & - & 0.83 & 0.33 \\ 0.33 & 0.17 & - & 0 \\ 0.83 & 0.67 & 1 & - \end{pmatrix}$$

where, for instance, p_i^{12} has been computed by using Equation 4 as

$$p_i^{12} = \frac{1}{2}\left(1 + \frac{2-3}{3}\right) = 0.33 \quad (5)$$

(B) *Conducting the CRP*: Once each group member's preference ordering O_i has been conducted into a fuzzy preference relation P_i over the top-n commonly predicted items, a CRP is applied to such preferences for bringing them closer to each other gradually, until a high level of agreement among them is achieved. The consensus model utilized is an automatic model in which the values of preferences are automatically updated to bring them closer to each other (instead of implementing a feedback mechanism for individuals[14]) and it considers the following phases (see Figure 4), which are sequentially carried out at each round until consensus is achieved:

(1) *Consensus Measurement*: Fuzzy preference relations of group members, $P_i = (p_i^{lk})_{n\times n}$, $i = 1, \ldots, m$, are gathered and utilized to determine the level of agreement in the group. To do this, the following computations are carried out:
   (i) For each pair of members in the group g_i, g_j, $(i < j)$, a similarity matrix $SM_{ij} = (sm_{ij}^{lk})_{n\times n}$ is computed, with $sm_{ij}^{lk} \in [0, 1]$ being the degree of similarity between g_i and g_j's assessment on the pair of items (x_l, x_k).[56]
   (ii) A consensus matrix, $CM = (cm^{lk})_{n\times n}$ is obtained by aggregating pairwise similarity matrices. Each element $cm^{lk} \in [0, 1], l \neq k$, is computed by applying an aggregation operator (e.g., the arithmetic mean or the OWA operator[57]) on similarity values.
   (iii) *Computing Consensus Degrees*: Once obtained CM, its values are successively aggregated[47] to obtain an overall consensus degree in the group, $cr \in [0, 1]$.

(2) *Consensus Control*: Once the overall consensus degree for the group is computed in the consensus measurement phase, here it is checked to determine whether it indicates enough agreement or not. If the consensus degree is enough, the CRP finishes having reached consensus among members' preferences. Otherwise, preference values are updated in the consensus progress phase to bring them closer to each other. A consensus threshold $\mu \in [0, 1]$ whose value fixed a priori indicates the minimum degree of agreement required among group members is used in this phase. The larger μ, the higher the consensus degree required. Furthermore, a parameter $Maxround$ can be used to limit the number of consensus rounds carried out without having reached μ.

(3) *Consensus Progress*: In this phase, group members' assessments p_i^{lk} which are farthest from consensus, are identified. A set of updates on the values of such identified assessments are then applied, with the aim of increasing consensus in the following rounds. The following steps are carried out in this phase:
   (i) A collective preference $P_c = (p_c^{lk})_{n\times n}$ is obtained, by aggregating individual assessments on each pair of items.
   (ii) A proximity matrix $PP_i = (pp_i^{lk})_{n\times n}$ whose values indicate the closeness degree between each member's preference relation and the collective preference, P_c, is computed for each $g_i \in G$.[57]
   (iii) Based on proximity matrices, some identification rules[47] are applied to identify group members g_i and pairs of items (x_l, x_k) whose assessments p_i^{lk} are not close enough to consensus. A set of direction rules are then applied to automatically update such identified assessments,[54] to increase the level of consensus in the group.

Once members' preferences have been updated, another CRP round starts by applying again the consensus measurement phase on them. After reaching consensus, the collective preference P_c, which reflects a high agreement level in the group, is used in the following step to deliver a list of agreed recommendations to the group.

(C) *Obtaining Agreed Recommendations List*: A consensus preference ordering is determined from the consensus preference relation P_c by applying a selection criterion,[12,29] to rank items from the best one to the worst one. More specifically, we consider the nondominance criterion[29] that assigns each item $x_l \in X$ a nondominance degree $ND(x_l) \in [0, 1]$ as follows:

(a) Construct a strict consensus preference relation $\tilde{P}_c = (\tilde{p}_c^{lk})_{n\times n}$, where:

$$\tilde{p}_c^{lk} = \begin{cases} \tilde{p}_c^{lk} - \tilde{p}_c^{kl} & \text{if } \tilde{p}_c^{lk} > \tilde{p}_c^{kl}, \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

(b) Compute $ND(x_l)$ as

$$ND(x_l) = 1 - \max_k\{\tilde{p}_i^{kl}\} \tag{7}$$

The consensus preference ordering, denoted by

$$O_C = \{o_C(1), \ldots, o_C(n)\}$$

is then obtained by arranging items from their highest nondominance degree to their lowest one, i.e., if $o_C(l) < o_C(k)$ then $ND(x_l) \geq ND_{x_k}$.

As a result of conducting the consensus phase, members of the group are provided with a set of highly agreed recommendations (given by O_C) over the top-n commonly predicted items to all of them.

4. CASE STUDY

The case study to validate the proposal previously presented is deeply described in this section. To do this, the data set utilized in the study is first introduced, followed by the details about the experiment performed and the GRSs technique to be compared with our proposal. Afterward, the experiment results are shown and discussed. Finally, an example to visually demonstrate how the proposal is aiming to improve agreement on group recommendations, is shown.

4.1. Data Set

The data set used in this case study is the MovieLens data set, collected by the GroupLens Research Project[a] at the University of Minnesota. Specifically, ml-100k data set is used, and it consists of a hundred thousand ratings statements given by 943 users over 1682 movies in {1,2,3,4,5} domain.

4.2. Experiment Description

The data set considered does not contain information about possible groups of users; therefore, the group formation technique utilized is a random selection. Thirty different groups of five members each are formed to perform the case study.

The validation technique applied is 20% item holdout, which has been adjusted to be used on group recommendation by selecting the 20% of items rated by

[a] http://grouplens.org/

each group as a test set. Multiple executions have been performed to obtain more significant results.

Finally, the evaluation measures considered for this experiment are Area Under receiver operating characteristic Curve (AUC) and precision.[58] AUC is used to evaluate classifiers by computing ratio between the true-positive and false-positive rate as a threshold variable is adjusted, which builds a curve. In the case of recommender systems, the threshold variable is the size of the recommendation list. Regarding precision, this measure shows the ratio between the true-positive rate and the number of recommended items. For both measures, the higher the value, the better the GRS performance.

4.3. Group Recommender Systems Comparison Analysis

The aim of this case study is to compare our proposal of consensus-driven GRS model with different group recommendation techniques focused on delivering satisfactory recommendations for all members. Therefore, the use of average aggregation as a baseline technique would not be correct since it does not take into account individual satisfaction, delivering recommendations that members might see as deviated toward other members' preferences.

Therefore, we take group recommendation with recommendation aggregation using minimum as the baseline technique, taking UBCF as the single user recommender system. As UBCF has a number of improvements,[21] Pearson correlation coefficient is taken as similarity measure. Data sparsity can lead to poor values for similarity measure due to a small number of co-rated items; therefore, a correction with relevance factor of 20 is applied to penalize similarities computed with less than 20 corated items. Weighted sum is taken as a prediction technique to aggregate neighborhood ratings. For the sake of comparability of results, the selection of top-n items over which the CRP is carried out is applied as described in Section 3.

Our proposal uses the above mentioned UBCF recommender system, but it performs the recommendation aggregation based on a CRP in which the consensus degree desired (given by the consensus threshold μ) needs to be set; hence, the consensus degree values considered are $\{0.8, 0.85, 0.9\}$.

4.4. Results

The result of AUC (see Figure 8) indicates that applying the CRP clearly improves the baseline results for the three consensus threshold values tested and states that the optimal value of consensus degree in this data set is 0.8, decreasing steadily as consensus degree increments.

Based on the precision measure, the results are now examined in a more detailed way. As shown in Figure 9, different configurations for the CRP module produce variations in recommendation lists delivered to groups. comparing consensus techniques, the results show that applying a consensus degree of 0.8 benefits the recommendation results in comparison with recommendations subject to a higher consensus level. If the results are analyzed regarding the list sizes considered, the results show that for recommendation lists of size lower than four elements the

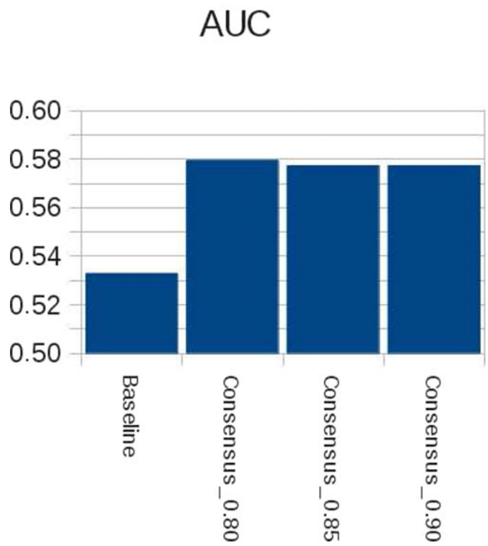

Figure 8. AUC for evaluated configurations.

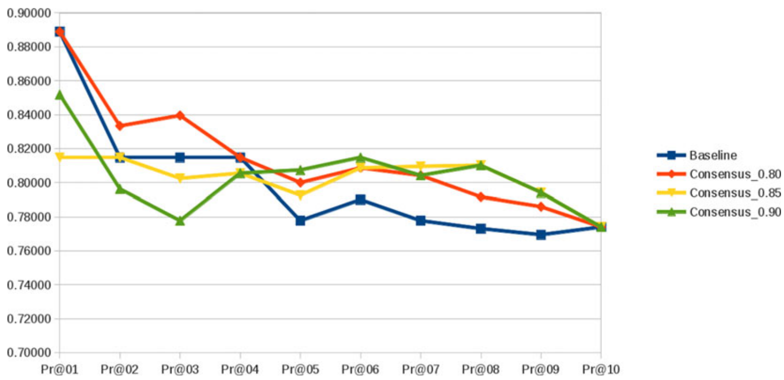

Figure 9. Precision at certain recommendation list sizes for evaluated configurations.

optimal consensus degree value is 0.8. For recommendation lists of size 4 or more, the consensus configurations shown in the precision diagram present similar values for precision, revealing a similar behavior in these cases.

REMARK 4. *In Figure 9, the output of the GRSs depicted are given in the same set of 10 items. Thus, the only difference between configurations is the ordering of those 10 items that the GRSs provided as the output. Precision does not take into account whether a good item is ranked in the first or last position within the ordering. This causes that all the GRSs evaluated in our case study present the same precision at list size of 10 (see Pr@10 in Figure 9).*

4.5. Graphical Visualization

Once the case study has been presented, we consider necessary to graphically depict the results on an intuitive manner to visualize users' preferences with respect to the baseline GRS recommendations and the agreed recommendations (AR). For this reason, in this section we describe an specific example with a group of users extracted from the MovieLens data set.

There are previous approaches on recommendation visualization techniques for individual recommender systems, such as the one presented by Kagie et al. in Ref. 59. These proposals represent the items in the recommendation set to show similarities between products, e.g., the closer two items are depicted with each other, the more similar they are. Following similar ideas, we represent group members' preferences to visualize where their opinions are regarding the group preference. As this problem has already been addressed in GDM, we use a Self-Organizing Maps-based[60] tool for visualizing preferences, so-called MENTOR.[61]

To apply a CRP before comparing graphically the differences between the baseline GRS output and the AR, we use a multiagent-based consensus support system[54] to compute the consensus ranking (see consensus phase, Section 3.3).

A group of five users from MovieLens data set, $U = u_1, ..., u_5$, is taken, and each one has a different prediction ranking. The recommendation lists are reduced taking the first 10 common films of the user rankings, selected using top-n member items, which are used as alternatives of a GDM problem, $X = \{x_1, ..., x_{10}\}$:

- x_1: Three Colors: Red.
- x_2: The Fugitive.
- x_3: A Space Odyssey.
- x_4: Manon of the Sprius.
- x_5: Delicatessen.
- x_6: Nikita.
- x_7: The Princess Bride.
- x_8: Raiders of the Lost Ark.
- x_9: Return of the Jedi.
- x_{10}: The Godfather II.

Once performed the initial recommendation phase, a first ranking is obtained from individual recommendations. A CRP is then applied to rerank the recommendations to improve agreement. A comparison among the baseline GRS rank and the agreed recommendations is shown in Table I, showing the differences between both rankings. For instance, "Return of the Jedi," which is the first one in the GRS ranking, changes to the second position, whereas "The princess bride," which is the second one in the GRS ranking, shifts toward the top of the AR. We can see the most drastic change in "The Fugitive," which change from the fourth to the ninth position.

Regarding the user's visualization (Figure 10), the consensus recommendation is situated in the center of the graphic, because CRPs take equidistances to bring users' opinions closer to the group opinion. On the other hand, GRSs attempt to minimize users' misery delivering less satisfactory recommendations to the group without taking into account individuals recommendations when computing group recommendations.

Table I. Comparison between GRS ranking and CR.

Baseline Group Recommendation	Agreed Recommendation
Return of the Jedi	The Princess bride
The Princess Bride	Return of the Jedi
Manon of the Sprius	Manon of the Sprius
The Fugitive	Three Colors: Red
Raiders of the Lost Ark	Raiders of the Lost Ark
The Godfather II	Nikita
Three Colors: Red	A Space Odyssey
Nikita	The Godfather II
Delicatessen	The Fugitive
A Space Odyssey	Delicatessen

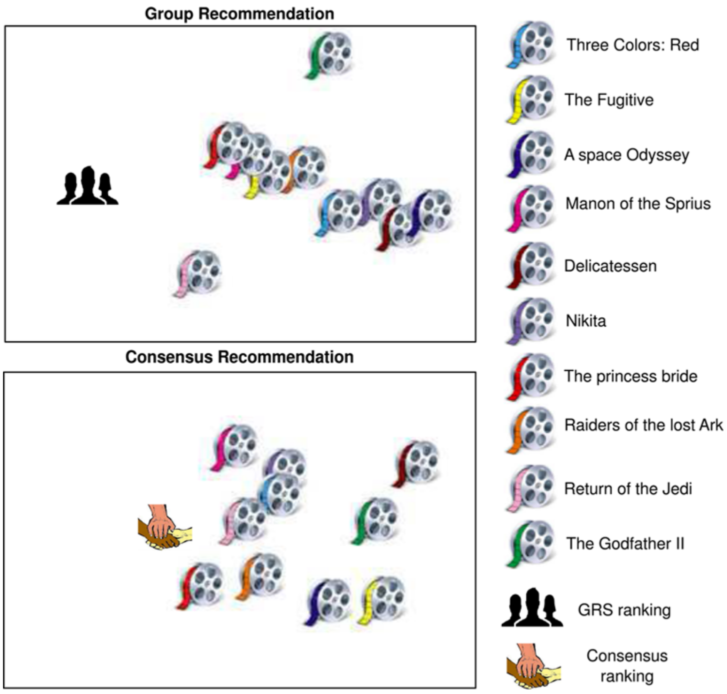

Figure 10. Users' visualization regarding group recommendation and consensus recommendation.

5. CONCLUDING REMARKS

In this paper, a novel technique for group recommendations based on CRPs has been proposed, to provide existing GRSs with the added value of improving group members' satisfaction regarding the items recommended. The case study results showed that applying CRPs in group recommendation clearly improves the results compared with baseline group recommending techniques.

The results suggest that bringing consensus into recommendation processes is a promising future research direction; hence, the exploration of other approaches developed in group decision making area might benefit recommendation.

Future works are mainly focused on developing a graphical tool, which allows users to visualize their position regarding items and other users in GRSs.

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4.4. Improving group recommendations with consensus reaching processes

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Improving group recommendations with consensus reaching processes

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Abstract Recommender systems help users when large amounts of information are available, by filtering it based on their preferences or needs. These systems have been successfully used in diverse areas, such as e-commerce, tourism and so on. Group Recommender Systems subsequently address the problem of recommending items to groups of users. In order to compute group recommendations, aggregation processes have been initially applied on individual recommendation lists. Notwithstanding, this process does not take into account group dynamics such as the influence of group on individual preferences or consensus reaching. To overcome these limitations, we attempt to deliver highly agreed group recommendations. Inspired by Consensus Reaching approaches from Group Decision Making, this chapter presents a group recommender system model that implements a consensus approach to deliver group recommendations under a high level of agreement amongst group members.

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1 Introduction

Recommender systems were proposed [1] to overcome the problem of filtering large amounts of information, thus delivering users only the information that suits their preferences or needs. Certain products or services are targeted to individual users by using traditional recommender systems, but there exist other products or services, such as music [2], movies [3] and tourist points of interest [4], that have social features, hence they shall be enjoyed by a group of users. In this situation, it became necessary to extend individual recommender systems to recommend to groups of users.

Given the necessity of delivering recommendations targeted to groups of users with individual and different interests, Group Recommender Systems (GRS) [5] have arisen as one of the most challenging aspects of research in the field of recommender systems. Jameson stated in [6], that in group recommendations there exist four basic recommending subtasks: i) acquiring group members' preferences, ii) generating recommendations, iii) explaining group recommendations and iv) assisting to make the final choice. Regarding the process to generate group recommendations, two extensions of individual recommender systems have been proposed [7]: i) *rating aggregation*, in which individual ratings are combined to obtain a group profile that represents the group preferences and ii) *recommendation aggregation*, in which each member's individual recommendations are obtained, and these recommendation lists are aggregated to obtain a suitable recommendation list for the group.

In this work we focus on recommendations aggregation in group recommendation models because our aim is to meet individual users' needs in a direct way. A desirable feature of these predictions would be to minimize the misery of members, in terms of possible disagreements of the members regarding the recommended products. In such a way the minimum operator has been traditionally used in some works to carry out the recommendation aggregation process [3], but this does not guarantee that there exists a high level of agreement amongst the group members over the recommendations received, but rather a minimum level of disagreement only.

Our objective in this chapter is to provide recommendations to groups where members present a high agreement level regarding the recommendations. Therefore, we consider applying a consensus reaching process (CRP) (see section 2.2) for Group Decison Making (GDM) to increase the satisfaction of the group regarding the recommendation. To do so, the CRP will be integrated them into the group recommendation process. In a GDM problem, several individuals or experts attempt to find a common solution for a decision problem composed by a set of alternatives [8, 9]. Hence, each expert expresses his/her preferences on each alternative. In this context, a number of consensus-based approaches [10, 11] have been proposed to reach a high level of agreement among experts before making a decision [12].

This chapter is set out as follows: in Section 2, some preliminaries on GRSs and consensus reaching for the resolution of GDM problems, are reviewed. Section 3 presents our proposal, the consensus-driven GRS, describing in detail its different phases. Section 4 shows a case study to evaluate the proposed GRS technique and compare it with baseline techniques. Finally, in Section 5 some concluding remarks are pointed out.

2 Preliminaries

This section firstly reviews some basic concepts on GRSs, followed by a brief overview of Consensus Reaching Processes in GDM.

2.1 Group Recommender Systems

At present, there exist a huge amount of information in the Internet, making it very difficult for users to filter the information they actually need. In order to overcome this problem, recommender systems (RS) were proposed [1] to filter information, thus delivering to users only the information that suits their preferences or needs.

In the following, some basic concepts about RSs and group recommender systems (GRS), describing the inputs and basic techniques for group recommendation.

Traditional RSs (individual user) use three sources of information:

- *Users*: Who may provide profile information about themselves, such as age or gender.
- *Items*: Which may have content information, such as textual description or metadata.
- *Ratings*: It is the set of users' ratings over the items, which describe how satisfied a user is regarding a particular item.

RSs aim at selecting and filtering items, as well as predicting ratings for unseen items by the user, in order to make recommendations using the available data sources. There exist many RS techniques, e.g. Demographic recommendation [13], content-based recommendation [14, 15] and collaborative filtering [16, 17], which rely on users' data, items or ratings, respectively. Formally, recommender systems attempt to determine the item or set of items that maximize a given utility function.

$$Recommendation(I,u) = \arg\max_{i_l \in I}[Utility(i_l,u)] \tag{1}$$

where $U = \{u_1,\ldots,u_q\}$ is the set of users of the system, $I = \{i_1,\ldots,i_t\}$ is the set of items and $R \subseteq U \times I \to D$, is the set of users' ratings over the items, expressed in the rating domain D.

A simple and popular technique for collaborative filtering RS is based on the K-nearest neighbors approach [18] (see Figure 1). In this technique, users' preferences are represented by a vector, which contains empty values representing the items that the user has not experienced yet. Memory-based collaborative filtering is based on finding the k-most similar users (neighbors) to the target one, computing the similarity between user's ratings. One of the most suitable measures to compute similarity between users is the Pearson correlation coefficient, since it is not affected by users who rate items on a consistently high or low basis [19]. The computed neighbourhood is used to predict the rating for non-rated items by the user. There exists a number of methods to predict such ratings [20, 21]. Finally, the recommendation list is constructed as an ordered list of non-rated items or *predictions*, ranked by decreasing order of their prediction value.

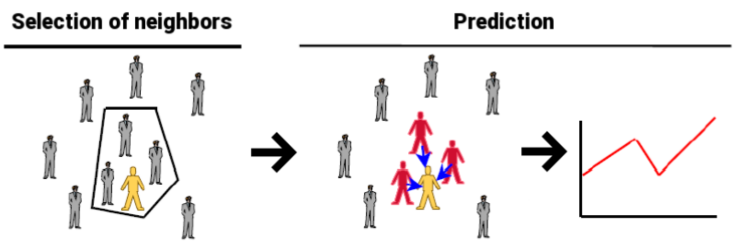

Fig. 1 Collaborative filtering using nearest neighbors

In GRSs, recommendations are targeted to groups of users ($G = \{g_1, \ldots, g_m\} \subseteq U$, with $m \ll q$). GRSs can operate in different modes, such as finding the most suitable group of users for a target item [22], or recommending possible groups to a user, as a suggestion for joining them [23]. In this chapter, we focus on item recommendation targeted to groups. Since we apply collaborative filtering approaches, our prediction function is based on rating information. Formally, group recommendation consists in finding the item (or set of items) that maximizes the rating prediction for the group of users:

$$GroupRecommendation(I, G) = \arg\max_{i_l \in I}[Prediction(i_l, G)] \quad (2)$$

There exist two basic GRS approaches [7]:

- *Rating aggregation* [2], which consists in aggregating individual ratings of each member to compute an aggregated group rating profile or *pseudo-user* and perform individual recommendation using this profile as input (as illustrated in Figure 2).
- *Prediction aggregation* [24, 3, 25], which aggregates each member's recommendations list into a list targeted to the group (as illustrated in Figure 3).

GRSs have a known limitation: recommendations might not meet the individual preferences of all members in the group, i.e. they might not take into account the satisfaction of each group member. To overcome this limitation, prediction aggregation minimizing member's misery [3] has been applied, but it does not take into

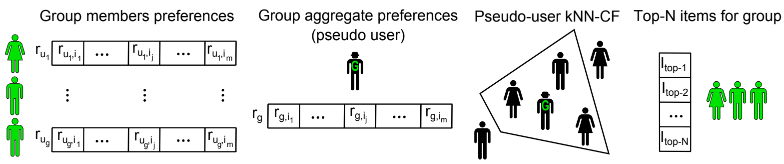

Fig. 2 Rating aggregation scheme using k-nearest neighbours collaborative filitering (kNN-CF) approach as individual RS.

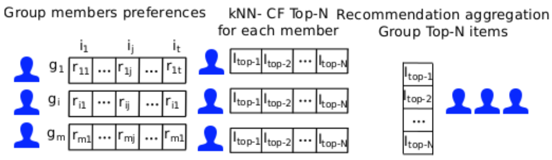

Fig. 3 Prediction aggregation scheme using k-nearest neighbours collaborative filitering (kNN-CF) approach as individual RS.

account group dynamics such as group influence on individuals behavior. Our proposal tries to completely overcome this drawback by applying consensus reaching processes.

2.2 Consensus Reaching Processes in GDM

GDM problems imply the participation of multiple experts, with different knowledge and experience, who have to solve a decision problem to make a common decision. Formally, a GDM problem is composed by [8, 9]:

- A set $X = \{x_1,\ldots,x_n\}(n \geq 2)$, of *alternatives* or posible solutions to the problem.
- A set $E = \{e_1,\ldots,e_m\}(m \geq 2)$, of participants or *experts*, who express their opinions or preferences over the set of alternatives X.

Usually, most real-life GDM problems are defined in uncertainty environments, which are characterized by existence of vague and imprecise information. One of the most used preference structures in GDM problems under uncertainty is the *preference relation* [26]. A preference relation P_i associated to expert e_i is defined by a membership function $\mu_{P_i} : X \times X \to Y$, with Y being the information domain and utilized to assess alternatives, for the example the $[0,1]$ interval. A preference relation is represented for X finite as an $n \times n$ matrix:

$$P_i = \begin{pmatrix} - & \dots & p_i^{1n} \\ \vdots & \ddots & \vdots \\ p_i^{n1} & \dots & - \end{pmatrix}$$

where each assessment p_i^{lk} represents the preference degree of the alternative x_l over x_k, $l,k \in \{1,...,n\}, l \neq k$, according to e_i, interpreted as follows:

- $p_i^{lk} > 0.5$ indicates e_i's preference of x_l over x_k.
- $p_i^{lk} < 0.5$ indicates e_i's preference of x_k over x_l.
- $p_i^{lk} = 0.5$ indicates e_i's indifference between x_l over x_k.

Classically, the process to find a solution for a GDM problem consists of an alternative selection process, which is composed of two phases [27] (Figure 4).

1. *Aggregation*: In this phase experts' preferences are combined by using an aggegation operator [28].
2. *Exploitation*: Here, a selection criterion [29] is applied to obtain an alternative or a subset of alternatives, as the solution for the GDM problem.

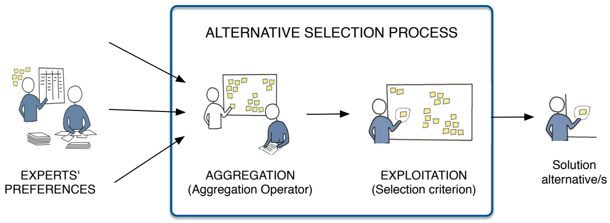

Fig. 4 Selection process in GDM problems.

When applying the selection process in a GDM problem exclusively, it may occur that one or more experts feel that their opinions have not been taken into account sufficiently to reach the solution achieved, therefore these experts may not feel identified with the solution [8]. There exist some situations in which it is necessary a high agreement level amongst the participant experts. Therefore, it arises the need for applying a Consensus Reaching Process (CRP), that introduces a new phase in the GDM resolution process to reach a high level of agreement between experts before making the decision [12].

A CRP is a dynamic and iterative process, which is coordinated by a human figure known as *moderator*. The moderator is responsible for supervising and guiding experts over the course of this process [12]. A large number of consensus models have been proposed, having each one different features [11]. Figure 5 depicts a general scheme which encompasses most of these existing approaches, having the following phases:

1. *Consensus Measurement*: All experts' preferences, $P_i, i \in \{1,\ldots,m\}$, are gathered to calculate the current agreement level, by means of consensus measures.
2. *Consensus Control*: The Current Consensus Degree (CCD) compared with the Consensus Threshold (CT) defined at the beginning of the CRP. If the CCD is greater than the CT, then the CRP finishes, having achieved consensus. On the other hand, if the CCD is lower than the CT, the CRP continues with another round, until consensus is achieved or the number of consensus rounds exceeds the maximum number of rounds allowed.
3. *Consensus Progress*: In order to increase the agreement level in the following rounds of the CRP, different procedures can be applied [11]:
   - *Feedback Generation*: In consensus models that implement this mechanism [30, 31], the moderator suggests by means of a feedback mechanism how experts should modify their preferences, in order to bring them closer to the group opinion.
   - *Automatic Updates*: These consensus models [32, 33] do not incorporate any feedback mechanism. Instead, they implement approaches in which experts provide their initial preferences, and automatic changes on preference values or importance weights are applied across the CRP in order to increase the agreement level.

Remark 1. In our proposal of GRS, we will use a consensus model based on an automatic preferences updating mechanism because recommender systems attempt to minimize the interaction with users.

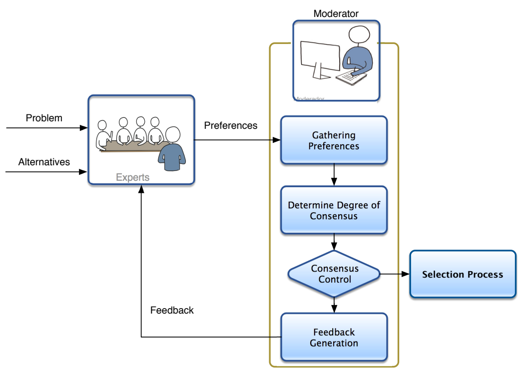

Fig. 5 General CRP scheme.

3 Consensus-driven Group Recommender System

This section introduces a novel consensus-based GRS model that provides group recommendations under a high level of consensus, based on their individual recommendations. In the following subsections it is described in further detail the phases of the underlying methodology in our proposal (See figure 6).

1. *Recommendation phase*: In this phase, individual recommendations for each group member are obtained applying a collaborative filtering algorithm. The resulting recommendations over the top-n commonly predicted items are represented as preference orderings at the end of this phase, and they are used as input for the consensus phase.
2. *Consensus phase*: The ordered recommendations for individuals of the group are transformed into preference relations, and a CRP is then applied to bring individual preferences closer to each other and obtain a collective preference under a high level of consensus, that will be used to compute an agreed recommendations list for the group.

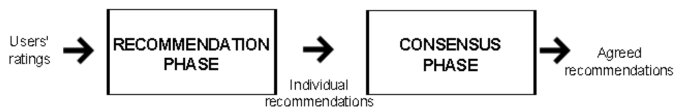

Fig. 6 General scheme of the proposal

3.1 Recommendation Phase

In this phase the RS computes individual predictions for each member in the group, $g_i \in G$. Such predictions are ordered from the best to the worst one. A ranking or preference ordering, O_i, is then generated for each group member, g_i, over the top-n items $x_l \in X$ that have been commonly predicted to *all* the members in the group. In Figure 7, the steps included in this phase are depicted, and described in detail below:

Remark 2. In Figure 7 r_{il} are the ratings for all users u_i and items i_l, but only some of them may have been rated by each user. The same situation applies with predictions for group members over all items in I.

1) *Applying single user collaborative filtering algorithm*: Firstly, it is computed for each group member $g_i \in G$, the individual predictions, $\tilde{r}_{il}$, over items $i_l \in I$ not yet rated by him/her, by applying a collaborative filtering algorithm. In our case, the collaborative filtering algorithm applied is User Based Collaborative Filtering (UBCF) algorithm [34] with K-nearest neighbors.

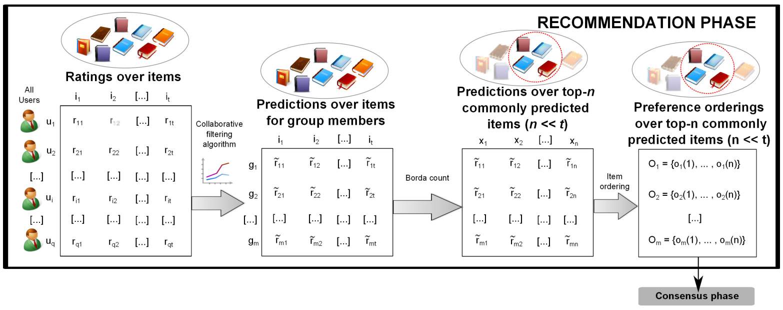

Fig. 7 Scheme of computations carried out in the Recommendation phase.

2) *Filtering of group member predictions over top-n commonly predicted items*: Group members' predictions, $\tilde{r}_{il}$, computed by the UBCF, do not cover all the existing items, i.e. two different members g_i and g_j may receive different predictions of items from I, hence only some of the existing items would be recommended to all group members simultaneously. In this step, the subset of items that have been commonly predicted for all the members in group G, is determined. Moreover, it is only considered the top-n commonly predicted items for the group, $X = \{x_1, \ldots, x_n\}$, $X \subset I$, with $n \ll t$, that are computed as:

   (i) Construct a set I^G, such that $X \subseteq I^G \subset I$, of *all* items commonly predicted for every member in G:

$$I^G = \{i_l \in I : \forall g_i \in G\ \exists \tilde{r}_{il} \in \tilde{D}\} \tag{3}$$

   being $\tilde{D}$ the prediction domain, e.g. the continuous interval $\tilde{D} = [r_{min}, r_{max}]$.

   (ii) For each group member $g_i \in G$, rank commonly predicted items in I^G in descending order of the prediction values, $\tilde{r}_{il}$.

   (iii) Given $n \in \mathbb{N}$, $n \leq |I^G|$, fixed a priori, a selection of common items from all individual recommendation lists is applied. A number of ranking aggregation techniques are applicable for this prefiltering such as Borda or cumulative voting [35]. Our proposal applies single transferable voting [36] over the ordered items recommended to group members, in order to determine X.

   (iv) Once the item set X has been determined, it is necessary to filter predictions, keeping those $\tilde{r}_{il}$ accomplishing $g_i \in G$ and $x_l \in X$.

3) *Constructing preference orderings from top-n common predictions*: Let $\tilde{r}_i = [\tilde{r}_{i1}\ \tilde{r}_{i2}\ \ldots\ \tilde{r}_{in}]$ be each group member's vector of predictions over the top-n commonly predicted items. We then obtain its corresponding preference ordering $O_i = \{o_i(1), \ldots, o_i(n)\}$, being $o_i(\cdot)$ a permutation over the index set $\{1, \ldots, n\}$, such that $o_i(l) < o_i(k)\ iff\ \tilde{r}_{il} > \tilde{r}_{ik}$ [37]. Hence, items in X are ordered for each g_i according to the prediction value.

3.2 Consensus Phase

Once generated the individual predictions expressed as preference orderings over the top-n items recommended to the group members, this phase aims at obtaining a collective ordered list of predictions for the group, which reflects a high level of consensus. To do this, a CRP is applied by using an automatic consensus model for GDM problems with preference relations [38]. Hence, individual predictions of group members must be firstly expressed as preference relations so that they can be managed by the consensus model.

The following two steps are performed in this phase (see Figure 8):

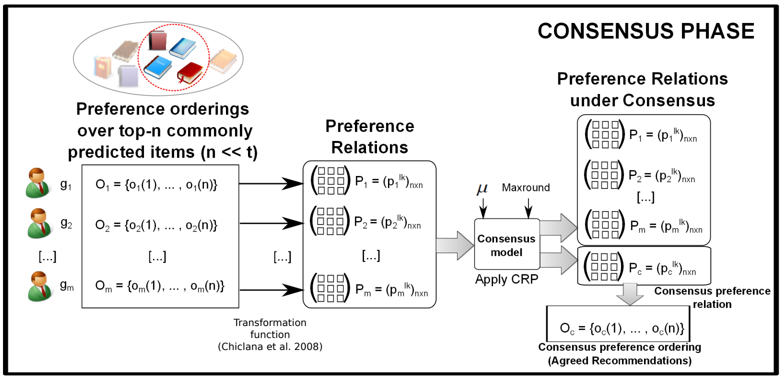

Fig. 8 Scheme of computations carried out in the Consensus phase

1) *Representing individual recommendations as preference relations*: In this phase individual recommendations are firstly expressed as preference relations in the unit interval before applying a CRP on them, since a consensus model for GDM with preference relations is utilized. Hence, a transformation function is used to express each preference ordering O_i as a preference relation P_i. Chiclana et al. in [39], developed a number of transformation functions to deal with multiple preference representation formats in decision making problems under uncertainty. In particular, they proposed the following method to construct the equivalent preference relation $P_i = (p_i^{lk})_{n\times n}$ to a preference ordering $O_i = \{o_i(1),\ldots,o_i(n)\}$:

$$p_i^{lk} = \frac{1}{2}\left(1 + \frac{o_i(k) - o_i(l)}{n-1}\right) \tag{4}$$

2) *Conducting the CRP*: A CRP is applied to bring preferences closer to each other gradually, until a high level of agreement amongst them is achieved. The consensus model utilized to apply the CRP is an automatic model in which assessments p_i^{lk} are automatically updated to bring them closer to each other (instead of

implementing a feedback mechanism for individuals [11]), and it considers the following phases (see Figure 5), which are iteratively performed at each round until consensus is achieved:

(i) *Consensus Measurement*: Preference relations of group members, $P_i = (p_i^{lk})_{n\times n}$, $i = 1,\dots,m$, are gathered and utilized to determine the level of agreement in the group.
(ii) *Consensus Control*: The computed consensus degree for the group is checked to determine whether it indicates enough agreement or not. If the consensus degree is enough, the CRP finishes having reached consensus amongst members' preferences. Otherwise, the Consensus Progress phase is applied to bring preferences closer to each other. A consensus threshold $\mu \in [0,1]$ and a parameter *Maxround* $\in \mathbb{N}$ are utilized (see Section 2.2)
(iii) *Consensus Progress*: In this phase, group members' assessments p_i^{lk} which are farthest from consensus, are identified. A set of updates on the values of such assessments are then applied, with the aim of increasing consensus in the following rounds.

Once members' preferences have been updated, another CRP round is conducted by applying again the consensus measurement phase on them. After having reached consensus, the resulting collective preference P_c, which reflects a high agreement level in the group, is used in the following step to obtain an agreed ordered list of recommendations for the group.

3) *Obtaining Agreed Recommendations List*: Applying a selection criterion [29, 40], a consensus preference ordering is determined from the consensus preference relation P_c, to rank items from the best one to the worst one. In this case, we consider the non-dominance criterion [40], that assigns each item $x_l \in X$ a non-dominance degree $ND(x_l) \in [0,1]$, as follows:

a. Construct a strict consensus preference relation $\tilde{P}_c = (\tilde{p}_c^{lk})_{n\times n}$, where:

$$\tilde{p}_c^{lk} = \begin{cases} \tilde{p}_c^{lk} - \tilde{p}_c^{kl} & \text{if } \tilde{p}_c^{lk} > \tilde{p}_c^{kl}, \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

b. Compute $ND(x_l)$ as:

$$ND(x_l) = 1 - \max_k\{\tilde{p}_i^{kl}\} \tag{6}$$

The consensus preference ordering, denoted by,

$$O_C = \{o_C(1),\dots,o_C(n)\}$$

is then obtained by ranking items in descending order of their non-dominance degree, i.e. if $o_C(l) < o_C(k)$ then $ND(x_l) \geq ND_{x_k}$.

Finally, members of the group are provided with an agreed ordered list of group recommendations (given by O_C).

4 Case Study

A case study to validate the proposal previously presented, has been developed and is deeply described in this section. In order to do this, firstly the dataset utilized in the study is introduced, followed by the details about the experiment performed and the GRSs technique to be compared with our proposal. Finally, the experiment results are shown and discussed.

4.1 Data set

In this case, the dataset used is the MovieLens data set, collected by the GroupLens Research Project[1] at the University of Minnesota. Particularly, ml-100k data set is used, and it consists of a hundred thousand ratings statements given by 943 users over 1682 movies in $\{1,2,3,4,5\}$ rating domain.

4.2 Experiment description

We have formed 30 different groups of 5 members each in order to perform the case study, because the dataset utilized does not contain information about possible groups of users.

The validation technique applied is 20% item hold-out, which has been adjusted to be used on group recommendation by selecting the 20% of items rated by each group as test set. We have performed 10 executions in order to obtain more significant results.

Finally, the evaluation measures considered for this experiment are Area Under receiver operating characteristic Curve (AUC) and precision [41]. In the case of recommender systems, the threshold variable is the size of the recommendation list.

- AUC: It is used to evaluate classifiers by computing ratio between true-positive and false-positive rate as a threshold variable is adjusted, which builds a curve. In this case the threshold is the recommendation size. Its value ranges from 0 to 1 and indicates the quality of the ranking provided. AUC values of 1, 0.5 and 0 correspond to perfect, random and inverse of perfect ranking, respectively. In a perfect sorting all relevant products are higher in the list.
- Precision: This measure shows the ratio between the true-positive rate and the number of recommended items.

For both measures, the higher the value, the better the GRS performance.

[1] http://grouplens.org/

4.3 Group Recommender Systems comparison analysis: Results

The purpose of this case study is to compare our proposal of consensus-driven GRS model with different group recommendation techniques. We have discarded the use of average aggregation as baseline technique because it does not take into account individual satisfaction, delivering recommendations that members might see as deviated towards other members' preferences.

Hence we take group recommendation with prediction aggregation minimizing member's misery as baseline technique, taking UBCF as the single user recommender system. As UBCF has a number of improvements [18], Pearson Correlation Coefficient is taken as similarity measure. Data sparsity can lead to poor values for similarity measure due to a small number of co-rated items, therefore a correction with relevance factor of 20 is applied to penalize similarities computed with less than 20 co-rated items. The prediction technique to aggregate neighborhood ratings taken is Weighted Sum. For the sake of comparability of results, the selection of top-n items over which the CRP is carried out, is applied as described in Section 3.

Our proposal uses the above mentioned UBCF recommender system but it performs the recommendation aggregation based on a CRP in which the consensus degree desired (given by the consensus threshold μ) needs to be set, therefore the consensus degree values considered are $\{0.8, 0.85, 0.9\}$.

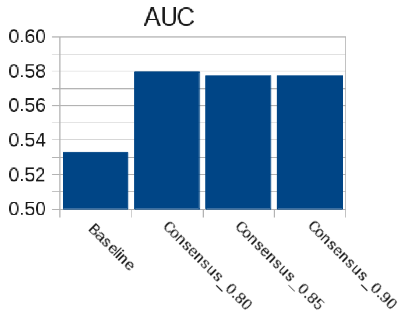

Fig. 9 Area under Receiver Operating Characteristic Curve (AUC) for evaluated configurations.

As we can see in Figure 9, the result of AUC shows that applying the CRP clearly improves the baseline results for the three consensus threshold values tested, and states that the best value of consensus degree in this case and in this dataset is 0.8, decreasing gradually as consensus degree increments.

To examine the results in a further detail, we use the precision measure over the group recommendations. As we can see in Figure 10, different configurations for the CRP module produce variations in recommendation lists delivered to groups. Comparing consensus techniques, the results show that applying a consensus degree of 0.8 benefits the recommendation results in comparison with recommendations subject to a higher consensus level.

Analyzing the results regarding the list sizes considered, we can see that for recommendation lists of size lower than 4 elements the best consensus degree value is 0.8. For recommendation lists of size 4 or more, the consensus configurations shown in the precision diagram present similar values for precision, exposing a similar behavior in these cases.

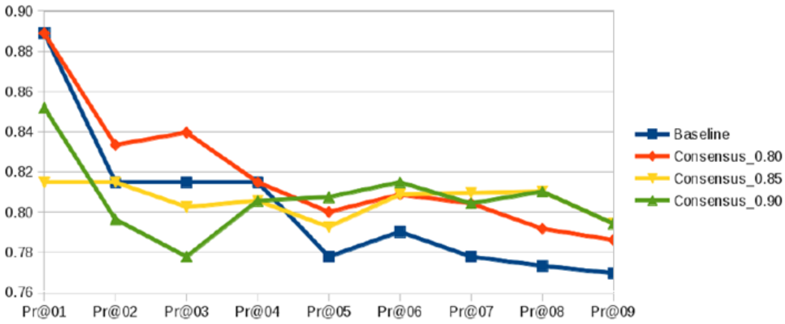

Fig. 10 Precision at certain recommendation list sizes for evaluated configurations.

5 Concluding Remarks

In this chapter, a novel technique for group recommendations based on consensus reaching processes has been proposed, in order to provided existing Group Recommender Systems with the added value of improving group members' satisfaction regarding the items recommended. The case study results showed that applying consensus reaching processes in group recommendation clearly improves the results compared with baseline group recommending techniques.

The results suggest that bringing consensus into recommendation processes is a promising future research direction, hence the exploration of other approaches developed in group decision making area might benefit recommendation.

Future works are mainly focused on developing a graphical tool which allows users to visualize their position regarding items and other users in GRSs.

Acknowledgements

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Capítulo 5

Conclusiones y Trabajos Futuros

Este capítulo cierra la memoria de investigación haciendo una revisión de las diferentes conclusiones obtenidas de las propuestas que se han realizado en la misma, así como exponiendo las líneas de investigación sobre trabajos futuros que podrían realizarse partiendo de los resultados presentados en ella. Finalmente, se indican el conjunto de publicaciones derivadas de la investigación realizada.

5.1. Conclusiones

Los avances tecnológicos derivados del auge de Internet han propiciado importantes cambios dentro del área de la TDG. Si hace unos años eran comunes los procesos de TDG en los que participaban un número reducido de expertos, hoy en día, es normal que estos procesos se realicen con grandes grupos. Este cambio de paradigma trae consigo problemas en algunos modelos de consenso que fueron diseñados para PACs clásicos, ocasionando que éstos disminuyan su rendimiento de manera considerable. Este hecho hace necesario que se tenga que llevar a cabo un proceso de adaptación.

El incremento del número de expertos participantes en este proceso también conlleva a que pueda existir una mayor diversidad de perfiles entre los expertos, lo que a priori dificulta que exista una rápida convergencia al consenso. Por otra parte, es posible que algunos de los expertos traten de aprovechar la complejidad de gestionar a un gran número de participantes, para intentar manipular la solución del PACGE mediante la adopción de comportamientos no cooperativos.

La gestión de expertos con comportamientos no cooperativos en PACGEs ha sido uno de los principales objetivos de esta tesis, habiendo desarrollado dos propuestas, que a diferencia de las existentes hasta la fecha en que fueron publicadas, permiten a los expertos que han sido penalizados, poder recuperar el peso de sus opiniones. La evaluación de estas propuestas muestra cómo estos enfoques incentivan a los usuarios a adoptar un comportamiento cooperativo pese a no haber cooperado en un principio, resultando esto en beneficio de todo el grupo, puesto que este cambio de comportamiento mejora la convergencia al consenso.

Recientemente, se han publicado algunos enfoques con la intención de dar respuesta a estas cuestiones [36], lo que hace indicar que la línea de investigación en este ámbito está todavía en sus primeros estadios.

El segundo objetivo de esta tesis ha sido la mejora de la satisfacción de los usuarios de SRG. Tradicionalmente, los SR estaban diseñados para recomendar a sus usuarios, de manera individual, un abanico de productos que pudieran ser de su interés. Sin embargo, el carácter social de algunos productos, hizo necesario la aparición de SR que pudiesen recomendar productos a grupos de usuarios. Principalmente, los esfuerzos se han centrado en minimizar la disatisfacción de los usuarios, no obstante aplicando estas técnicas podemos encontrar casos en los que la satisfacción de los usuarios disminuye de manera drástica. Por tanto, resulta necesario el desarrollo de modelos que permitan maximizar la satisfacción de los usuarios de SRG.

En esta tesis se ha abordado este objetivo mediante una propuesta en la que se obtienen recomendaciones a grupo consensuadas. Para ello, se ha añadido una fase de consenso a los SRG con agregación de recomendaciones, de manera que se llegue a un alto nivel de acuerdo antes de realizar la recomendación de grupo. La evaluación de esta propuesta arroja resultados positivos, mejorando notablemente a la referencia, y estableciéndose aquí un punto de partida para futuros trabajos.

5.2. Trabajos Futuros

En base a la investigación realizada, proponemos a continuación las líneas de trabajos futuros. Estos trabajos están enfocados tanto a la ampliación de las propuestas presentadas como a señalar algunas cuestiones todavía sin resolver que han surgido durante el desarrollo del doctorado. Concretamente estas cuestiones son relativas a los PACs en general y a los PACGEs en particular.

5.2.1. Mejora en la detección y tratamiento de nuevos comportamientos

Actualmente, nuestras propuestas solamente permiten la gestión de comportamientos analizando las recomendaciones que se les hacen a los expertos y posteriormente observando la obediencia o no a dichas recomendaciones. Esto reduce la aplicación de estos trabajos exclusivamente a modelos de consenso con recomendaciones. Una posible línea de trabajo sería llevar a cabo la gestión de comportamientos sin hacer uso de las recomendaciones (por ejemplo, utilizando la evolución de la distancia de las opiniones de los expertos a lo largo del proceso de negociación). Esto haría posible la aplicación de estas técnicas a todos los modelos de consenso tanto a los que tienen recomendaciones como a aquellos que no las tienen.

Por otra parte, sería bastante interesante afinar en la detección de los distintos tipos de comportamientos presentes en los PACs. En los trabajos que conforman esta tesis aparecen los tipos de comportamiento: cooperativo, no cooperativo, híbrido, mixto, manipulador e indefinido. Sin embargo, en el futuro, sería conveniente realizar un estudio en profundidad de las estrategias de negociación y los patrones de actuación empleados normalmente por los expertos participantes en PACs, para así poder detectar y gestionar sus comportamientos con una mayor precisión.

5.2.2. Consenso ético

La RAE define la *ética* como "*el conjunto de normas morales que rigen la conducta de la persona en cualquier ámbito de la vida*". Del mismo modo, encontramos la definición de *moral* como "*perteneciente o relativo a las acciones de las personas, desde el punto de vista de su obrar en relación con el bien o el mal y en función de su vida individual y, sobre todo, colectiva*". La ética ha sido uno de los temas filosóficos más comentados a lo largo de la historia de la humanidad, desde la antigua Grecia hasta nuestros días. En la actualidad, el crecimiento de la tecnología en general y de la inteligencia artificial en particular, han hecho que existan multitud de sistemas autónomos en casi todos los ámbitos de nuestras vidas. Este hecho a ocasionado que numerosas personalidades de este ámbito del conocimiento consideren de vital importancia la aplicación de la ética en los sistemas que emplean inteligencia artificial [12, 73, 117]. Un ejemplo puede ser el coche autónomo de Google[1]. Imagine que el coche detecta una avería en los frenos y sólo puede pararse girando para la izquierda, dónde hay un muro de piedra y el choque podría causar la muerte de las personas que van en el coche, o para la derecha, dónde hay una familia paseando con sus hijos y el atropello puede causar la muerte de todos sus miembros, pero minimizar el daño de los ocupantes del vehículo[2]. ¿Qué acción debería llevar a cabo el coche? Dar respuesta a esta pregunta es un debate abierto en el que se está trabajando en la actualidad.

Uno de los principales objetivos de esta tesis ha sido la gestión de expertos con comportamientos no cooperativos ya que este tipo de actitudes pueden repercutir negativamente en la solución del PACGE. En las distintas propuestas se asume que los expertos en cuestión rompen el contrato de colaboración para intentar desviar la solución del PACGE hacia sus intereses personales. Sin embargo, se pueden dar casos en los que los participantes no siguen las recomendaciones debido a razones éticas. Por ejemplo, imagine un grupo de trabajadores que deciden llevar a cabo un PACGE para elegir el restaurante donde celebrar una comida de empresa. Si a una

[1] https://www.google.com/selfdrivingcar/

[2] http://moralmachine.mit.edu

persona vegetariana se le recomienda votar a favor de un restaurante que sólo sirve platos con carne o a una persona alérgica al marisco se le recomienda que cambie sus preferencias para ir a una marisquería, el comportamiento no cooperativo resulta la única opción. Sin embargo, es posible que estos participantes quieran cooperar para llegar a un consenso, pero actualmente no tienen otra opción que les permita no ir en contra de sus principios o salud.

Estos casos empeoran más aún con las distintas propuestas para gestionar comportamientos no cooperativos, ya que éstas no distinguen entre los expertos que se comportan así por *ética* y aquellos que lo hacen para manipular el PACGE. De este modo, el común denominador de todos estos enfoques radica en la reducción del peso de la opinión del experto dentro del PACGE. Así, los participantes anteriores tienen dos opciones: (I) seguir las recomendaciones (comportamiento cooperativo) e ir en contra de sus principios/salud para que su opinión cuente a la hora de calcular el grado de consenso, o (II) ser coherentes y rechazar las recomendaciones (comportamiento no cooperativo) aunque su opinión pierda peso o apenas cuente en el cálculo del grado de consenso. Ante este tipo de vicisitudes podemos encontrar un paralelismo con el primer ejemplo, en el que la aplicación de la ética es absolutamente necesaria.

Dada la trascendencia de estos casos, resulta primordial profundizar en esta línea de investigación.

5.2.3. Desarrollo de métricas para comparar modelos de consenso

Tal y como se ha recogido en el capítulo 2, en la literatura podemos encontrar numerosos modelos de consenso [74, 96, 129], que emplean distintas metodologías y operadores. A pesar de ello, a día de hoy es difícil saber qué modelo es más conveniente en cada situación, ya que apenas existen métricas con las que se puedan comparar los distintos modelos. Por ejemplo, sería útil conocer:

1. ¿Qué modelo converge más rápido al consenso?

2. ¿Cuál consigue un nivel de acuerdo más alto en el menor número de rondas?

3. ¿En qué modelo se produce un consenso con las opiniones de los expertos más cohesionadas?

4. ¿Qué modelo funciona mejor partiendo de unas opiniones muy alejadas?

Esto resulta esencial ya que sabiendo el modelo que se ajusta mejor a nuestras necesidades se podrá optimizar el proceso de TD, lo cual tendrá un efecto positivo en los expertos de cara a futuros procesos de TD.

Debido a la importancia y a la necesidad de dar respuesta a estas cuestiones, se hace imprescindible trabajar a corto plazo en esta línea de investigación.

5.3. Publicaciones Derivadas

Para finalizar, a continuación se muestra la lista de publicaciones derivadas de los resultados presentados en esta memoria:

Publicaciones:

- En Revistas Internacionales Indexadas:
  - F. J. Quesada, I. Palomares and L. Martínez. *Managing experts behavior in large-scale consensus reaching processes with uninorm aggregation operators.* Applied Soft Computing, vol. 35, pp. 873 - 887, 2015.
  - J. Castro, F. J. Quesada, I. Palomares and L. Martínez. *A Consensus-Driven Group Recommender System.* International Journal of Intelligent Systems, vol. 30, no. 8, pp. 887–906, 2015.
- Como Capítulos de Libro Indexados:
  - F. J. Quesada, I. Palomares and L. Martínez. *Using Computing with Words for Managing Non-cooperative Behaviors in Large Scale Group*

Decision Making. Granular Computing and Decision-Making, vol. 10: Springer International Publishing, pp. 97-121, 2015.

- F. Moya, F. J. Quesada, J. Castro, R. M. Rodríguez, I. Palomares and L. Martínez. *Improving group recommendations with consensus reaching processes*. Soft computing and Hybrid Systems for Knowledge Discovery and Decision-making, Atlantis Computational Intelligence Series (ACIS), In Press.

- En Congresos Internacionales:

  - F. J. Quesada, I. Palomares and L. Martínez. *A Multi-agent System for Performing Consensus Processes in Heritage problems*. University of Jaén, Baeza (Spain), I International Meeting of Young Researchers on Heritage - PatrimoniUN10, November 19th-21st, 2014.
  - I. Palomares, F. J. Quesada and L. Martínez. *An Approach based on Computing with Words to Manage Experts Behavior in Consensus Reaching Processes with Large Groups*. IEEE Computational Intelligence Society, Beijing (China), 2014 International Conference on Fuzzy Systems (FUZZ-IEEE), July 6-11th, 2014.

- En Congresos Nacionales:

  - F. J. Quesada, I. Palomares and L. Martínez, *Gestión de Expertos con Comportamientos no Cooperativos en Procesos de Consenso en Grandes Grupos*. Zaragoza (Spain), Universidad de Zaragoza, ESTYLF, pp. 139-144, February 5-7th, 2014.
  - J. Castro, F. J. Quesada and L. Martínez. *Uso de procesos de alcance de consenso para mejorar la recomendación a grupos*. XVI Conferencia de la Asociación Española para la Inteligencia Artificial (CAEPIA'15), Albacete (Spain), November 9-12th, 2015.

Apéndice A

English Summary

This appendix contains an English summary of the thesis entitled: *Procesos de Alcance de Consenso ante Nuevos Retos en Toma de Decisión en Grupo: Grandes Grupos y Recomendación* (*Consensus Reaching Processes faced with New Challenges in Group Decision Making: Large Groups and Recommendation*) as partial fulfilment for obtaining the International PhD.

First of all, the motivation for carrying out this research and the proposed objectives are outlined. Secondly, it is highlighted the structure of the whole thesis. After that, the proposals and results derived from the research are described. The appendix finalises with concluding remarks, future work and the publications related to this research.

A.1. Motivation

Decision Making (DM) is a usual process in human beings' daily life, which lies in choosing the best option among a set of possible alternatives in a specific environment [111]. In particular, most of real DM processes are carried out in uncertain environments [83]. This makes necessary the use of approaches and tools that allow the adaptation to these environments, standing out Fuzzy Logic [148]. Apart from that, the frequent need of multiple viewpoints in the DM process has entailed the creation of Group Decision Making (GDM) problems, where several experts have to reach a common solution for the decision problem [17].

Traditionally, GDM problems under uncertainty have been solved by a selection process of alternatives [47, 116] in which the best alternative was selected. However, the fact of achieving a solution without carrying out a previous negotiation process between the participants, might lead to not having an acceptable level of agreement on the solution. This fact may entail the solution not being accepted by all experts [118]. Consensus Reaching Processes (CRPs) were developed in order to tackle this problem. In CRPs every participant expert promises to fulfil a collaboration contract [87], with the aim of collaborating and reaching a high level of agreement prior to making a decision. To do so, it is necessary to carry out a discussion process between experts, in which they modify their preferences according to the CRP's needs. This process is normally guided under the supervision of a human moderator [17, 54, 118].

Throughout last decades, there have been a lot the contributions in the CRPs' area, standing out for their relevancy the consensus models included in [33, 53, 129]. In a similar way, there are some approaches with the aim of integrating CRPs in decision support systems [3, 101, 102, 154].

The concept of *consensus* has evolved along the time. Initially, it was considered in a classical view as a total agreement between all experts in all of their preferences. In practice, usually, this agreement was difficult to achieve. For this reason, the classical view was evolving until more flexible approaches [59, 64], in which fuzzy consensus measures were used to evaluate different partial agreement levels. The idea of "*soft consensus*" stands out among these approaches. It was proposed by J. Kacpryk and is based on the fuzzy majority [59]. Hence, there is consensus when "the majority of participant experts agreed in their opinions over the relevant alternatives". A key aspect of the consensus models under these approaches is the choice of appropriate consensus measures to evaluate the achieved global agreement level [54]. These consensus measures are usually based on the use of similarity measures between experts and aggregation operators [7, 18, 157].

Classically, GDM problems were resolved by a limited number of experts, who carried out the DM process gathered in the same place. However, nowadays this has considerably changed thanks to the peak of the Internet and the development of the mobile technology. Thus, it is currently possible to carry out DM processes without the need of participants to be physically present in the same place. The elimination of the presence barrier and the development of social media, have opened the scope, offering a wide variety of new scenarios in the field of DM, standing out the possibility of considerably increase the number of participants in these processes. However, these advances have also entailed the appearance of new challenges in Group Decision Making with Large Groups (GDMLG), also denoted as Large Scale Group Decision Making (LSGDM) [21, 22, 159], where the number of participants is too much larger than in the classical view, in which it was only considered a limited number of experts [13, 17–19, 21].

In [24] it is proposed that we can consider a LSGDM problem when the number of experts is superior to 20, although the idea is that there could be hundreds or thousands of experts. Examples of these kinds of problems might be found in multinational companies, electronic democracy, [66], social media [128], crisis management [65] and emergency scenarios [138] among others. Liu et al. classified in [82] the principal lines of research related to LSGDM problems, standing out the following categories: (i) LSGDM based on cluster methods, (ii) LSGDM applied to CRPs, (iii) LSGDM methods and (iv) LSGDM support systems.

In these problems, the increase of the number of participant experts adds complexity to control and supervise the DM process, in contrast to classic GDM problems, where this management is easier. Labella et al. conducted in [75] a comparison between classic consensus models for few experts, applying them to LSGDM problems, using AFRYCA 2.0 [74]. They conclude that in general, the performance of these models is directly affected when the CRP is applied to problems with a large number of experts. Another important aspect to keep in mind is that the larger number of experts is, the more probable they have a great diversity regarding to experts' profiles and objectives is. This fact, makes more difficult controlling and supervising the CRP, which might be used by some experts for trying to deviate the solution of the CRP toward their own interests, not fulfilling the collaboration contract. For this reason, it appears the need of managing the behaviours of these experts, in order to avoid the manipulation of the CRPs. Currently, there are few proposals that consider the management of these kinds of behaviour derived from not fulfilling the collaboration contract. The first approach, and also the most drastic one, was formulated by R. Yager in [142], proposing to penalise all experts who deny cooperating, eliminating them from the DM process. Palomares et al. proposed applying penalisation to experts who do not obey the recommendations given by the moderator of the CRP, reducing the weight of their opinions before to computing the consensus degree [98]. The main disadvantage of this proposal lies in the fact that the experts who have been penalised, cannot recover the weight of their opinions, even though they adopt a cooperative behaviour in future rounds. This situation might entail that these experts do not feel satisfied with the solution despite having correct their behaviours, or that they reconsider future cooperation if they do not have any benefit in return.

The satisfaction of experts or users with respect to a particular solution, might be drastically affected when large groups are involved. An example can be found in Group Recommender Systems (GRS).

GRS in contrast to Recommender Systems (RS) should consider the preferences of all group's members in order to obtain recommendations which satisfy all the group and not only a particular individual. Currently, GRS are based on aggregation models which average satisfaction and minimise dissatisfaction of group's members, however, the highest expectation of the users of a GRS is the maximisation of satisfaction as group.

In the face of the difficulties previously considered about CRP in LSGDM and GRS, the main motivation of our research has been focused on overcoming the following challenges:

- Currently, managing non-cooperative behaviours in CRPs with large groups is not able to generate satisfactory solutions due to its personalised view in users' attitude changes cannot be reflected in the result.

- Current GRS only consider the individual recommendations of group's members to compute the group recommendation by simple aggregation processes which do not have the aim of maximising the satisfaction of the group.

A.2. Objectives

Keeping in mind the motivation and the previous considerations, the general purpose of this research is concentrated on improving CRPs in LSGDM, paying special attention to the management of non-cooperative behaviours and the application of CRPs in GRS in order to enhance the satisfaction of group's members with respect to the obtained recommendations.

From this general purpose we formulate the following objectives which are specific to our research:

- *Developing models for managing non-cooperative behaviours of experts in Large Scale Consensus Reaching Processes (LSCRPs) by weighting their opinions with uninorms and hyper-similarity operators, allowing that opinions of the penalised experts can recover importance if they correct their behaviours and start cooperating again with the group.*

- *Improving satisfaction of group's members in GRS by generating recommendations based on CRPs, obtaining recommendations in the group with high agreement degree.*

A.3. Structure

In order to fulfil the objectives proposed in the previous section, and according to article 23.3 of the current regulations for PhD studies in the University of Jaén, corresponding to the programme establishes in RD. 99/2011, this thesis is presented as a set of papers published by the doctoral student, which contain the research carried out during the PhD studies.

These publications constitute the core of the thesis. They are: two scientific papers published in International Journals indexed by the database *JCR* (Journal Citation Reports) and produced by *ISI* (Institute for Scientific Information), as well as, a published book chapter and a book chapter in press. Thus, the thesis consists of four publications, two articles published in high standing journals and two book chapters.

Below, it is succinctly described the structure of the thesis:

- **Chapter 1**: Presents a general introduction of the research problems that have been addressed in the thesis, as well as the motivation and the objectives pursued in the research that has been carried out.
- **Chapter 2**: Reviews all theoretical concepts used in the formulated proposals, having in mind the previous works in which the research of this thesis is supported. First of all, the problems of DM and GDM are addressed, as they are the basis to go in depth in CRPs in LSGDM and in GRS, which are addressed at the end of the chapter.
- **Chapter 3**: Summarises the proposals over which the thesis is substantiated, highlighting the main results obtained in our research. Each proposal is briefly described, highlighting its main features.
- **Chapter 4**: Constitutes the core of the thesis, containing the publications obtained as a result of the research that has been carried out. For each publication, it is indicated its quality index.

- **Chapter 5**: Expounds the concluding remarks and some possible future works.

- **Appendix A**: Presents a summary in English of the thesis, introducing the motivation and the objectives, a summary of the proposals and the results, and the concluding remarks and further works. The appendix finishes with the publications fruit of our research.

- **Appendix B**: Describes the main concepts of the Computing with Words (CW) paradigm. This paradigm is used in the proposals set out in section A.4.

- **Appendix C**: Introduces the basic notions of the aggregation operators, *uninorms*, which are used in the proposal described in section A.4.1.1.

- **Appendix D**: Expounds the properties of the similarity measure, which is used in the proposal presented in section A.4.1.2.

A.4. Discussion of Results

This section summarises the proposals that have been carried out to satisfy the research goals introduced in sections A.1 and A.2. Each approach is briefly described, focussing on the analysis of the obtained results. These approaches are presented in depth in chapter 4.

The proposals summarised in these section are:

1. *Managing Experts with non-cooperative Behaviours in LSCRPs*

   a) *Approach based on Uninorms.*

   b) *Approach based on the hyper-similarity measure.*

2. *GRS based on consensus.*

A.4.1. Managing Experts with non-cooperative Behaviours in Large-scale Consensus Reaching Processes

The first goal of the thesis has been the improvement of the management of experts with non-cooperative behaviours in LSCRPs. In previous chapters, we have seen how all the proposals for managing experts with this kind of behaviours converge in the idea of penalising these experts. This task is carried out by reducing the weight of their opinions without allowing them to recover the weight that they have lost [98]. In some cases, this process entails the elimination of these experts from the CRP [142]. Our research focuses on managing the behaviours developed by experts along the CRP, applying a penalisation to those who disobey the recommendations, but allowing them to recover the weight of their opinions if they amend their behaviour and cooperate. To do so, it is proposed the addition of a new phase to the LSCRP. This phase is called "*management of experts' behaviours*" and is allocated between the "*gathering preferences*" phase and the "*determine degree of consensus*" phase, as it is depicted in figure A.1.

It is necessary to clarify that the phase "*management of experts' behaviours*" takes place from the second round, once experts have changed their preferences after knowing the feedback provided by the moderator. This phase is composed of two sub-phases, as we can appreciate in figure A.2:

- *Detecting behaviours.* The kind of behaviour is determined based on whether experts have changed their preferences attending to the feedback or not.
- *Managing behaviours in LSCRP by weighting experts.* The weight of experts' opinions is updated prior to carry out the phase in which the consensus is computed. The update considers both the kind of behaviour of experts and the weight of their current opinions.

Below, we summarise the two proposals that we have developed in our research for managing experts with non-cooperative behaviours in LSCRPs. These

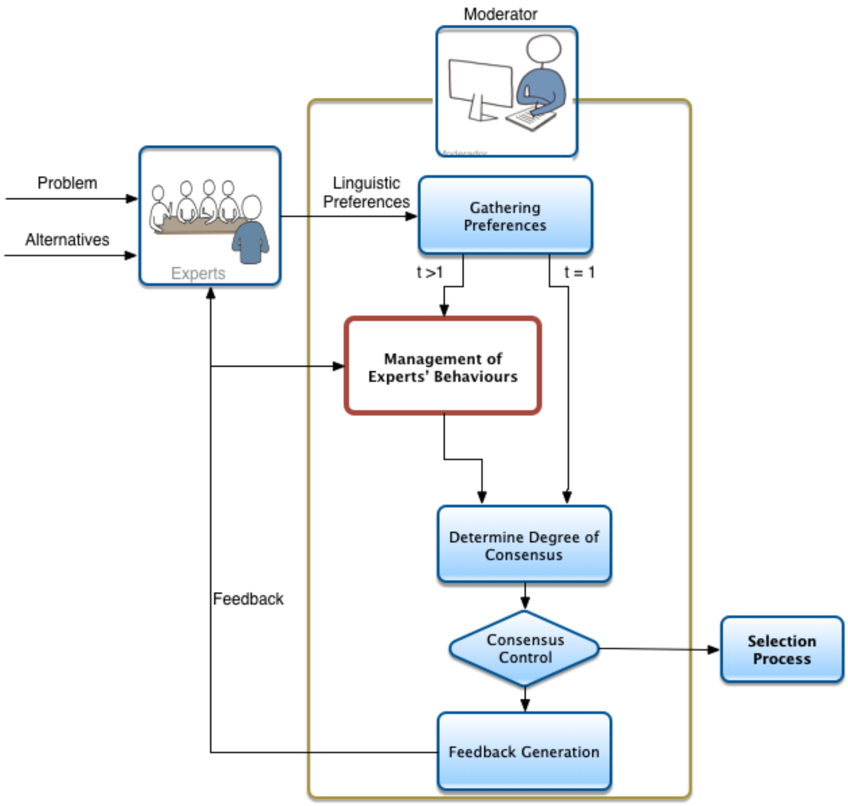

Figure A.1: General scheme of managing experts' behaviours in large scale consensus reaching processes with recommendations.

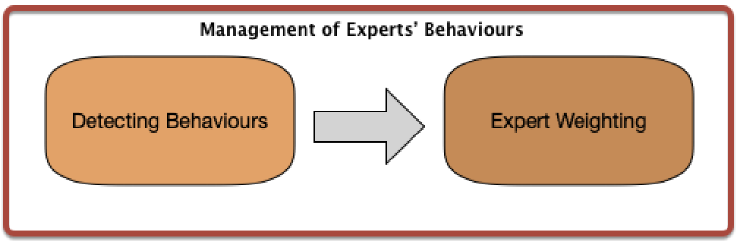

Figure A.2: Management of experts' behaviour sub-phases.

approaches are explained in depth in chapter 4. The first proposal uses uninorm operators [146], detailed in appendix C, whereas the second uses the hyper-similarity measure [145], described in appendix D. Both proposals take advantage of the CW paradigm, detailed in appendix B. In particular, this paradigm is used to define the linguistic term *cooperative*, whose membership function is used to determine the kind of behaviour adopted by each expert.

A.4.1.1. Proposal based on Uninorms

Uninorms [146] are operators used to carry out aggregation processes. Among their features, described in detail in appendix C, it stands out the *reinforcement* [109], which allows reinforcing tendencies regarding the values of a variable throughout the time. Uninorms operators have *full reinforcement*, as they benefit from both the *downward reinforcement* of the t-norms and the *upward reinforcement* of the t-conorms. This characteristic is used in the proposal in order to reinforce experts' behaviours in the way that depending on the tendency of the behaviour of the expert (cooperative or non-cooperative), his/her opinion will gain or lose importance respectively.

The proposal has the following phases, as it is depicted in figure A.3:

- *Measuring cooperativeness.* Fuzzy set theory and CW are used in order to evaluate the cooperation degree of each expert, $e_i \in E$, in the current round, $t \in \mathbb{N}$, of the LSCRP. This phase has two sub-phases:

    1. *Computing cooperation coefficient.* The cooperation coefficient evaluates an expert's behaviour on the amount of the feedback received and the amount of assessments that he/she modified according to such feedback. The behaviour is more cooperative the more the expert obey the feedback and modify his/her preferences according to it, or the less the number of recommendations is, because this indicates that the opinion of the expert is close to the consensus.

    Definition 1 *Let $\#ADV_i^t$ be the total number of assessments that have been advised to modify and $\#ACP_i^t$ be the amount of assessments that the expert e_i accepted to modify according to the feedback received at round t, the cooperation coefficient of e_i at round t, $CC_i^t \in [0,1]$, is defined as:*

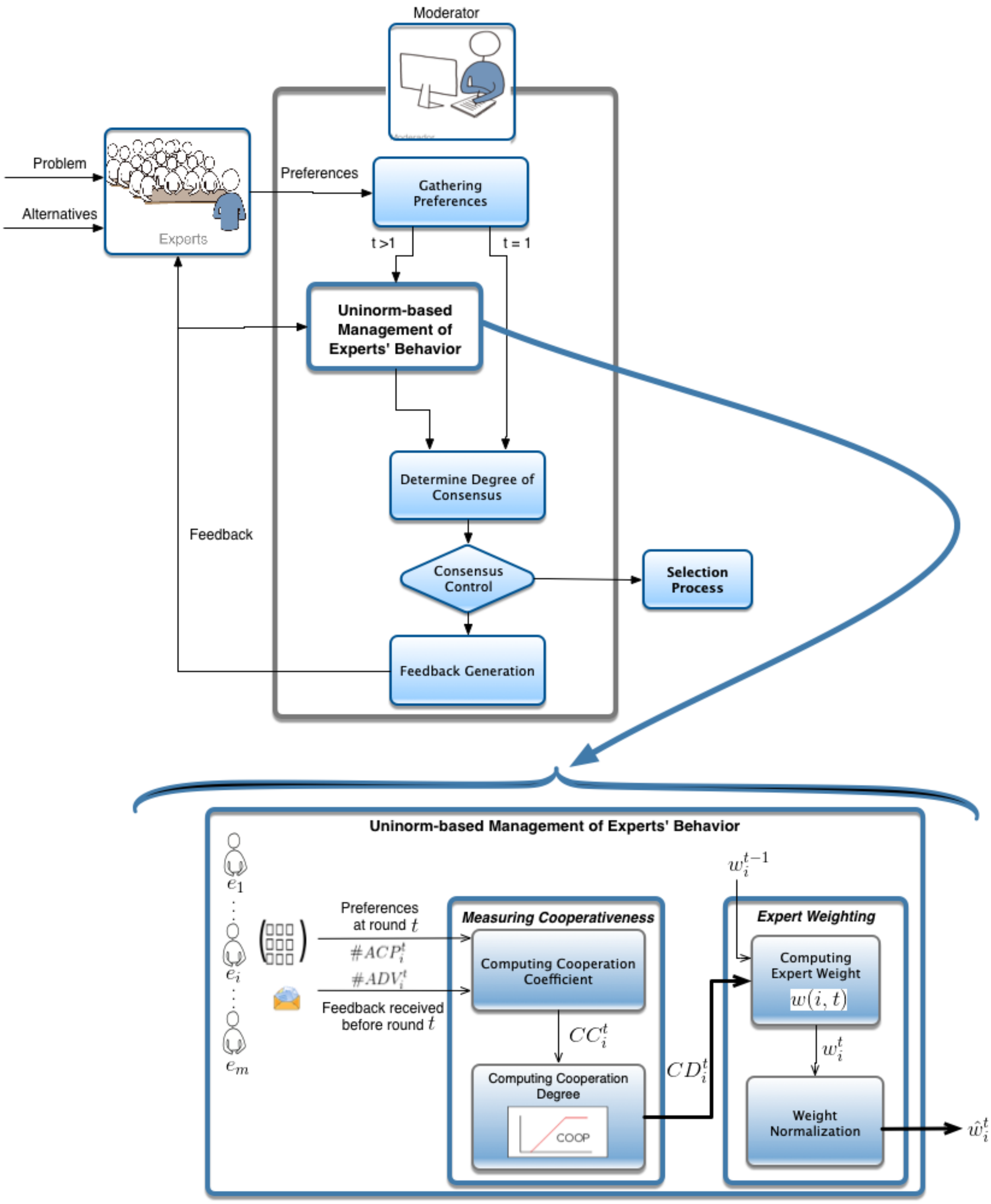

Figure A.3: Scheme of the uninorm-based method for managing experts' behaviours in large scale consensus reaching processes.

$$CC_i^t = \begin{cases} 1 & \text{if } \#ADV_i^t = 0, \\ \eta \frac{\#ACP_i^t}{\#ADV_i^t} + (1-\eta)\left(1 - \frac{\#ADV_i^t - \#ACP_i^t}{n(n-1)}\right) & \text{otherwise.} \end{cases} \tag{A.1}$$

being $n(n-1)$ the total number of assessments in P_i. The parameter $\eta \in [0,1]$ is utilised to control the penalising degree for high values of $\#ADV_i^t$, in the way that, the higher the value of η is, the higher penalisation will be applied when $\#ADV_i^t$ is high.

2. *Computing cooperation degree.* Based on the cooperation coefficient in this phase it is computed the degree of cooperation of each expert at the current round. This degree shows to what extent the expert satisfies the notion of "*cooperativeness*". To do so, the CW paradigm is used in order to define the linguistic term *cooperative.*

Definition 2 *Let "cooperative" be a linguistic term, whose semantics are given by the fuzzy set $COOP \in [0,1]$, with the following non-decreasing membership function:*

$$\mu_{COOP}(y) = \begin{cases} 0 & \text{if } y < \alpha, \\ \frac{y-\alpha}{\beta-\alpha} & \text{if } \alpha \leq y < \beta, \\ 1 & \text{if } y \geq \beta. \end{cases} \tag{A.2}$$

with $\alpha, \beta, y \in [0,1], \alpha < \beta$. The cooperation degree of e_i at round t, denoted by, CD_i^t, correspond to the membership degree of CC_i^t to the fuzzy set $COOP$. For example, $CD_i^t = \mu_{COOP}(CC_i^t) \in [0,1]$. Figure A.4 shows the membership function of the term "cooperative" along the CRP.

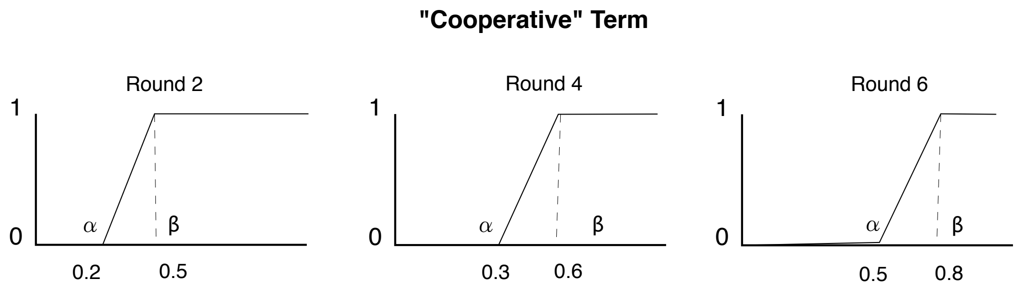

Figure A.4: Evolution of the fuzzy membership function associated to the linguistic term "*cooperative*" along the consensus reaching process.

- *Expert weighting.* Once the cooperation degrees of experts have been obtained, the following step is assigning to each expert an importance weight for the following round, according to his/her cooperation degree. This phase has the following sub-phases:

  1. *Computing experts' weights.* Uninorm aggregation operators are used to compute importance weights of experts at each round. The main reason of utilising these operators is for taking advantage of their full reinforcement property. This property allows us to consider the accumulated behaviour along the previous round and the behaviour of the current round. Thus, the uninorm operator will reinforce upwards or downwards if the expert cooperates or not.

     Definition 3 *The function $w(i,t)$ return the weight of the opinion of expert e_i at round t, denotes by $w_i^t \in [0,1]$ and computed as follows:*

     $$w_i^t = w(i,t) = \begin{cases} g & \text{if } t = 1, \\ U(CD_i^t, w_i^{t-1}) & \text{if } t > 1. \end{cases} \tag{A.3}$$

     being $CD_i^t, w_i^{t-1} \in [0,1]$, U a uninorm operator [19, 109] and $g \in]0,1[$ its neutral element, according to which an input value above g is viewed as a good behaviour in the aggregation, and vice versa.

     Figure A.5 shows an example of the kind of reinforcements. In image *(a)* we can see how the values are over the neutral element, g, so the weight is reinforced upward. On the contrary, in graphic *(b)* we can see how when the operator's value is below g, it is reinforced downward. Nonetheless, after two rounds in which the value is over g, the reinforcement is upward.

  2. *Weight normalization.* Using the weight of each expert computed in the previous phase, in this phase it is carried out the normalisation of these weights. This fact is essential because it allows penalised experts to recover importance of their opinions. To do so, it is used the following formula:

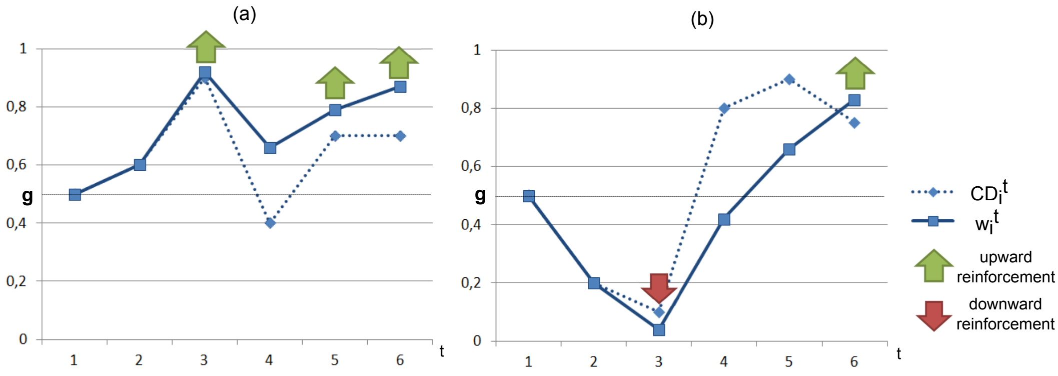

Figure A.5: Example of upward and downward reinforcements.

$$\hat{w}_i^t = \frac{w_i^t}{\sum_{i=1}^{m} w_i^t} \tag{A.4}$$

with $\hat{w}_i^t \in [0,1]$ y $\sum_i \hat{w}_i^t = 1$. Once the weights have been normalised, they are taken into account in the current round both to compute the collective preference P_c by aggregating experts' preferences, and to compute the consensus degree.

The normalised weights will be used in each round to compute the consensus degree, in the way that the opinions of experts with cooperative behaviours will have much importance than those that do not cooperate.

Evaluation

Two experiments were carried out in order to evaluate this proposal:

- The first one consisted in the comparison between (I) a consensus model that does not penalise experts with non-cooperative behaviours, (II) a model that penalised experts with this kind of behaviour, but does not allow them to recover the weight that they have lost, and (III) our proposal based on uninorms.
- The second evaluation was focussed on the update of experts' weights along the LSCRP. In this case, there were selected 3 kind of experts with the

following behavioural profiles: (I) a kind of expert that always has a cooperative behaviour, (II) a kind of expert that always adopts a non-cooperative behaviour, and (III) a kind of expert that has a non-cooperative behaviour at the firsts rounds of the LSCRP, but afterwards he/she adopts a cooperative behaviour.

In order to carry out these evaluations, it was executed an illustrative example of a LSCRP with 30 experts. These experts had different behavioural profiles that were defined in advance.

The analysis between consensus models points out our approach as the best one regarding the convergence to consensus. Thus, the consensus degree reaches the consensus threshold one round before the other models.

Regarding the evolution of the weights of experts' opinions, we can see how the expert that does not cooperate at the beginning and decides cooperating afterwards, he/she recovers the weight of his/her opinion, reaching the same level of importance that the experts that always cooperate.

Visually analysing figure A.6, we can see how the opinion of the expert that always cooperates (expert 22), always gets closer to the collective opinion. The opinion of the expert that never cooperates (expert 23), gets away from the collective preference more and more at each round. In the case, in which the expert does not cooperate at the beginning and cooperates afterwards (expert 24), we can see how at the beginning his/her opinion gets away from the collective preference, but when he/she starts cooperating, his/her opinion starts getting closer.

This proposal is described in detail in article (see Section 4.1):

F. J. Quesada, I. Palomares and L. Martínez. *Managing experts behavior in large-scale consensus reaching processes with uninorm aggregation operators.* Applied Soft Computing, vol. 35, pp. 873 - 887, 2015.

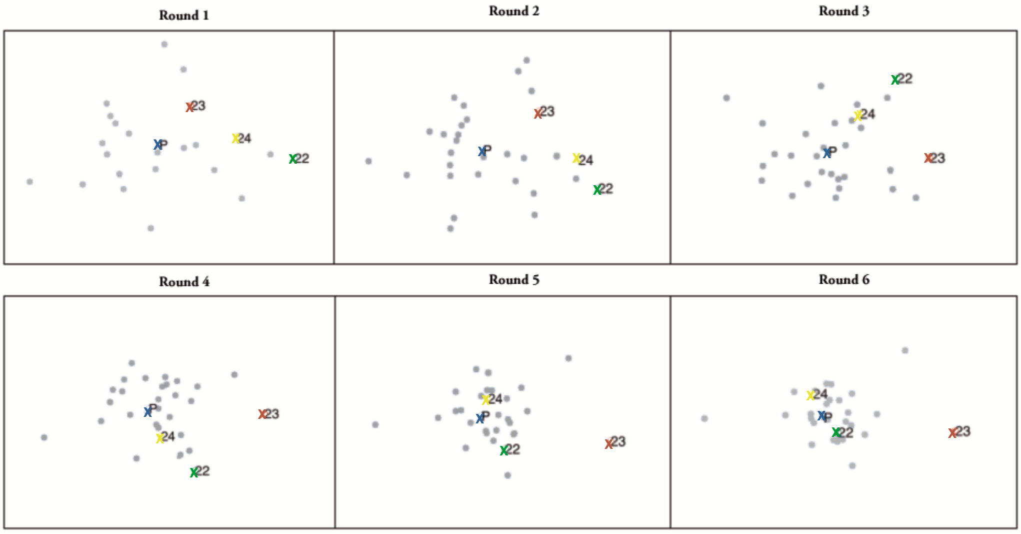

Figure A.6: Evolution of experts' opinions along the consensus reaching process.

A.4.1.2. Proposal based on using hyper-similarity measure

The main characteristic of this proposal is the use of the hyper-similarity measure, described in appendix D. This approach takes advantage of the amplification property. In particular, it is used to amplify extreme values when aggregating attribute values in order to amplify experts' behaviours considering the trajectory of each expert's behaviour. In the way that the behaviour of previous rounds (behavioural trajectory) could be useful to compute the weights of experts' opinions. As opposed to the previous proposal in which uninorms were used, in this proposal the trajectory of expert's behaviour along all the round of the LSCRP, has a bigger influence to compute the weight of the opinion of each expert. Figure A.7 depicts the scheme of the LSCRP with the phase of *non-cooperative behaviour management* using the hyper-similarity measure. We can see how the scheme is similar to the previous proposal (see section A.4.1.1), except for the sub-phase of *expert weighting*. The particularities of this proposal are described below:

- *Expert weighting.*

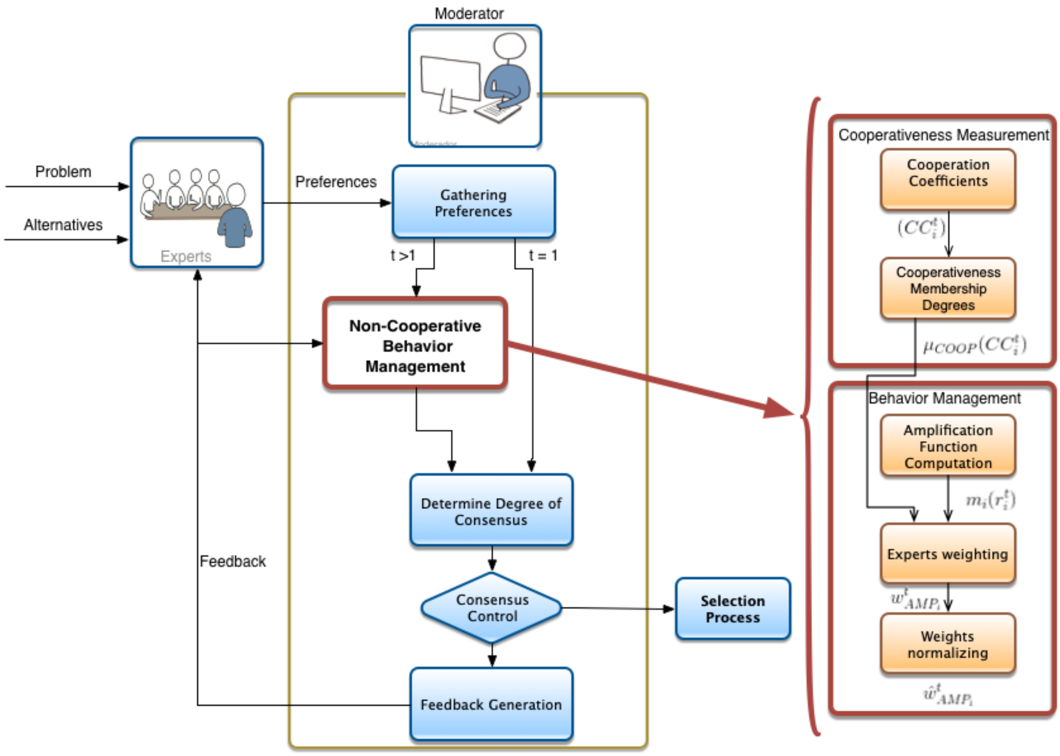

Figure A.7: Scheme of the hyper-similarity based method for managing experts' behaviours in large scale consensus reaching processes.

1. *Computing amplification function.* The amplification function will be used to reinforce the weight of each expert's opinion considering the closeness of their preferences, P_i, with respect to the opinion of the group, P_c. Thus, if the opinion of an expert remains close to the opinion of the group along the LSCRP, this indicates that the expert has had a cooperative behaviour along the process. However, if an expert's opinion moves further from the group opinion at some stage, it means that this expert is having a non-cooperative behaviour. Based on this idea, a cooperation rate is computes from the received recommendations in a particular round.

Definition 4 *Let r_i^t be the rate of e_i's assessments p_i^{lk} which are close to consensus at round t. $r_i^t \in [0, 1]$ is defined as follows:*

$$r_i^t = 1 - \frac{\sharp ADV_i^t}{n(n-1)} \tag{A.5}$$

The lower the number of received recommendations of change, ADV_i^t, is, the closer the opinion of the expert to the consensus, P_c, is and vice versa.

Definition 5 *Once r_i^t is computed and based on Yager's ideas [145], an amplification function $m(r_i^t)$ is defined as:*

$$m(r_i^t) = r_i^t + 1, m(r_i^t) \in [1, 2] \tag{A.6}$$

2. *Computing experts' weights.* To compute the weights of experts' opinions there are used both the amplification value, $m(r_i^t)$, and the value of the similarity measure $\mu_{COOP}(CC_i^t)$ computed in the phase in which the *cooperativeness* is computed[1]. Thus, the weights of experts' opinions will be computed by the following formula:

$$w_{AMP_i}^t = \frac{m(r_i^t)\mu_{COOP}(CC_i^t)}{2}, \tag{A.8}$$

   it is necessary to divide by 2, to have the result of $w_{AMP_i}^t$ within the unit interval, being $w_{AMP_i}^t \in [0, 1]$.

3. *Weight normalization.* We re-normalised experts' weights to allow the recovery of opinions importances:

$$\hat{w}_{AMP_i}^t = \frac{w_{AMP_i}^t}{\sum_1^n w_{AMP_i}^t} \tag{A.9}$$

   this weight will be used to compute the consensus degree.

[1]The phase for computing the cooperativeness measure is similar to the one described in section A.4.1.1, having in common the same linguistic term *cooperative*, but using the following formula:

$$CC_i^t = \begin{cases} 1 & \text{if } \#ADV_i^t = 0, \\ \frac{\#ACP_i^t}{\#ADV_i^t} & \text{otherwise.} \end{cases} \tag{A.7}$$

Evaluation

The evaluation of this proposal has as objective observing the evolution of the weights of experts opinions along the LSCRP using different models. To do so, it was defined a consensus problem in which there were 30 experts with different behavioural profiles:

- *Cooperative* ($e_1 - e_{21}$). These experts always apply all changes suggested on their assessments, as indicated in the feedback they receive along the LSCRP.

- *Manipulative* (e_{22}). Experts who alternate null and full cooperation along the LSCRP in order to manipulate its solution.
- *Non-Cooperative* (e_{23}). Experts with this behaviour always disobey the recommended changes.
- *Undefined* ($e_{24} - e_{30}$). The behaviour of these experts can change along the LSCRP, applying or ignoring suggested changes to a variable degree.

This consensus problem was executed in the following models: (I) a model that penalises experts with non-cooperative behaviours, but it neither allows that their opinions can recover importance nor has into account the expert's behavioural trajectory [98], (II) a model in which experts' opinions can recover weight, but that it does not consider experts' behavioural tends [100], and (III) this approach, that allows weight recovery of experts' opinions and that considers the trajectory of behaviours.

The analysis of the results of this proposal, as we can see in figure A.8, shows how it is essential to consider experts' behavioural trajectory along the LSCRP, because this allows rewarding experts that keep a continuous cooperative behaviour, whereas it penalises experts who alternates cooperation and non-cooperation. This approach is described in detail in the book chapter (see Section 4.2):

F. J. Quesada, I. Palomares and L. Martínez. *Using Computing with Words for Managing Non-cooperative Behaviors in Large Scale Group Decision*

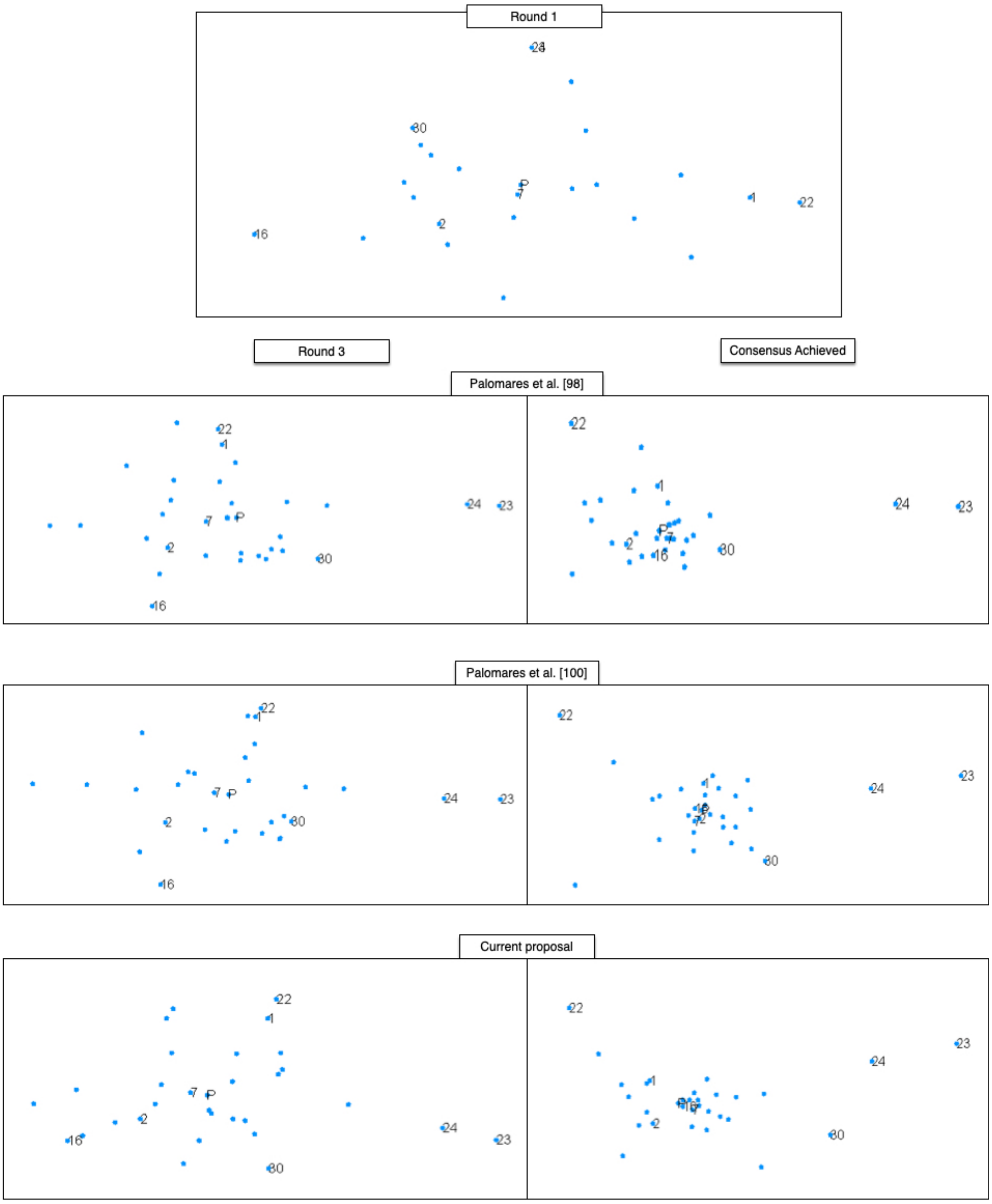

Figure A.8: Comparison between proposals of the evolution of experts' opinions along the consensus process

Making. Granular Computing and Decision-Making, vol. 10: Springer International Publishing, pp. 97-121, 2015.

A.4.2. A Consensus-Driven Group Recommender System

The second objective of our research focussed on improving users' satisfaction in GRSs by generating agreed recommendations.

To do so, we propose to modify GRSs general scheme by adding a consensus phase. In section 2.3 there are identified two aggregation methods for the RS, these are: *aggregation of ratings* and *aggregation of recommendations*. Due to our main goal lies in improving users' satisfaction, we used the *aggregation of recommendations*. In this new phase, recommendations are generated with a high agreement degree between the members of the group. Thus, the system has two phases as it is depicted in figure A.9:

Figure A.9: General scheme of group recommender systems based on consensus.

1. *Recommendation phase.* Recommendations for all group members are generated using the ratings of every user.

2. *Consensus phase.* Based on the recommendations of each user, generated in the previous phase, an automatic CRP is carried out, with the aim of generating recommendations with high consensus degree for the group.

Recommendation phase

This phase includes the necessary computations to generate a set of recommendations for each member in the group $g_i \in G$. This process consist of the following steps, as it is depicted in figure A.10:

1. *Ratings over items.* In this step each user u_i give for each item i_l the correspondent rating r_{il}. Once the process has concluded, we have a matrix of users U and items I that have been rated.

2. *Predictions over items for group members.* Based on the matrix obtained in the previous step, in this step it is generated for all the members of the group G, the predictions of items $i_l \in I$ that have not been rated by the users yet. To do so, it is applied the User-based k-nearest neighbours collaborative filtering algorithm (UBCF) [121]. This algorithm generates prediction values, $\tilde{r}_{il}$, over these items for all users. Once applied the UBCF, only those predictions $\tilde{r}_{il}$ corresponding to group members $g_i \in G$ are taken into account in the following computations carried out in this step.

3. *Predictions over top-n commonly predicted items.* After generating the predictions for all members of the group $g_i \in G$, it is necessary to carry out a filtering process in order to eliminate those items which are not common to all the members of the group. The main reason of doing this task lies in the possibility that different members of the group G could receive recommendations over different subsets of items; so it is necessary to extract a subset of items that have been simultaneously recommended to all the members of the group. Due to there might be cases in which the number of common recommendations could be excessively high, in this proposal we only extract the top-n commonly predicted items for the group, $X = \{x_1, \ldots, x_n\}, X \subset I$, with $n << t$. This subset has been generated by applying a number of rank aggregation techniques such as Borda or the cumulative voting [6].

4. *Preference orderings over top-n commonly predicted items.* Based on the predictions for each user, $\tilde{r}_i = [\tilde{r}_{i1}, \tilde{r}_{i2}, \ldots, \tilde{r}_{in}]$, over the top-n commonly predicted items, then it is obtained its corresponding preference ordering $O_i = \{o_i(1), \ldots, o_i(n)\}$. Thus, items in X are ordered from the item having the highest prediction value, to the item having the lowest one.

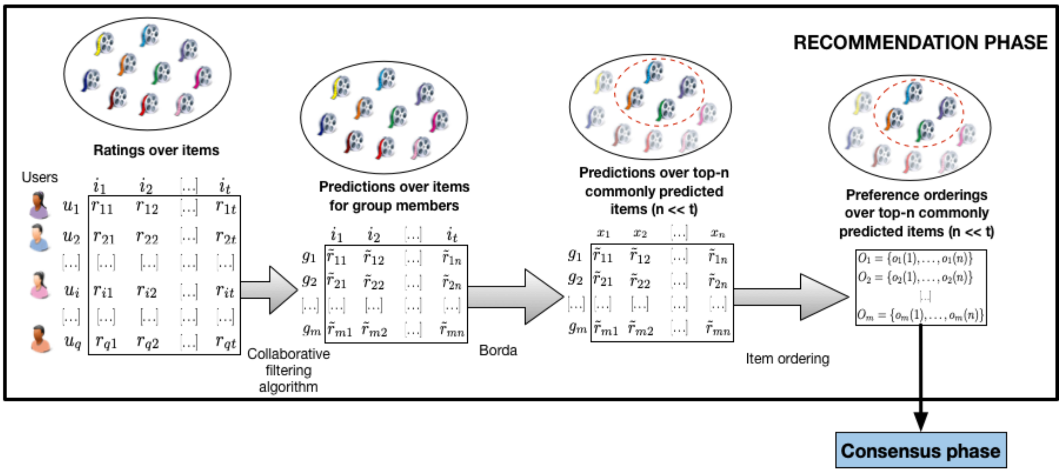

Figure A.10: Scheme of the recommendation phase.

Consensus Phase

After the recommendation phase, in this phase it is carried out a CRP with the aim of obtaining an ordering of recommendations with a high agreement level.

This phase has the following sub-phases as it is illustrated in figure A.11:

1. *Representing individual recommendations as fuzzy preference relations.* Based on the preference orderings generated in the recommendation phase, in this sub-phase, the preference orderings are transformed into fuzzy preference relations. To do so, we use the transformation function proposed by Chiclana et al. in [28]:

$$p_i^{lk} = \frac{1}{2}\left(1 + \frac{o_i(k) - o_i(l)}{n-1}\right) \tag{A.10}$$

2. *Conducting the CRP.* The fuzzy preference relations obtained in the previous sub-phase will be used as input for the CRP, together with the consensus threshold, $\mu \in [0,1]$, and the maximum round number $maxRounds \in \mathbb{N}$. The applied CRP is an automatic process in which preferences will be modified according to the received recommendations. The process concludes when the consensus threshold has been reached (consensus is achieved) or when the

process reach the maximum round number (consensus is not achieved). Once the consensus level reach the minimum agreement level determined by the consensus threshold, we can confirm that all preference relations achieve consensus. At this point, the following step should be the aggregation of all preference relations, forming an agreed preference relation or collective preference relation, $P_c = (p_c^{lk})_{n\times n}$.

3. *Obtaining agreed recommendation list.* Finally, the alternative selection process is carried out. To do so, the collective preference relation is transformed into a preference ordering, $O_c = \{o_c(1), \ldots, o_c(n)\}$, that indicates a ranking of agreed recommendations for the group.

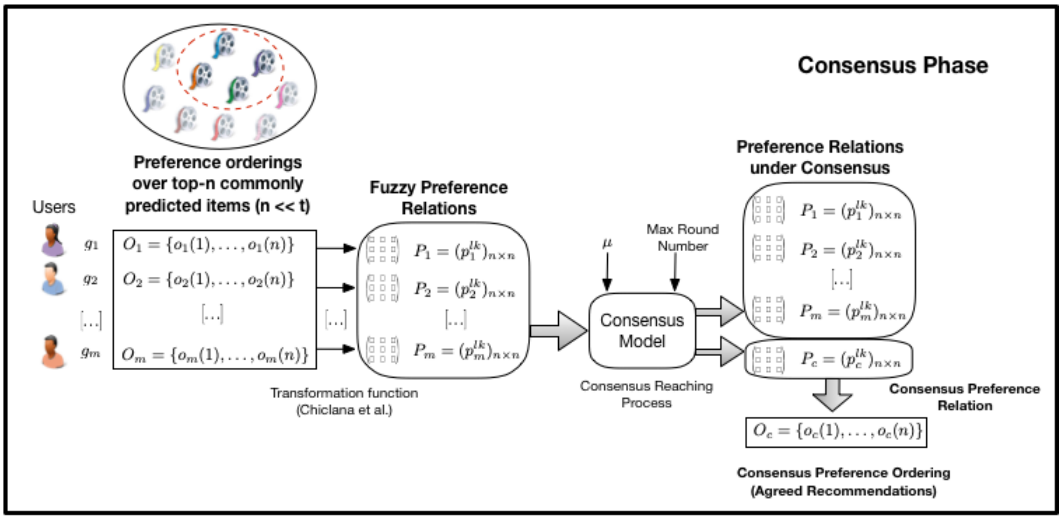

Figure A.11: Scheme of the consensus phase.

Evaluation

This proposal has been evaluated by a case of study that generates recommendations of films to 30 groups of 5 users, regarding several degrees of minimum agreement (0,80, 0,85 y 0,90). *MovieLens 100k*[2] has been the dataset

[2]Generated by GroupLens Research Project, University of Minnesota. Accessible in http://grouplens.org/datasets/movielens/

that we used. To do so, we have studied both the area under the receiver operating characteristic curve and the precision.

Regarding the first parameter, the analysis of the results shows how applying a CRP to generate a group recommendation significantly improves the results with respect to the baseline. Although all configurations with consensus have a better performance than the reference, the system reaches its optimum performance in the configuration with the consensus threshold equal to 0.80.

Concerning the precision, the configuration with the consensus threshold equal to 0.80 is again the one that presents the best performance. In particular, for the list with 4 or fewer recommendations, this configuration is the best of the studied, having a similar performance as the other configurations with lists that have more recommendations.

Considering the obtained results, we can claim that including a CRP in the generation of group recommendations entails significant improvements, implying an increase in the satisfaction of the members of the group.

This proposal is described in detail in the article (see Section 4.3):

J. Castro, F. J. Quesada, I. Palomares and L. Martínez. *A Consensus-Driven Group Recommender System.* International Journal of Intelligent Systems, vol. 30, no. 8, pp. 887–906, 2015.

And in the book chapter (see Section 4.4):

F. Moya, F. J. Quesada, J. Castro, R. M. Rodríguez, I. Palomares and L. Martínez. *Improving group recommendations with consensus reaching processes.* Soft computing and Hybrid Systems for Knowledge Discovery and Decision-making, Atlantis Computational Intelligence Series (ACIS), Accepted.

A.5. Conclusions and Future Works

This section closes the English appendix of the thesis with a review of different conclusions that we have obtained from the presented proposals. After that, a number of possible future works are presented, as well as the publications fruit of our research.

A.5.1. Concluding Remarks

The technological advances produced by the boom of the Internet have contributed to significant changes within the GDM area. Few years ago, GDM processes involve a small number of participants, whereas nowadays these processes usually involve large groups. This change has provoked problems in some consensus models, which were designed for classic CRPs, causing that they considerably reduce their performance. For this reason, it is necessary to carry out an adaptation process.

The increase of the number of participant experts in this process, also entails a wide diversity of profiles between experts, which in principle makes difficult the existence of a quick consensus convergence. Moreover, it is possible that certain experts try to take advantage of the complexity of managing a large number of participants, and try to bias the solution of the LSCRP by adopting non-cooperative behaviours.

Managing experts with non-cooperative behaviours in LSGDM has been one of the main objectives of this thesis, having developed two proposals, which allow penalised experts to recover the importance of their opinions. The evaluation of these proposals shows how these approaches incentive users to adopt a cooperative behaviour even though they have not cooperated at first. This fact, benefits the group because this change of behaviour improve consensus convergence.

Recently, some approaches have been published with the aim of answering these questions [36], which denotes that this line of research is still in its firsts stages.

The second objective of the thesis has been the improvement of the satisfaction of GRS' users. Traditionally, RS were designed to recommend individually to their users a range of items which might be of their interest. However, the social character of certain products, entails the appearance of RS which could recommend items to a group of users. Mainly, the efforts have focused on minimising users' dissatisfaction. Nonetheless, despite applying these techniques there are cases in which users' satisfaction plummets. Therefore, it is necessary to develop models which allow maximising the satisfaction of GRS' users.

In the thesis we have addressed this objective by developing a proposal which obtain group recommendations with a minimum consensus degree. To do so, we have added a consensus phase into GRS with aggregation of recommendations, in order to reach a high level of agreement prior to compute the group recommendation. The evaluation of this proposal shows positive results, considerably improving the reference and establishing here a starting point for future works.

A.5.2. Future works

Based on the carried out research, a number of possible future works are proposed. These works are focussed both on widening the presented proposals, and on pointing out several unsolved questions that have arisen during the development of the PhD studies. Concretely, these questions are relative to CRPs in general and LSCRPs in particular.

A.5.2.1. Improving detection and management of new behaviours

Currently, our proposals only allow managing behaviours by analysing the recommendations that experts receive and after that, observing whether they comply with the recommendations or not. This reduces the application of these works exclusively to consensus models with feedback. A possible line of work might be carrying out the management of behaviours without using recommendations (e.g.

using the evolution of distances between experts' opinion along the negotiation process). This makes possible the application of these techniques to all consensus models either the ones that use feedback or the ones that do not.

Apart from that, it might be really interesting to refine the detection of the different kinds of behaviours that appear in CRPs. In the works that makes up this thesis we consider the following behaviours: cooperative, non-cooperative, hybrid, mixed, manipulative and undefined. However, in the future, it might be convenient studying in depth negotiation strategies and patterns normally used by participant experts in CRPs, in order to detect and manage behaviours with a higher precision.

A.5.2.2. Ethical Consensus

The RAE (Academy of the Spanish Language) defines *ethic* as "*the set of moral rules which govern human behaviour in any sphere of its life*". In a similar way, it defines *morals* as *"belonging or relative to people actions from the point of view of their behave with respect to good and evil, and according to their individual life, and especially, their collective life"*. Ethics has been one the most commented philosophical topics throughout history, from the Ancient Greece until our days. Currently, the technological development in areas such as Artificial Intelligence, has entails the existence of multitude of autonomous systems in almost all spheres of our lives. This fact has provoked that notorious personalities of this field of knowledge, have considered vital the application of ethics to artificial intelligence systems [12, 73, 117]. An example might be Google self-driving car [3]. Let's imagine that the car detects that the brakes are failing and that it only can stop turning to the left where there is a wall of stones and the crash might cause that occupants will die, or turning to the right where there is a family strolling, and the accident might cause the death of all the family members, but minimizing the damage of the occupants [4]. What action should the car carries out? The answer of this question is an open debate in which researchers are currently working.

[3] https://www.google.com/selfdrivingcar/

[4] http://moralmachine.mit.edu

One of the main objectives of this thesis has been the management of experts with non-cooperative behaviours because this kind of attitudes may negatively affect the solution of the Consensus Reaching Process with Large Groups (CRPLG). In the different proposals it is assumed that these experts break the collaboration contract in order to move the CRPLG solution towards their own interests. However, there are cases in which participants do not follow the recommendations because of ethical issues. For example, let's imagine a group of employees who want to carry out a CRPLG to choose the restaurant for next Christmas lunch. If a vegetarian receives a recommendation in which he/she has to vote for a restaurant that only serves meat or if an employee allergic to seafood is recommended to change his/her preferences to go to a shellfish bar, the non-cooperative behaviour is the only option. However, it is possible that these participants want to cooperate to reach a consensus, but currently they do not have other alternative which allow them not going against their principles or health.

These cases become even worse with the approaches to manage non-cooperative behaviours because they do not differentiate between experts who adopt this behave because of ethical reasons and those who try to manipulate the CRPLG. The common denominator between all of these approaches lies in reducing the weight of experts' opinions in the CRPLG. Thus, in the case of the previous participants they only have two options: (i) following the recommendations (cooperative behaviour) and go against their principles/health so that their opinion has a significant impact when computing the consensus degree, or (ii) being coherent and reject the recommendations (non-cooperative behaviour) even though their opinion lose weight and it does not count in the computation of the consensus degree.

Due to the importance of these scenarios, it is essential to delve into this line of research.

A.5.2.3. Development of metrics to compare consensus models

In chapter 2, it has seen how we can find many consensus models [74, 96, 129], which use different methodologies and operators. In spite of this, currently, it is difficult to know which model is more convenient in each scenario because there are not metrics to compare models. For example, it might be useful knowing:

1. Which model converges faster to consensus?

2. Which model reaches the highest level of consensus in the minimum number of rounds?

3. Which model reaches consensus with more cohesion between experts' opinions?

4. Which model has a better performance the starting expert's opinions are far between them?

Answering these questions is essential because knowing which model fits better to each scenario, we can optimise the DM process which will have a positive effect to participant experts with regard to future DM processes.

Due to its significance, it is indispensable working in this line of research in the short term.

A.6. Publications

Finally, below it is shown a list of publications derived from the results that have been presented in the thesis:

Publications:

- Indexed International Journals:

- F. J. Quesada, I. Palomares and L. Martínez. *Managing experts behavior in large-scale consensus reaching processes with uninorm aggregation operators*. Applied Soft Computing, vol. 35, pp. 873 - 887, 2015.
- J. Castro, F. J. Quesada, I. Palomares and L. Martínez. *A Consensus-Driven Group Recommender System.* International Journal of Intelligent Systems, vol. 30, no. 8, pp. 887–906, 2015.

▪ Indexed Book Chapters:

- F. J. Quesada, I. Palomares and L. Martínez. *Using Computing with Words for Managing Non-cooperative Behaviors in Large Scale Group Decision Making.* Granular Computing and Decision-Making, vol. 10: Springer International Publishing, pp. 97-121, 2015.
- F. Moya, F. J. Quesada, J. Castro, R. M. Rodríguez, I. Palomares and L. Martínez. *Improving group recommendations with consensus reaching processes.* Soft computing and Hybrid Systems for Knowledge Discovery and Decision-making, Atlantis Computational Intelligence Series (ACIS), In Press.

▪ International Conferences:

- F. J. Quesada, I. Palomares and L. Martínez. *A Multi-agent System for Performing Consensus Processes in Heritage problems.* University of Jaén, Baeza (Spain), I International Meeting of Young Researchers on Heritage - PatrimoniUN10, November 19th-21st, 2014.
- I. Palomares, F. J. Quesada and L. Martínez. *An Approach based on Computing with Words to Manage Experts Behavior in Consensus Reaching Processes with Large Groups.* IEEE Computational Intelligence Society, Beijing (China), 2014 International Conference on Fuzzy Systems (FUZZ-IEEE), July 6-11th, 2014.

▪ National Conferences:

- F. J. Quesada, I. Palomares and L. Martínez, *Gestión de Expertos con Comportamientos no Cooperativos en Procesos de Consenso en Grandes Grupos*. Zaragoza (Spain), Universidad de Zaragoza, ESTYLF, pp. 139-144, February 5-7th, 2014.
- J. Castro, F. J. Quesada and L. Martínez. *Uso de procesos de alcance de consenso para mejorar la recomendación a grupos.* XVI Conferencia de la Asociación Española para la Inteligencia Artificial (CAEPIA'15), Albacete (Spain), November 9-12th, 2015.

Apéndice B

Computación con Palabras

En numerosas situaciones de la vida cotidiana, los seres humanos utilizamos palabras y expresiones, para comunicarnos, razonar y comprender nuestro entorno. Estos términos lingüísticos tan comunes en nuestro día a día, no son utilizados por los ordenadores, ya que necesitan utilizar símbolos mucho más formales para representar el conocimiento [144].

El paradigma de la CCP fue propuesto por L. A. Zadeh [153] basándose en la teoría de conjuntos difusos [148], de manera que los conceptos pertenecientes a un vocabulario concreto puedan ser modelados mediante conjuntos difusos. El principal objetivo es establecer un nexo de comunicación comprensible entre los seres humanos y los sistemas computacionales. Esta metodología incrementa el uso de lenguaje natural en comunicaciones, razonamiento y toma de decisiones llevadas a cabo por estos sistemas, lo que facilita la comunicación persona-ordenador, ya que ofrece un entorno de trabajo en el que los conceptos son modelados mediante conjuntos difusos, de manera que puedan ser entendidos por ambas partes.

Los elementos fundamentales en la CCP son la *Variable Lingüística* y el *Término Lingüístico.*

Definición 6 *[149–151] Entendemos por* variable lingüística *a la 5-tupla* $(H, T(H), U, G, M)$ *donde* H *es el nombre de la variable;* $T(H)$ *simboliza el conjunto de términos lingüísticos o valores de* H*, siendo cada valor una variable difusa* *normalmente denotada como X que oscila entre el universo del discurso U; G es una regla sintáctica (normalmente dada por una gramática) para generar nombres de términos lingüísticos en H; y M es una regla semántica para asociar cada elemento en H con su respectivo significado, $M(X)$, dado por un conjunto difuso en U.*

Basándonos en la definición de variable lingüística dada por L. A. Zadeh, podemos entender el concepto *término lingüístico* como la palabra o frase utilizada para expresar el valor de una variable. Gracias a los términos lingüísticos, los seres humanos pueden comprender y razonar de una mejor manera las diferentes características de su entorno. Por ejemplo, considerando la variable *distancia*, algunos posibles términos lingüísticos para expresar el valor de dicha variable podrían ser: "*cercana*", "*media*", "*lejana*" o "*muy lejana*".

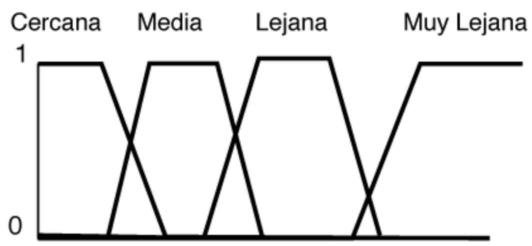

Figure B.1: Ejemplo de términos lingüísticos para el atributo *distancia*

Debido a la inherente vaguedad e imprecisión que presentan los términos lingüísticos, los conjuntos difusos [153] constituyen una herramienta adecuada para formalizar los conceptos asociados a dichos términos. En la figura B.1 podemos ver cómo la semántica de los términos lingüísticos pertenecientes a la variable *distancia* están representados como conjuntos difusos con funciones de pertenencia trapezoidales [69]. De este modo, usando la teoría de los conjuntos difusos logramos que los ordenadores sean capaces de comprender y llevar a cabo procesos computacionales y de razonamiento sobre estos conceptos. Así, siendo $\tau \in T(H)$ un término lingüístico (p. ej. "cercana") perteneciente a un vocabulario asociado a la variable lingüística H (p. ej. *distancia*), podemos expresar τ como un subconjunto difuso del dominio $Y \in U$ de H. Dado un valor $y \in Y$, su grado de pertenencia para τ, $\mu_\tau(y) \in [0,1]$ indica el grado de compatibilidad del valor y con el término lingüístico τ. Estas ideas son usadas en las propuestas para gestionar expertos con comportamientos no cooperativos en PAC.

Apéndice C

Uninormas

Las uninormas son operadores de agregación definidos por R. Yager y A. Rybalov en [146], que proveen la unificación de los operadores t-norma y t-conorma. Debido a que para comprender el concepto de uninorma es necesario conocer estos operadores, a continuación se revisarán las definiciones formales de t-norma y t-conorma.

Definición 7 *[146]. Una norma triangular o* t-norma T *es una correspondencia,*

$$T : [0,1] \times [0,1] \to [0,1]$$

teniendo las siguientes propiedades para todo $a, b, c, d \in [0,1]$*:*

i) Conmutativa*:* $T(a,b) = T(b,a)$.

ii) Monótona*:* $T(a,b) \geq T(c,d)$ *if* $a \geq c$ *and* $b \geq d$.

iii) Asociativa*:* $T(a, T(b,c)) = T(T(a,b), c)$.

iv) Elemento neutro*:* $T(a,1) = a$.

Las t-normas son operadores de agregación conjuntivos, por lo que presentan la siguiente propiedad:

$$T(a_1, \ldots, a_n) \leq min_i[a_i]$$

A partir de esta propiedad, podemos ver que el valor agregado nunca es mayor que el menor a_i. Además, si todos los valores de a_i son bajos, entonces estos valores se reforzarán unos a otros de manera que el resultante valor agregado será aún menor. Esta propiedad es conocida como *refuerzo hacia abajo o negativo* [147].

Algunos ejemplos conocidos de t-normas son:

- Mínimo: $T_{min}(a, b) = min(a, b)$.
- Producto: $T_{prod}(a, b) = ab$.
- T-norma de Łukasiewicz: $T_{Luk}(a, b) = max\{0, a + b - 1\}$.

Definición 8 *[146]. Una conorma triangular o* t-conorm S *es una correspondencia,*

$$S : [0, 1] \times [0, 1] \to [0, 1]$$

presentando las siguientes propiedades para todo $a, b, c, d \in [0, 1]$*:*

i) Conmutativa*:* $S(a, b) = S(b, a)$.

ii) Monótona*:* $S(a, b) \geq S(c, d)$ *if* $a \geq c$ *and* $b \geq d$.

iii) Asociativa*:* $S(a, S(b, c)) = S(S(a, b), c)$.

iv) Elemento neutro*:* $S(a, 0) = a$.

Las t-conormas son operadores de agregación disyuntivos, por lo que presentan la siguiente propiedad.

$$S(a_1, \ldots, a_n) \geq max_i[a_i]$$

A partir de esta propiedad podemos ver que el valor agregado es siempre al menos tan alto como el mayor a_i. Además, si todos los valores de a_i son altos, entonces estos valores se reforzarían entre ellos, dando lugar a un valor aún mayor. Esta propiedad se denomina, *refuerzo hacia arriba o positivo* [147].

Algunos ejemplos de t-conormas son:

- Máximo: $S_{max}(a,b) = max(a,b)$.
- Suma probabilística: $S_{prob}(a,b) = a + b - ab$.
- T-conorma de Łukasiewicz: $S_{Luk}(a,b) = min\{a+b, 1\}$.

Como se ha comentado anteriormente, las uninormas son operadores que proveen una generalización de las t-normas y las t-conormas [40, 146]. Así, las uninormas se definen como:

Definición 9 *[146] Una uninorma es una correspondencia,*

$$U : [0,1] \times [0,1] \to [0,1]$$

teniendo las siguientes propiedades para todo $a, b, c, d \in [0,1]$*:*

i) Conmutativa*:* $U(a,b) = U(b,a)$*.*

ii) Monótona*:* $U(a,b) \geq U(c,d)$ *if* $a \geq c$ *and* $b \geq d$*.*

iii) Asociativa*:* $U(a, U(b,c)) = U(U(a,b), c)$*.*

iv) Elemento neutro*:* $\exists g \in [0,1]$ *:U(a,g)=a.*

A diferencia de las t-normas y las t-conormas donde los elementos neutros eran 1 y 0 respectivamente, las uninormas pueden tomar como elemento neutro cualquier valor dentro del intervalo unidad. Por este motivo, dependiendo de si el valor a agregar (a,b) es mayor o menor que g, el comportamiento será conjuntivo, disyuntivo o intermedio [19].

J. C. Fodor y otros introducen en [43] dos tipos generales de uninormas con elemento neutro g:

$$U(a,b) = \begin{cases} (a) \quad gT_U\left(\dfrac{x}{g}, \dfrac{y}{g}\right) & \text{si } 0 \leq a, b \leq g, \\ (b) \quad g + (1-g)S_U\left(\dfrac{x-g}{1-g}, \dfrac{y-g}{1-g}\right) & \text{si } g \leq a, b \leq 1, \\ (c1) \quad max(a,b) & \text{si } min(a,b) \leq g \leq max(a,b), \\ (c2) \quad min(a,b) & \text{si } min(a,b) \leq g \leq max(a,b). \end{cases} \tag{C.1}$$

con T_U y S_U siendo cualquier operador t-norma y t-conorma, respectivamente. La principal diferencia entre ambos tipos de uninormas está en el uso tanto del elemento (c1), que define a la llamada familia de uninormas $\mathcal{U}_{\mathbf{max}}$, como del elemento (c2), que define a la familia de uninormas $\mathcal{U}_{\mathbf{min}}$ [7].

Para cualquier $g \in]0,1[$, podemos observar que las uninormas presentan refuerzo hacia arriba cuando el valor de entrada es mayor que g y refuerzo hacia abajo en caso contrario, por lo tanto presentan un *refuerzo completo*. Este comportamiento hace adecuada la utilización de uninormas para reforzar comportamientos en PACs.

En estos ejemplos se muestran las uninormas utilizadas en la propuesta 3.1.1:

Ejemplo 1 *A partir de la t-norma producto y la t-conorma suma probabilística, revisadas anteriormente, la siguiente uninorma es definida en base a la familia general de uninormas definida por Fordor et. al. (Ec.(C.1)) [43]:*

$$U(a,b) = \begin{cases} \dfrac{ab}{g} & si\ 0 \leq a,b \leq g, \\ \dfrac{a+b-ab-g}{1-g} & si\ g \leq a,b \leq 1, \\ M_U(a,b) & si\ min(a,b) \leq g \leq max(a,b). \end{cases} \tag{C.2}$$

con $M_U(a,b)$ siendo un operador intermedio.

Ejemplo 2 *La uninorma cross-ratio [27] es un ejemplo de una uninorma continua en*$[0,1]^2 \backslash \{(0,1),(1,0)\}$*, con elemento neutro $g = 0.5$:*

$$U(a,b) = \begin{cases} 0 & si\ (a,b) \in \{(0,1),(1,0)\}, \\ \dfrac{ab}{ab+(1-a)(1-b)} & en\ otro\ caso. \end{cases} \tag{C.3}$$

Apéndice D

Hipersimilitud

La ideas sobre las que se fundamenta el concepto de hipersimilitud fueron expuestas por Yager y Petry en [145], donde hacen uso del alineamiento de valores de atributos mediante la hipersimilitud para facilitar una TD más intuitiva.

Principalmente sugieren que cuando las alternativas son valoradas de acuerdo a ciertos atributos, aquellos atributos para los que una determinada alternativa toma un valor extremo, éste debe tener una mayor importancia en el proceso de TD. Por ejemplo, para un valor extremo de una persona con un exceso de peso considerable, el atributo *peso* jugará un rol más importante a la hora de caracterizar el atributo *peso* que una persona con un peso medio (p. ej. el número de personas y la media de peso máximo que se puede admitir dentro de un ascensor). Así, el principal objetivo de su propuesta se centra en estudiar el efecto de estos valores extremos a la hora de amplificar la importancia de dicho atributo dentro un escenario de TD.

Definición 10 *[145] Siendo $m_i(S1)$ el efecto de amplificación asociado con el atributo A_i para la situación $S1$, entonces:*

$m_i(S1) = f(Dev(A_i(S1)))$

Siendo $Dev(A_i(S1))$ la desviación de $A_i(S1)$ con respecto a la normal.

La función $f : [0, \infty]$ presenta las siguientes propiedades:

1. $f(0) = 1$

2. $f(a) \geq f(b)$ si $a > b$; f es monótona

Estas ideas se usarán en la propuesta descrita en la sección 3.1.2 para reforzar el peso de un experto en base a su trayectoria de comportamiento.

Apéndice E

Glosario

Glosario

Computación con Palabras

Paradigma que propone la representación de los conceptos de un vocabulario mediante conjuntos difusos.

Conjunto Difuso

Conjunto en el que los elementos pueden tener una pertenencia parcial al grupo. Éste valor queda determinado por medio de la función de pertenencia.

Consenso

Acuerdo producido de mutuo consentimiento entre los miembros de un grupo o varios grupos.

Operador de Agregación

Función que produce un valor a partir de la combinación de valores. En los problemas de PACs se utiliza dentro de la *fase de agregación*, para combinar las preferencias de los expertos.

Operador de Similitud

Función que produce un valor, indicando la similitud o distancia entre dos elementos. En los PACs se usa en la fase de *determinación del grado de consenso*.

Preferencia Colectiva

Preferencia que expresa la opinión del grupo en un PAC.

Relación de Preferencia

Estructura utilizadas para modelar las preferencias de los expertos en procesos de TDG. Puede tener forma de matriz en la que el experto valora sus preferencias entre las alternativas del problema.

Sistema de Recomendación

Sistema que mediante el filtrado de información sugiere a los usuarios del sistema ciertos artículos o elementos que pueden ser de su interés.

Sistema de Recomendación Colaborativo

Sistema de Recomendación que busca relaciones entre los usuarios del sistema para generar predicciones, considerando para ello las valoraciones que estos han hecho sobre ciertos elementos.

Soft Consensus

Enfoque basado en la teoría de conjuntos difusos en el que el consenso se produce cuando la mayor parte de los expertos están de acuerdo en las alternativas importantes.

T-conorma

Operador de agregación disyuntivo que además de tener las propiedades *conmutativa, monótona, asociativa y elemento neutro*, presenta la propiedad de *refuerzo positivo*.

T-norma

Operador de agregación conjuntivo que además de tener las propiedades *conmutativa, monótona, asociativa y elemento neutro*, presenta la propiedad de *refuerzo negativo*.

Término Lingüístico

Palabra o frase empleada para expresar el valor de una variable lingüística.

Uninorma

Operador de agregación que provee una unificación de los operadores t-norma y t-conorma.

Variable Lingüística

5-tupla formada por el nombre de la variable, el conjunto de valores lingüísticos que puede tomar la variable, el universo del discurso, la regla sintáctica y la regla semántica por las que se rige la variable.

Apéndice F

Acrónimos

Acrónimos

CCP	Computación con Palabras.
CD	Consensus Degree.
CP	Colective Preference.
CRP	Consensus Reaching Process.
CRPLG	Consensus Reaching Process with Large Groups.
CT	Consensus Threshold.
CW	Computing with Words.
DM	Decision Making.
FCBU	Filtrado Colaborativo Basado en Usuarios.
FPR	Fuzzy Preference Relation.
GC	Grado de Consenso.
GDM	Group Decision Making.
GDMLG	Group Decision Making with Large Groups.
GRS	Group Recommender Systems.

HRS	Hybrid Recommender System.
KBRS	Knowledge-Based Recommender System.
LSCRP	Large Scale Consensus Reaching Processes.
LSGDM	Large Scale Group Decision Making.
MAS	Multi-Agent System.
PAC	Proceso de Alcance de Consenso.
PACGE	Proceso de Alcance de Consenso a Gran Escala.
PACGG	Proceso de Alcance de Consenso con Grandes Grupos.
PC	Preferencia Colectiva.
PPG	Proceso de Personalización de Grupo.
PR	Preference Relation.
RAE	Real Academia Española.
RP	Relación de Preferencia.
RPD	Relación de Preferencia Difusa.
RS	Recommender Systems.
SMA	Sistemas Multi-Agente.
SR	Sistemas de Recomendación.
SRBCono	Sistemas de Recomendación Basados en Conocimiento.
SRBCont	Sistemas de Recomendación Basados en Contenido.

SRBU	Sistemas de Recomendación Basados en Utilidad.
SRC	Sistemas de Recomendación Colaborativos.
SRD	Sistemas de Recomendación Demográficos.
SRG	Sistemas de Recomendación a Grupos.
SRH	Sistemas de Recomendación Híbridos.
TD	Toma de Decisión.
TDG	Toma de Decisión en Grupo.
TDGGE	Toma de Decisión en Grupo a Gran Escala.
TDGGG	Toma de Decisión en Grupo con Grandes Grupos.
UBCF	User-based k-nearest neighbours collaborative filtering algorithm.
UC	Umbral de Consenso.

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